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AVERAGING PARAMETERS IN mN - nk MULTIPLE POROSITY - PERMEABILITY GEOTHERMAL RESERVOIRS

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ABSTRACT

Many geothermal fields are naturally fractured systems. In these reservoirs, physical parameters can be heterogeneous and randomly distributed. Classic double porosity models the flow between matrix and fractures, under the hypothesis that petrophysical properties are uniform in each medium. Fractures have the largest permeability and drive the fluid toward the wells. The matrix, with smaller permeability, only acts as a source of fluid for the fractures. Under the concept of multiple porosity- permeability, M continuous porous media interact with each other. Every medium has its own parameters and its own interporosity flow, which can be non-isothermal, stationary, or transitory. Double porosity models can be classified as special cases of this general theoretical concept, applicable to all class reservoirs. In such heterogeneous systems, the numerical simulation of heat and mass coupled flows requires to average physical properties, which sometimes are highly contrasting, independently of the method used in the solution of equations. One crucial problem in multiple porosity reservoirs, is what kind of average should be used to represent parameters in the global transport processes among different media. Irregular spatial distribution of petrophysical parameters affects both mass and energy flows, and the thermodynamic evolution and mechanical behavior of the whole system. Its influence can be as decisive as relative permeabilities or capillary pressure.

INTRODUCTION

Many productive fields in the world (oil, gas, water, geothermal) are associated to faulting and fracturing. In these systems, fault zones act as open conduits for fluid flow from depth. Mainly because of economic reasons, it is important to correctly understand the behavior of such reservoirs. The international experience shows that storage capacities and permeabilities in those systems vary widely, depending on the degree and intensity of fracturing and on the effective porosity. If the original fluid is stored in a well connected fractured network, a very high initial extraction rate can induce to overestimate the long term well production, by considering porosity and permeability larger than they are in fact. Cases of wells in this type of reservoirs have been observed, that produce high initial quantities of fluid, and decline drastically after a short time. A lot of attention has been devoted to develop realistic models that describe the flow in naturally fractured reservoirs under exploitation, in order to characterize accurately their depletion history. But, to the best of our knowledge very little attention has been focused on the problem of averaging petrophysical parameters in heterogeneous fractured geothermal reservoirs.

In 1863 E. Andrews introduced in geology the notion of "*fractured porosity*" (Golf-Racht, 1982), to distinguish reservoirs whose porosity is no longer simple. The original concept of double porosity was enunciated 40 years ago in hydrology (Barenblatt et al., 1960) and applied soon after to petroleum engineering (Warren and Root, 1963). Both classic models consider that the fluid moves toward the producing wells only through interconnected fractures. After these two pioneer works, an enormous amount of literature has been written on double porosity, for both stationary flow and transitory flow in the matrix and between the matrix and fractures.

Cinco Ley and Samaniego (1985) reported the transient behavior of pressure in fractured reservoirs with multiple porosity, for different sizes of the matrix blocks. Double porosity was extended to geothermal reservoirs by Pruess and Narasimhan (1985), by means of the concept of "*Multiple Interacting Continua*". Yu-Shu and Pruess (1988) applied this model to the study of two-phase geothermal processes and to water imbibition in matrix blocks with oil, pointing out several limits to the traditional double porosity.

Closmann (1975), enlarged for the first time the concept of double porosity when describing an aquifer with stationary flow in the network of fractures and with two different types of matrix: a *bad* one having smaller permeability and smaller porosity and a *good* one having better conditions. He concluded that the flow is dominated by the fractures and that the fracturing permeability and the volume fraction of *fissures* are the most important factors in the behavior of the aquifer.

Liu Ci-qun (1981), found an exact solution for the radial transient flow of a slightly compressible liquid in a medium which he called of *triple porosity*. The three media were the fractured network and two different types of matrix, although the flow in the matrix was considered negligible. His model resulted more realistic than the double porosity to describe liquids in carbonated formations.

Abdassah and Ershaghi (1986), observing anomalies, during the transitory period, in the slopes of some oil wells tests, introduced another model of *triple porosity*. Their model considers the reservoir formed by two different matrix blocks, separated by a set of orthogonal fractures.

Liu and Chen (1990) found the exact solution for the radial transient flow of slightly compressible isothermal liquid, contained in a cylindrical reservoir formed by N different media. They assumed a pseudo steady state interporosity flow regime, and a constant production well located in the center of the field. They called this model a *multiple porosity and multiple permeability medium*.

Mao Bai (et al., 1993) presented an array of deformation-dependent flow models of various porosities and permeabilities to characterize the behavior of isothermal naturally fractured reservoirs. The rock matrix has low permeability and fracture flow is dominant. The fluid has slight compressibility.

Uninterpretable pressure tests by current available theory and other data from reservoirs with large faults, drove us to introduce the concept of *triple porosity - triple permeability* in volcanic geothermal systems (Suárez and Samaniego, 1995). The three media, matrix, fractured network and fault are considered as three continua which interact through interporosity transport functions, depending on the form and size of the blocks, on the intensity of fracturing and on the distance to the fault.

Al-Ghamdi and Ershaghi (1996), proposed a dual fractures model plus the matrix. One of the systems is a compound of normal fractures or macrofractures, while the other medium consists on a net of microfractures with smaller permeability that sometimes is confused with another type of matrix.

As physically expected, the authors of these models concluded that for small times, the flow characteristics are controlled by the fractures. Some portions of the transition period are similar to those predicted by double porosity theory. But the final portion of the transition zone is different, showing slope changes whose duration is a function of the interporosity quotients and storages of both matrices. Triple porosity models predict three, instead of two, parallel straight lines in the traditional semi-log plots.

LIMITS OF APPLICATION OF THE DOUBLE POROSITY (DP)

The DP classic models explain the flow between matrix and fractures. The matrix blocks surrounded by fractures can have several geometries and any size. Fractures have very little storage, but provide the high permeability conduits to drive the fluid toward the wells. Matrix blocks have higher porosity and constitute the largest storage, but have smaller permeability, acting only as a source of stationary fluid for the fractures. Fundamental parameters of the DP models are the intensity of the interporosity flow between a matrix and fractures and the storage ratio of fractures divided by the total storage of the system. Each medium has its own pressure, but at the beginning both pressures are equal. When fluid extraction starts through the fractures, a pressure difference appears between both continua. The largest pressure is in the matrix, pushing the stored fluid toward the fractures. The classic double porosity production mechanism is established.

When quick phase changes exist in the liquid water contained in the reservoir, it is necessary to take into account the transient flow inside the matrix blocks and in the matrix-fractures boundary. DP models the flow between two different media under the hypothesis that the petrophysical properties in each medium are uniform. This is another important restriction, because very mixed reservoirs exist. A prototype of heterogeneous system is the Los Humeros, Mexico geothermal field, where data and pressure tests analysis (Suárez, 1995), indicate local high permeability and low global permeability, with steam segregation and abrupt phase changes. The DP is also insufficient to explain the behavior of volcanic geothermal fields, crossed by big open faults (Suárez and Samaniego, 1995). The same limitations can be found in isothermal multiple porosity models with pseudo-stationary interporosity flow.

THE MULTIPLE POROSITY - PERMEABILITY CONCEPT IN GEOTHERMAL RESERVOIRS

Any medium that exhibits well-differentiated discontinuities in its distribution of porosities must be considered as a multiple porosity continuum. However this is merely a definition, because a unified theory of porous media based on this notion does not exist. Under the concept of multiple porosity, M (∃ 2) continuous porous media interact with each other (Fig. 1). Each one has its own parameters and its own interporosity flow, which can be stationary, pseudo-stationary or transient. The saturating fluid can be non isothermal, in one or two phases or contain multiple components. Double porosity models and the simplified extensions achieved by some authors, can be classified as special cases of this general theoretical concept.

Fractured reservoirs form a category of complex systems, in which the analysis and comprehension of every one of its components does not guarantee that the complete system can be understood as a whole. Understanding the interaction among all its parts is the key for the global description of the complex system. For example, in a hydrothermal reservoir heat, permeabilities and phase changes influence the amount and composition of the extraction rate and of the fluid geochemistry. None of these phenomenons can be understood isolated, by itself.

The non-isothermal multiple porosity-permeability model describes an interconnected global phenomenon that also produces multiple effects on other interdependent phenomena at a larger scale. This is the more general concept applicable to all-class reservoirs, conforming the highest degree of complex system in geothermal engineering. However, it is not possible to give a unique practical definition of this idea. Volcanic reservoirs can contain more than two systems of this type. Similar observations were done in fractured petroleum reservoirs (Mao Bai et al., 1993). Some more specific multiple porosity-permeability geothermal models are described next.

One Porosity - One Permeability

This is the simplest porous media. It possesses a continuous homogeneous distribution of empty spaces and a single type porosity, with a single permeability. This is a Single porosity-Single permeability model.

One Porosity - Double Permeability

If the medium is homogeneous, has scarce fractures or it is crossed by a single open fault, then the system will have a single dominant porosity and two permeabilities.

Double Porosity - One Permeability

When the medium is very fractured, the fractures add a secondary porosity to the original porosity, breaking the medium into blocks. If the matrix has high permeability, then its global behavior is equal to that of a medium with unique permeability and two different porosities. In general, if it is not possible to distinguish between the fracture permeability and the matrix permeability, we have a Double porosity- Single permeability model. A fractured reservoir with low global permeability but high storage, fits also in this model.

Double Porosity - Double Permeability

In the conventional model of Double Porosity-double Permeability, the matrix has the highest porosity and the lowest permeability while the fractures have the lowest porosity and the highest permeability. As an important historical antecedent, the main characteristic of the DP model is the clear distinction that it makes between two types of flows: one in the fractures and the other one intergranular. Its general formulation allows the treatment of flow through the matrix blocks, through the fractures and through the contact boundary between both media. DP always supposes the existence of a transfer function that describes the fluid exchange between both continua.

Triple Porosity - Double Permeability

An obvious extension of DP is triple porosity. A reservoir with fractures of homogeneous properties, interacting with two types of separated matrix blocks, each one with different porosities but similar permeabilities, defines a Triple Porosity - Double Permeability model. Another example of this image is formed by a network of dominant fractures intercepting a less permeable system of cracks, nested inside a matrix with different porosity. In this case the mechanism of flow is in the direction of matrix-fissures toward the fractures. A severely fractured reservoir with moderate permeability can be represented by this model. Its smaller global permeability could be produced by a higher tortuosity or by partial self-sealing of fissures.

Triple Porosity - Triple Permeability
Direct observations of fractured geothermal fields with large faults, show that fracturing intensity is higher close to the fault than in a farther fractured network. A remarkable permeability contrast coexists among the matrix blocks, the fractures and the faults (Table 1). This model is based on the fact that the initial response to fluid extraction is detected immediately in the fault, then after a while it becomes notorious in the fractures and much later in the fresh rock (matrix). The system's global permeability depends directly on the distance to the fault.

Tetra Porosity - Tetra Permeability
The observation on the microscope of thin sections of cores and cuttings extracted from volcanic fields show, visible fractures apart, the existence of microfractures connected to the matrix and to the fractured network. The net of microfractures conforms another continuum overlapped to the previous ones and has intermediate permeability values. It is the case of volcanic reservoirs with matrix -microfracture - fracture - fault flow. This model describes wide areas of the Los Azufres and of the Los Humeros, Mexico geothermal fields. The interporosity flows are transitory and depend on several factors including tortuosity and mineralization.

m Porosity - n Permeability
In the previous last model, if two different types of matrixes are detected, then the transfer could be matrix1- matrix2 - microfractures - fractures - fault. If the matrix is so heterogeneous that three or more porosities can be distinguished inside it (Table 1), then we will have models of Five or Six Porosities, and so forth. The notion of multiple porosity - multiple permeability in fractured reservoirs, arises in this natural way. To name all of them in a compact form we introduce the notation ***mN -nk Reservoirs*** (m Porosity - n Permeability).

AVERAGES OF PETROPHYSICAL PARAMETERS

Systematic analysis of information from Mexican geothermal fields under exploitation, clearly points out that a general trend in the thermodynamic evolution of producing wells does not exist (Suárez, 1995). Pressure depletions, as well as temperature variations, steam quality and geochemical fluid evolutions are different in every well. This should also be observed in other fields of this planet and not only in geothermal, but also in oil and in gas reservoirs. Thus, different theoretical and practical approaches are necessary to understand such variety of behavior. A sample of the natural heterogeneity of volcanic reservoirs, is condensed in Table 1. In this paper we only treat a single aspect of the phenomenon of mN-nk reservoirs: in what way the interaction among several different overlapping continua can be represented, when fluid and heat cross the interfaces of all of them?

During its geologic genesis, the reservoir acquires certain parametric values. Nevertheless, those primary values are altered later on by unpredictable random physical processes. In this way porosity, permeability, elasticity, thermal conductivity, density and mobility of fluids become heterogeneous. On the other hand, thermal, mechanical, electric and transport properties of rocks are determined by their geochemical and mineralogical compositions. Experimental evidences (Contreras et al., 1990) show the existence of correlations between such composition and petrophysical properties.

However, these correlations are, theoretically and practically, difficult to model. Heterogeneities in the fractured matrix, illustrated in the previous section, induce to consider practical mechanisms of interaction among different parts of the reservoir. Their effective treatment is achieved on the basis of averages at the contact interfaces. Mathematical statistics provides large number of averages to estimate the effective mean of different quantities.
In reservoir engineering some of those means can have physical meaning, but others only have a mathematical definition. The decision about what kind of average should be used in every particular problem, corresponds to both, laboratory measurements and field tests. In order to present clear ideas, we illustrate the general theory with simple averages. Table 2 summarizes some of them.

Equation 1

Equation 2

TREATMENT OF THE INTERPOROSITY FLOW

In *mN -nk Reservoirs*, a crucial problem is to quantify the fluid crossing the interfaces among m media. To simplify the analysis, let us suppose a stationary flow of an isothermal liquid, passing through a medium **i** to another medium **j** at their common boundary **S** $_{\mathbf{ij}}$ (Fig. 2). The parameter to represent the interaction between both continua is the mass flow per second per unit volume of fractured rock (q_{ij}). Neglecting gravity this term can be deduced directly from Darcy's Law:

Equation 3

Δ is density and μ is fluid viscosity, both could be constants or not.)x = $D_i + D_j$ represents the distance between one point in the medium **i** at pressure p_i and another point in the medium **j** at pressure p_j. This distance is an unknown critical variable because its numerical value can lead to abrupt changes in flow conditions. In general $\forall_{ij}$ is a dimensionless constant which only depends on the geometry of the boundary S_{ij}.

In the present work $\forall_{ij}$ contains implicitly the permeability average in that interface. For example if the continuum **i** is a matrix block and the continuum **j** is a fracture, then the effective permeability k_{ij} should be interpreted as an average at the block/fracture boundary. If Δ and μ are constants, equation (3) is the basic model of Barenblatt and coauthors (1960); it also contains implicitly the discretization of the IFD method (Pruess and Narasimhan, 1985) for stationary flow. If there is no pressure difference there is no flow and $q_{ij} = 0$. If $q_{ij} > 0$ is constant, then we can calculate either, the pressure difference)p (k_{ij}) as a function of k_{ij}, or the pressure in any one of the continua. If for example p_i is known:

Equation 4

Let us compare what happens when k_{ij} is obtained from different averages of both permeabilities. Using data from the Los Humeros geothermal field (Suárez, 1995), we have: p_i = 120 bar, T = 320°C, Δ = 669.3 kg/m^3, μ = 78.7×10^{-6} Pa · s; enthalpy h=1460.9 kJ/kg corresponds to compressed liquid and q_{ij}=0.03 kg/s/m^3. Let)x =1 m be the distance defined before. In this case medium **i** is a matrix block with some microfractures, with k_i .10^{-15} m^2, and medium **j** is an intensely fractured zone close to a fault, with k_j . 10^{-12} m^2. Using equation (4) and some formulas of Table 2, we obtain the results of Table 3.

It is clear that the thermodynamic conditions of volume V_j are extremely sensitive to the type of average used to calculate pressure p_j. Assuming an isothermal flow when crossing the interface, the first and second pressure drawdowns, correspond to an abrupt phase change, liquid becoming 100% steam with an enthalpy h = 2710 kJ/kg and Δ = 63.6 kg/m^3, μ = 20.9×10^{-6} Pa·s. For a non-isothermal flow, the Hashin-Shtrikman formula shows that it suffices a slight decrement of fluid temperature (–0.6°C) to obtain two-phase conditions, because at 112 bar, the saturation temperature is 319.4°C. Both situations are commonly encountered in the Los Humeros, Mexico, geothermal field (Suárez, 1995).

DISCONTINUITIES IN THE COMMON INTERFACE

If the fluid is homogeneous Δ and μ are constants. But if the fluid suffers a sharp drop in pressure when crossing from one medium to the other, as in the previous example, then both parameters become discontinuous at the common boundary S_{ij}. In this case, equation (4) is only valid if appropriate averages for Δ and μ are included. Also equation (3) needs to incorporate a different expression for the discontinuous permeability, which is a function of a very short distance ($x-x_0$), where x_0 is the exact position of the common interface S_{ij}. In terms of distributions, its mathematical representation in one dimension becomes:

Equation 5

H(x) is the Heaviside generalized function. The data condensed in Table 1, is an example of the discontinuous nature of permeability in a real mN-nk reservoir. Figure 3 illustrates this fact in a semi-log plot of permeability values, assuming some hypothetical regular distribution in one dimensional space.

CONCLUSIONS

- The Double porosity models the flow between two different media under the hypothesis that the petrophysical properties in each medium are uniform. Under the concept of multiple porosity - permeability, M continuous porous media interact with each other. Every medium has its own parameters and its own interporosity non-isothermal, transitory flow.

- The multiple porosity - permeability model represents a global phenomenon in heterogeneous fractured geothermal reservoirs. Its overall condition produces multiple effects on other interdependent phenomena, in such a way that the whole system cannot be understood without taking into account the mutual interactions of its components.

- Examples of a real multiple porosity-permeability reservoir were presented. Los Humeros, Mexico geothermal field, with its contrasting permeabilities, steam segregation and abrupt phase changes, is a prototype of such heterogeneous systems.

- In this work we focused in the problem of interaction among media with different properties. The effective treatment of their interactions is achieved through appropriate averages of parameters at the contact interfaces. The decision about what kind of average to use should be particular to the specific problem, and should be based on both, laboratory measurements and field tests results.

- Geothermal fluid is extremely sensitive to geometric changes of the flow conduits in the reservoir. In a mN-nk model the fluid transfer can produce abrupt changes from liquid to two-phases or to steam at the interface of any medium. Main thermodynamic functions could become discontinuous in that boundary. Its correct mathematical representation needs the use of the theory of distributions.

- Equilibrium and evolution thermodynamics of fractured systems can be completely different under different averaging conditions. Analysis of data from Mexican geothermal fields does not show clear general trends in the thermodynamic evolution of producing wells. Different theoretical and practical approaches are necessary to understand such variety of behavior.

- The mN-nk model provides a solid and flexible theoretical and practical framework to improve pressure tests analysis and numerical simulation of heterogeneous reservoirs. This is the natural physical application of the general concept.

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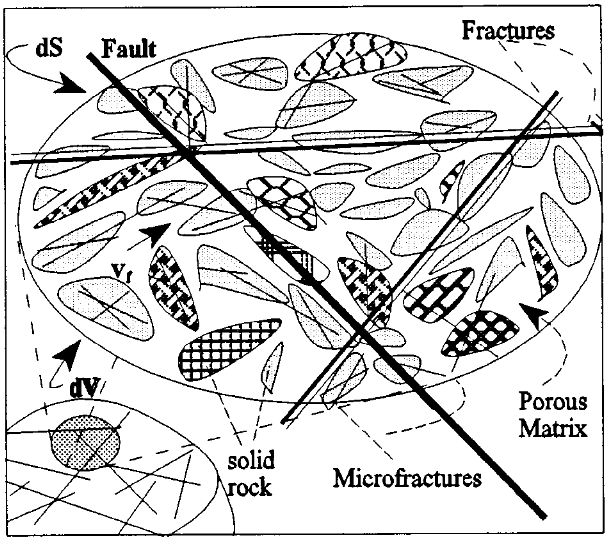

Fig. 1.- Differential Volume of a mϕ-nk medium showing a Fault ($\sim 10^{-11}$ m^2), fractures ($\sim 10^{-13}$ m^2), microfractures ($\sim 10^{-15}$ m^2) and matrix ($\sim 10^{-18}$ m^2).

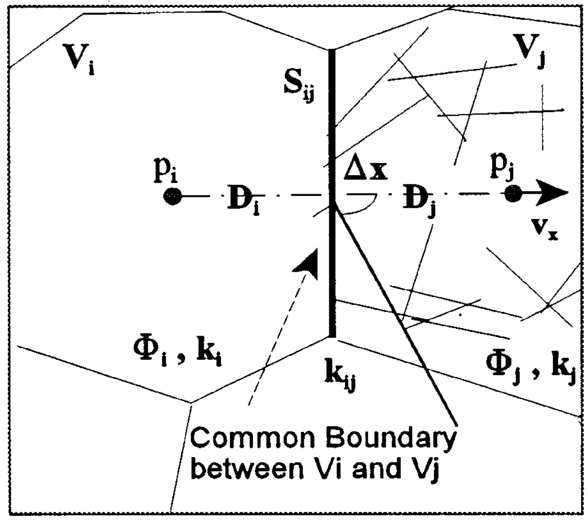

Fig. 2.- Interporosity flow between two media.

Fig. 3.- Idealized Distribution of Permeabilities in a mϕ - nk Medium

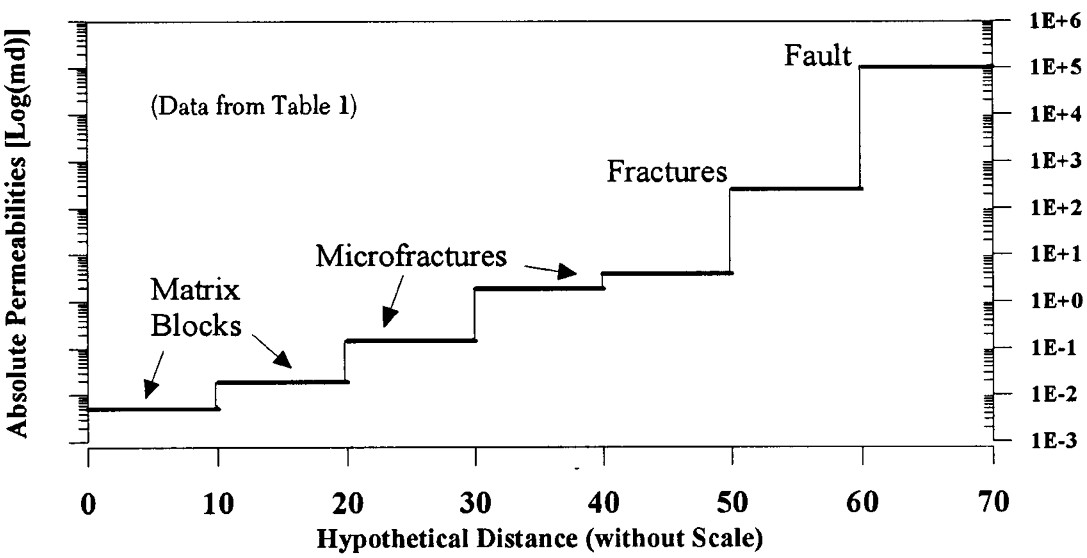

Table 1.- Some Petrophysical Properties of mϕ-nk Reservoirs [(@)]

Well	Depth (m)	Density (kg/m³)	Porosity (%)	Permeability (milidarcy)	K_{Ter} (W/m/°C)	C_P (J/kg/°C)
H-18	1750-1753	2340	14.7	0.01	2.42	921.1
H-02	616-619	2160	19.7	0.02	1.54	1046.7
H-19	1769-1771	2460	11.5	0.15	1.91	1172.3
H-26	1810-1813	2670	4.5	1.87	1.95	1004.8
H-24	2844-2847	2450	12.7	3.83	1.62	1046.7
Az-33	1350	2355	12	247.6	1.93	1165.0
H-28	1200	2430	12.3	101.3×10^3	1.99	1069.0

[@] *Petrophysical data were measured in a Terra-tek lab (Contreras et al., 1990) in liquid saturated cores at p=100 bar and T =25°C. Permeability values in the last two rows of this table were deduced from analysis of pressure tests. Wells H -* are in the Los Humeros and Az-33 is a well in the Los Azufres geothermal field.*

Table 2.- Some Useful Averages in Multiple Porosity - Permeability Reservoirs with Multicomponent Fluid [@]

NAME	FORMULA	NAME	FORMULA
General Arithmetic Mean	$A_A = \sum_{n=1}^{N} K_n / N$	General Geometric Mean	$A_G = \left(\prod_{n=1}^{N} K_n \right)^{\frac{1}{N}}$
Logarithmic Average	$A_L = \dfrac{K_i - K_j}{Ln(K_i) - Ln(K_j)}$	Weighted Geometric Mean	$A_{WG} = K_R^{(1-\phi)} K_L^{S_L \phi} K_V^{S_V \phi}$
Flow Continuity Mean	$\dfrac{D_i + D_j}{A_C} = \dfrac{D_i}{K_i} + \dfrac{D_j}{K_j}$	Parallel Average	$A_P = (1-\phi) K_R + S_L \phi K_L + S_L \phi K_V$
Lagrange Linear Mean	$A_{LI} = \dfrac{D_j K_i + D_i K_j}{D_i + D_j}$	Serial Average	$\dfrac{1}{A_S} = \dfrac{(1-\phi)}{K_R} + \dfrac{\phi S_L}{K_L} + \dfrac{\phi S_V}{K_V}$
Weighted Average ($\theta \in [0, 1]$)	$A_W = (1-\theta) K_i + \theta K_j$	General Weighted Average ($\theta_i \in [0, 1]$)	$A_{WA} = \sum_{n=1}^{N} \theta_n K_n \; ; \; (\sum_{n=1}^{N} \theta_n = 1)$
Budiansky [1]	$A_{BI} = \dfrac{-b_1 + \sqrt{b_1^2 + 8 K_i K_j}}{4}$	Hashin - Shtrikman [2]	$A_{HS} = \dfrac{K_H + K_S}{2}$

[@] *Letter K is any reservoir parameter. S can be phase saturation and ϕ is porosity; in general both represent a volumetric fraction of one component or phase. D_i, D_j are distances (Fig. 2) between the center of each region and its interface. Subscripts R, L and V mean rock, liquid and vapor phases respectively. If $D_i = D_j$ and for any values of the parameters involved, the following inequality always holds: $A_C < A_G < A_L < A_A$. The references for the last two formulas are: (1) Pritchett, (1995); (2) Hashin & Shtrikman, (1962). Budiansky and Hashin-Shtrikman formulas include the following definitions:*

$$b_1 = [3(1-\phi) - 2] K_j + (3\phi - 2) K_i \quad (1)$$

$$K_H = K_m + \phi \left[(K_f - K_m)^{-1} + \frac{1-\phi}{3K_m} \right]^{-1} \; ; \quad K_S = K_f + (1-\phi) \left[(K_m - K_f)^{-1} + \frac{\phi}{3K_f} \right]^{-1} \quad (2)$$

$$v_x = - \frac{k_{ij}}{\mu} \frac{d p_{ij}}{dx} \rightarrow \frac{\rho v_x}{\Delta x} \approx - \frac{\rho k_{ij}}{\mu \Delta x} \frac{\Delta p}{\Delta x}$$

$$if: \; \alpha_{ij} = \frac{k_{ij}}{(\Delta x)^2} \rightarrow q_{ij} = \alpha_{ij} \frac{\rho (p_i - p_j)}{\mu} \quad (3)$$

$$p_j = p_i - \frac{\mu}{\rho} \frac{q_{ij}}{\alpha_{ij}} \quad (4)$$

$$K_{ij}(x) = (k_j - k_i) H(x - x_0) + k_i$$

$$where: \; H(x - x_0) = 1 \; , \; if \; x \geq x_0 \quad (5)$$

$$and: \; H(x - x_0) = 0 \; , \; if \; x < x_0$$

Table 3.- Interporosity Flow Values at the Interface S_{ij}

Formula	k_{ij} (m^2)	β/k_{ij} (bar)	p_j (bar)
A_{BI} *	1.03×10^{-15}	34.22	85.8
A_C	2.00×10^{-15}	17.64	102.4
A_{HS}	4.36×10^{-15}	8.09	111.9
A_G	3.16×10^{-14}	1.12	118.9
A_L	1.45×10^{-13}	0.24	119.8
A_W	6.67×10^{-13}	0.05	119.95

* *In the use of the Budiansky formula we assume a volumetric fraction of fractures equal to 1%, and $\beta = q_{ij} \mu / \rho$.*