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A VIRTUAL NODE METHOD FOR TREATMENT OF WELLS IN MODELING MULTIPHASE FLOW IN RESERVOIRS

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Abstract

How to treat well boundary conditions is a difficult issue when formulating and coding a multiphase numerical reservoir simulator. The difficulty arises because the partial differential equations governing multiphase subsurface flow are of a mixed parabolic-hyperbolic type. Maximum changes in primary variables and mass/heat fluxes occur only at boundaries or well nodes. Consequently, these well nodes tend to be singular, leading to computational convergence problems. The conventional method of well treatment in geothermal or oil reservoir simulators is to use a sink/source term approach and distributes flow rates by a potential or mobility allocation scheme for a multilayered well. However, this traditional method cannot rigorously handle a backflow problem, which may occur in a multilayered well in heterogeneous formations. Any potential backflowing layer has its flow set to zero by the standard method, which may lead to a physically incorrect solution for well flow.

This paper presents a "virtual node" method for treating well boundary conditions. This method handles a wellbore either as a single node or several computational nodes that are screened and connected to many neighboring nodes for a multilayered well. The wellbore can be vertical, inclined, or horizontal, and the well borehole node is treated in the same way as any other nonwell node for flow calculations. Pumping/injection conditions are accounted for using sink or source terms to the well node only. The solution at the well is then obtained by solving mass-balance equations for the well node. It will be shown that the new method provides a natural, physically consistent, and numerically efficient approach to handling well flow problems. In addition, implementation of this new method for a three-phase flow reservoir simulator will be discussed, and an application example will be provided.

1. Introduction

Even with the recent continual progress made in both computational algorithms and computer hardware efficient and rigorous well treatment in numerical reservoir simulation remains a challenge. It is in general the most difficult, time-consuming part of a multiphase flow simulation, because of the nature of the partial differential equations governing multiphase subsurface flow, which are of a mixed parabolic-hyperbolic type. Maximum changes in primary variables

and mass fluxes occur at boundary or well nodes. Therefore, well nodes are often singular and make it difficult to achieve convergence during a simulation. In addition, multiphase interactions at a model boundary make describing boundary conditions for multiphase flow quite different from those under single-phase flow conditions. For example, specification of a given phase pressure at boundaries must be consistent with other phase pressures at the same location, which are correlated by capillary functions. In many cases, only one or two phases may be present at certain boundaries, and special caution must be taken to describe the pressure conditions for the nonexisting phases to avoid an artificial injection or pumping that could occur for nonexisting phases at that boundary. In that case, it is important to maintain physical consistency in describing inflow or outflow at a boundary node.

Strictly speaking, boundary condition in modeling multiphase flow is a constraint condition rather than a rigorous "boundary condition" as used in the mathematical sense in solving partial differential equations. The conventional method of well treatment in geothermal/oil reservoir simulators is to use a sink/source term approach and distribute flow rates by a potential or mobility allocation scheme for a multilayered well. As described in detail by Thomas (1982), the potential allocation scheme, which is probably the most common method used in the geothermal and petroleum industries, will estimate correct total fluid injection/production rates as long as the maximum and minimum pressure constraints are not violated. However, this traditional method cannot rigorously handle a backflow problem, which may occur at a multilayered well in heterogeneous formations. Any potentially backflowing layers have their backflow set to zero by this method, and therefore the solution may not always be physically correct. In addition, solving for borehole pressure explicitly in order to eliminate the wellbore mass balance equation from the entire equation system will introduce additional nonlinearity and increase convergence difficulties with this approach. The mobility allocation method, on the other hand, is easy to implement, but may result in physically incorrect solutions and poorer numerical performance as well.

This paper presents a "virtual node" method for treatment of well boundary conditions. The methodology proposed and implemented in this paper follows the work by Wu et al. (1996) and is based on a general, physically consistent approach. In this method, any type of well boundary condition is treated as a physical sink or source term to the wellbore node, i.e., the well boundary node either gains or loses mass over a time step. At the same time a wellbore node is regarded as a computational or "virtual" node, included in the mass-balance calculations with the rest of the nodes. This method handles a wellbore either as a single node or several computational nodes screened and connected to many neighboring nodes for a multilayered well. The wellbore can be vertical, inclined, or horizontal, and the well borehole node is treated in the same way as any other nonwell node. Production/injection conditions are accounted for using sink or source terms to the well. The solution for the well node is then obtained by solving mass balance equations for the well node. It

will be shown that the new method provides a natural, physically consistent, and numerically efficient approach to handling well flow problems. In addition, implementation of the new method to a three-phase flow reservoir simulator will be discussed and an application example will be provided.

2. Governing Equations

A multiphase system in an oil reservoir is assumed to be composed of three phases: oil, gas, and water. Although each of the three phases may contain several components, each is treated here as a single "pseudo-component" with averaged properties of the fluids such as in a "black-oil" model. The two liquid components, water and oil, are assumed to be present only in their associated phases; the gas exists in the gas phase and is also dissolved in the oil phase.

Each phase flows in response to pressure, gravitational and capillary forces according to the multiphase extension of Darcy's law. In an isothermal system containing three mass components, three mass balance equations are needed to fully describe the system. The mass balance equations for the three phases are written in an arbitrary flow region as follows:

For gas flow,

$$\frac{\partial}{\partial t}\left\{\phi\left(S_o\bar{\rho}_{dg}+S_g\rho_g\right)\right\}=-\nabla\bullet\left(\bar{\rho}_{dg}\,\vec{V}_o+\rho_g\,\vec{V}_g\right)+q_g \qquad (1)$$

For water flow,

$$\frac{\partial}{\partial t}\left\{\phi S_w\rho_w\right\}=-\nabla\bullet\left(\rho_w\,\vec{V}_w\right)+q_w \qquad (2)$$

For oil flow,

$$\frac{\partial}{\partial t}\left(\phi S_o\bar{\rho}_o\right)=-\nabla\bullet\left(\bar{\rho}_o\,\vec{V}_o\right)+q_o \qquad (3)$$

In Equations (1), (2) and (3), the Darcy velocity of phase β is defined,

$$\vec{V}_\beta=-\frac{k k_{r\beta}}{\mu_\beta}\left(\nabla P_\beta-\rho_\beta g\nabla D\right), \qquad (4)$$

where ρ_β is the density of phase β (β = g for gas, β =w for water, and β = o for oil) at reservoir conditions; $\bar{\rho}_o$ is the density of oil, excluding dissolved gas, at reservoir conditions; $\bar{\rho}_{dg}$ is the density of dissolved gas (dg) in the oil phase at reservoir conditions; ϕ is the effective porosity of the formation; μ_β is the viscosity of phase β; S_β is the saturation of phase β; P_β is the pressure of phase β; q_β is the sink/source term of phase (component) β per unit volume of formation; g is gravitational acceleration; k is absolute/intrinsic permeability (tensor) of the formation; $k_{r\beta}$ is relative permeability to phase β; and D is depth.

3. Constitutive Relations

The governing Equations (1), (2) and (3) of mass balance for three phases need to be supplemented with constitutive equations, which express all the secondary variables and parameters as functions of a set of primary thermodynamic variables of interest. The following relationships will be used to complete the description of describing multiphase flow through porous media.

In addition, there is one supplementary equation given by

$$S_w + S_o + S_g = 1 \qquad (5)$$

The capillary pressures are needed to relate pressures between the phases. The aqueous and gas phase pressures are related by

$$P_w = P_g - P_{cgw}(S_w), \qquad (6)$$

where P_{cgw} is the gas-water capillary pressure in a three-phase system, and is assumed to be a function of water saturation only. The oil phase pressure is related to the gas phase pressure by

$$P_o = P_g - P_{cgo}(S_w, S_o), \qquad (7)$$

where P_{cgo} is the gas-oil capillary pressure in a three-phase system, a function of two saturations of water and oil phases, respectively. For many reservoir formations, the wettability order is (1) aqueous phase, (2) oil phase, and (3) gas phase. The gas-water capillary pressure is usually stronger than the gas-oil capillary pressure. The oil-water capillary pressure, P_{cow}, in a three-phase system, may be defined as,

$$P_{cow} = P_{cgw} - P_{cgo} = P_o - P_w \qquad (8)$$

The relative permeabilities are assumed to be functions of fluid saturations only. The relative permeability to the water phase is described by

$$k_{rw} = k_{rw}(S_w) \qquad (9)$$

to the oil phase by

$$k_{ro} = k_{ro}(S_w, S_g) \qquad (10)$$

and to the gas phase by

$$k_{rg} = k_{rg}(S_g) \qquad (11)$$

When no three-phase relative permeability data are available, the oil relative permeability may be determined using the Stone Method II (Aziz and Settari, 1979).

The densities of oil, gas, and water; the gas-oil ratio; and the viscosities of fluids under reservoir conditions can in general be treated as functions of both fluid and bubble-point pressures.

4. Numerical Model

The virtual node method of this paper has been implemented into a general-purpose, three-phase reservoir simulator, the MSFLOW code (Wu, 1998). As implemented in the MSFLOW code, the continuum Equations (1), (2), and (3) can be discretized in space using an integral finite-difference or control-volume finite-element scheme. The time discretization is carried out with a backward, first-order finite-difference method. Then the discrete nonlinear equations of Node i are as follows:

For gas flow:

$$\left\{\left[S_o \bar{\rho}_{dg} + \phi S_g \rho_g\right]_i^{n+1} - \left[S_o \bar{\rho}_{dg} + \phi S_g \rho_g\right]_i^{n}\right\}\frac{V_i}{\Delta t} = \sum_{j\in\eta_i}\left(\bar{\rho}_{dg}\lambda_o\right)_{ij+1/2}^{n+1}\gamma_{ij}\left[\psi_{oj}^{n+1} - \psi_{oi}^{n+1}\right] + \sum_{j\in\eta_i}\left(\rho_g\lambda_g\right)_{ij+1/2}^{n+1}\gamma_{ij}\left[\psi_{gj}^{n+1} - \psi_{gi}^{n+1}\right] + Q_{gi}^{n+1} \qquad (12)$$

for water flow:

$$\left\{\left[\phi\, S_w \rho_w\right]_i^{n+1} - \left[\phi S_w \rho_w\right]_i^n\right\}\frac{V_i}{\Delta t} = \sum_{j\in\eta_i}\left(\rho_w \lambda_w\right)_{ij+1/2}^{n+1}\gamma_{ij}\left[\psi_{wj}^{n+1} - \psi_{wi}^{n+1}\right] + Q_{wi}^{n+1} \quad (13)$$

and for oil flow:

$$\left\{\left[\phi\, S_o \bar{\rho}_o\right]_i^{n+1} - \left[\phi\, S_o \bar{\rho}_o\right]_i^n\right\}\frac{V_i}{\Delta t} = \sum_{j\in\eta_i}\left(\bar{\rho}_o \lambda_o\right)_{ij+1/2}^{n+1}\gamma_{ij}\left[\psi_{oj}^{n+1} - \psi_{oi}^{n+1}\right] + Q_{oi}^{n+1} \quad (14)$$

where n denotes the previous time level; n+1 is the current time level; V_i is the volume of element i; and Δt is time step size; η_i contains the set of neighbor elements (j) or nodes of element i to which element i is directly connected. Subscript ij+1/2 denotes a proper averaging at the interface between two elements i and j. The mobility of phase β is defined as,

$$\lambda_\beta = \frac{k_{r\beta}}{\mu_\beta} \quad (15)$$

The flow potential term is

$$\psi_{\beta i}^{n+1} = P_{\beta i}^{n+1} - \rho_{\beta, ij+1/2}^{n+1}\, g\, D_i \quad (16)$$

and the transmissivity of flow terms is defined, if the integral finite difference scheme is used (Pruess, 1991), as

$$\gamma_{ij} = \frac{A_{ij}\, k_{ij+1/2}}{d_i + d_j}\,, \quad (17)$$

where A_{ij} is the common interface area between connected elements i and j, d_i is the distance from the center of element i to the interface between elements i and j, $k_{ij+1/2}$ is the averaged absolute permeability along the connection between elements i and j, and D_i is the depth to the center of element i. The sink/source term at element i, $Q_{\beta i}$ for phase β, is defined as

$$Q_{\beta i}^{n+1} = q_{\beta i}^{n+1} V_i \quad (18)$$

for i =1, 2, 3, …, N, with N being the total number of elements or grid blocks in the grid. If element i is a wellbore node, $Q_{\beta i}$ will be determined as described in the next section.

The Newton/Raphson iterations are used to solve Equations (12) to (14). For a three-phase flow system, 3*N coupled nonlinear equations must be solved, including three equations at each element for the three mass balance equations of gas, water, and oil, respectively. Three primary variables

Table 1 Choice of the Primary Variables and Associated Equations.

Equations	Primary variable	Physical variable
Gas (12)	$x_1 = P_o$	Oil pressure
Water (13)	$x_2 = S_o$	Oil saturation
Oil (14)	$x_3 = P_s$ or $x_3 = S_g$	Saturation pressure or gas saturation

(x_1, x_2, x_3) are selected for each element, - oil pressure, oil saturation and saturation pressure (or gas saturation), as shown in Table 1. An automatic variable switching scheme is used to handle the transition of free gas appearing and disappearing during simulations of oil production with oil, gas, and water three-phase flow conditions.

In terms of the three primary variables, the Newton/Raphson scheme gives rise to

$$\sum_k \frac{\partial R_i^{\beta,n+1}\left(x_{k,p}\right)}{\partial x_k}\left(\delta x_{k,p+1}\right) = -R_i^{\beta,n+1}\left(x_{k,p}\right) \quad (19)$$

for k=1, 2, and 3

where index k = 1, 2, and 3 indicate the primary variable 1, 2, and 3, respectively; p is the iteration level; and i =1, 2, 3, …, N. The primary variables are updated after each iteration:

$$\delta x_{k,p+1} = x_{k,p+1} - x_{k,p} \quad (20)$$

A numerical method is used to construct the Jacobian matrix for Equation (19). For a fully implicit element, the Jacobian is evaluated using a numerical differentiation, as outlined by Forsyth et al. (1995), and for an IMPES (IMplicit Pressure and Explicit Saturation) node, a simpler, semi-analytical approach is used for Jacobian calculations.

The flow along wellbore is handled as fully coupled with the reservoir. Instead of using Darcy's law, the flow equation along boreholes for horizontal wells (Dikken, 1990) is extended here to describing flow in horizontal, inclined, or vertical boreholes as

$$q_{ij,\beta}^w = \gamma_{ij}^w \tau_\beta^w \left(\psi_{\beta j} - \psi_{\beta i}\right) \quad (21)$$

where $q_{ij,\beta}^w$ is mass flux of phase β between two connected wellbore nodes i and j, the transmissivity of the wellbore, when comparing (21) with the wellbore flow equation of Dikken (1990), may be defined as

$$\gamma_{ij}^w = \frac{1.97588 \times d^{5/2}}{d_i + d_j} \quad (22)$$

and the wellbore mobility is given by

$$\tau_\beta^w = \frac{\left(S_\beta \rho_\beta\right)_{ij+1/2}}{\left(\bar{\rho}\right)^{1/2}} \left| \frac{\left(\psi_{\beta j} - \psi_{\beta i}\right)}{d_i + d_j} \right|^{-1/2}, \quad (23)$$

where d is the diameter of the wellbore. The averaged density of fluids is defined as

$$\bar{\rho} = \frac{1}{2}\left\{ \begin{array}{l} \left(S_g \rho_g\right)_i + \left(S_o \rho_o\right)_i + \left(S_w \rho_w\right)_i + \\ \left(S_g \rho_g\right)_j + \left(S_o \rho_o\right)_j + \left(S_w \rho_w\right)_j \end{array} \right\} \quad (24)$$

5. Well Treatment

The virtual node method (Wu et al., 1996) handles a wellbore as a single node or as several computational nodes that are screened and connected to many neighboring nodes for a multilayered well. A borehole node is treated in the same form as for any other nonwell node for flow calculations. The mass balance Equations (12) to (14) are still applicable to Well Node i, however, the coefficients for flow terms are evaluated differently. In this case, productivity index is used for wellbore-formation flow and

wellbore mobility (23) and transmissivity (22) are for wellbore-wellbore flow. It is assumed that a well node is subject only to pumping or injection operations without any other boundary conditions. Therefore, Equations (12) to (14), for Well Node i, are rewritten as

$$
\begin{aligned}
R_i^{g,n+1} = & \left\{ \left[\phi S_o \bar{\rho}_{dg} + \phi S_g \rho_g\right]_i^{n+1} - \left[\phi S_o \bar{\rho}_{dg} + \phi S_g \rho_g\right]_i^{n} \right\} \frac{V_i}{\Delta t} - Q_{g,w}^{n+1} - \\
& \sum_{j \in \eta_i} \left(\bar{\rho}_{dg} \lambda_o\right)_{i\,j+1/2}^{n+1} PI_{ij} \left[\psi_{oj}^{n+1} - \psi_{oi}^{n+1}\right] - \\
& \sum_{j \in \eta_i} \left(\rho_g \lambda_g\right)_{i\,j+1/2}^{n+1} PI_{ij} \left[\psi_{gj}^{n+1} - \psi_{gi}^{n+1}\right] - \\
& \sum_{k \in \eta_w} \left(\bar{\rho}_{dg} \tau_g^w / \rho_o\right)_{ik+1/2}^{n+1} \gamma_{ik}^w \left[\psi_{ok}^{n+1} - \psi_{oi}^{n+1}\right] - \\
& \sum_{k \in \eta_w} \left(\tau_g^w\right)_{ik+1/2}^{n+1} \gamma_{ik}^w \left[\psi_{gk}^{n+1} - \psi_{gi}^{n+1}\right]
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
R_i^{w,n+1} = & \left\{ \left[\phi S_\beta \rho_w\right]_i^{n+1} - \left[\phi S_w \rho_w\right]_i^{n} \right\} \frac{V_i}{\Delta t} - Q_{w,w}^{n+1} - \\
& \sum_{j \in \eta_i} \left(\rho_w \lambda_w\right)_{i\,j+1/2}^{m} PI_{ij} \left[\psi_{wj}^{n+1} - \psi_{wi}^{n+1}\right] - \\
& \sum_{k \in \eta_w} \left(\tau_w^w\right)_{ik+1/2}^{n+1} \gamma_{ik}^w \left[\psi_{wk}^{n+1} - \psi_{wi}^{n+1}\right]
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
R_i^{o,n+1} = & \left\{ \left[\phi S_o \bar{\rho}_o\right]_i^{n+1} - \left[\phi S_o \bar{\rho}_o\right]_i^{n} \right\} \frac{V_i}{\Delta t} - \\
& \sum_{j \in \eta_i} \left(\bar{\rho}_o \lambda_o\right)_{i\,j+1/2}^{n+1} PI_{ij} \left[\psi_{oj}^{n+1} - \psi_{oi}^{n+1}\right] - \\
& \sum_{k \in \eta_w} \left(\bar{\rho}_o \tau_o^w / \rho_o\right)_{ik+1/2}^{n+1} \gamma_{ik}^w \left[\psi_{ok}^{n+1} - \psi_{oi}^{n+1}\right] - Q_{o,w}^{n+1}
\end{aligned}
\tag{27}
$$

where PI_{ij} is a well productivity or injectivity index for the connection between Well Node i and neighboring node j; $Q_{\beta,w}^{n+1}$ is the total mass rate of pumping or injection at the well, to be determined in the following subsections for different pumping or injection specifications; j is the index of a neighboring formation node, connected to Well Node i; η_w is a set of wellbore nodes, connected with Well Node i along the borehole; and k is the index of the neighboring wellbore node to Well Node i.

In general, two types of flow terms appear in the well flow equations (25), (26), and (27) and are not described by Darcy's law. The first accounts for the radial flow between the wellbore and the formation, for which we need a productivity (injectivity) index or a well function. The other accounts for the flow between wellbore nodes and is described by Equation (21). If a wellbore is represented by more than one computational node, the sink/source term for the well needs to be added to only one of the well nodes.

There are many methods and equations, especially in petroleum literature, for evaluating the well index. For a vertical borehole, Thomas (1982) proposed the following well index formulation,

$$
PI_{ij} = \frac{2\pi k \Delta z_j}{\ln\left(\frac{r_e}{r_w}\right) + s - 1/2}, \tag{28}
$$

where Δz_j is the thickness of layer j, r_e is an effective radius of grid block j, r_w is the wellbore radius, and s is the skin factor. Various well indexes exist in the literature for vertical, inclined, or horizontal wells (Peaceman, 1978, 1982, 1991, and 1995; Lee et al., 1993; and Fung et al., 1991).

The injection rate for phase β at an injection well is evaluated by

$$Q_{\beta,w} = \sum_{j\in\eta_i} \left(\rho_\beta \lambda_\beta\right)_{i\,j+1/2} P\,I_{i\,j} \left[P_{\beta,\,j} - P_w - \rho_\beta\, g\left(D_j - D_w\right)\right] \tag{29}$$

The production rates at a production well are evaluated as follows.

For oil,

$$Q_{o,w} = \sum_{j\in\eta_i} \left(\overline{\rho}_o \lambda_o\right)_{ij+1/2} PI_{ij} \left[P_{o,j} - P_w - \rho_o\, g\left(D_j - D_w\right)\right] \tag{30}$$

For water,

$$Q_{w,w} = \sum_{j\in\eta_i} \left(\rho_w \lambda_w\right)_{i\,j+1/2} PI_{i\,j} \left[P_{w,\,j} - P_w - \rho_w\, g\left(D_j - D_w\right)\right] \tag{31}$$

For gas,

$$Q_{g,w} = \sum_{j\in\eta_i} \left(\rho_g \lambda_g\right)_{ij+1/2} PI_{ij} \left[P_{g,j} - P_w - \rho_g\, g\left(D_j - D_w\right)\right] + \sum_{j\in\eta_i} \left(\overline{\rho}_{dg} \lambda_o\right)_{ij+1/2} PI_{ij} \left[P_{o,j} - P_w - \rho_o\, g\left(D_j - D_w\right)\right] \tag{32}$$

In the above equation, the total mass rate is calculated from summation of the flow terms between Well Node i and all its neighbors, j. P_w is the well pressure, determined using an additional constraint equation; and D_w is the depth at which a pump or injection is located inside the wellbore.

The mobility terms in the well flow equations (25) to (27) are evaluated using the upstream weighting schemes. In other words, the well node is treated as a virtual node, like any other node in the grid, except that the sink/source term is determined by Equations (29) to (32). The following sections will provide detailed treatment for different pumping and injection scenarios.

5.1 Rate-Specified Pumping Well

There are in general two types of rate-specified pumping wells. In the first (a) total liquid (water and oil) volumetric production rate is specified, and in the second (b) one-phase pumping volumetric rate is specified, i.e., the pumping rate is given or fixed for only one individual phase of oil, gas, or water. Physically, the phase(s) of nonspecified fluids may also flow out and should be accounted for. The two rate-specified pumping scenarios are treated differently in this section. The following approach is not only rigorous and efficient in treating a pumping well, but also allows back flow to occur, which cannot be easily handled by other well treatment schemes. The back flow may occur at certain layers in a pumping well that penetrates multiple layers, cannot be controlled in general, and needs to be modeled.

a. Total Liquid Rate Specification

With a total volumetric pumping rate, Q_L (> 0), of liquid (water + oil) specified at the well, the well flowing pressure is evaluated by

$$P_w = \left\{-Q_L + \sum_{j\in\eta_i} \sum_\beta \left(\rho_\beta \lambda_\beta / \rho_\beta^o\right)_{ij+1/2} PI_{ij} \left[P_{\beta,j} - \rho_\beta\ \ g\left(D_j - D_w\right)\right]\right\} / \left\{\sum_{j\in\eta_i} \sum_\beta \left(\rho_\beta \lambda_\beta / \rho_\beta^o\right)_{ij+1/2} PI_{ij}\right\} \tag{33}$$

with β = o and w. The well pressure determined by Equation (33) is subject to a constraint,

$$P_w \geq P_{w,\min}, \tag{34}$$

where $P_{w,min}$ is the minimum well pressure allowed. Using Equation (34) enforces the physical constraint that it is

not always possible to produce liquid at the specified rate. If not, the pumping well is switched to a pressure-specified pumping operation.

The actual pumping rate, to be added to Equations (25) to (27) as a sink term, is evaluated using Equations (30) to (31) for oil and water, respectively, with the well pressure determined by Equations (33) and (34). In addition, gas may also flow out simultaneously with liquids at the well, which cannot be controlled, even though a liquid rate is specified. The gas production rate is determined by Equation (32).

In general, the well pressure, P_w, from Equations (30) and (31) should approach the well nodal pressure, P_i, from the simulation when the solution is converged and if zero capillary forces are specified for the well node.

b. Single-Phase Rate Specification

With a single-phase fluid volumetric pumping rate, Q_β(> 0), of oil, water, or gas, specified at the well, the well pressure is evaluated as follows,

$$P_w = \left\{ -\rho_{\beta,STC} Q_o + \sum_{j\in\eta_i} \left(\rho_\beta \lambda_\beta\right)_{ij+1/2} PI_{ij} \left[P_{\beta,j} - \rho_\beta g\left(D_j - D_w\right)\right] \right\} \Big/ \left\{ \sum_{j\in\eta_i} \left(\rho_\beta \lambda_o\right)_{ij+1/2} PI_{ij} \right\} \quad (35)$$

with β = o, w or g, where $\rho_{\beta,\,STC}$ is the density of phase β at standard conditions.

The well pressure from (35) is also physically subject to a constraint condition (34). Then the actual pumping rate for the phase is evaluated using Equations (30), (31) and (32) and added to Equations (25), (26) and (27). Again, nonspecified phases may be pumped out, and their flow terms should also be calculated accordingly, subject to the same well pressure.

5.2 Pressure-Specified Pumping Well

If a pumping well is operated at a specified well pressure, this pressure is directly substituted into Equations (30) to (32) for P_w for evaluating sink terms of the three phases. This type of pumping condition is generally named as production under "deliverability." In this case, any phase or all of the three phases may be produced, depending on the conditions at the nodes that are connected to the well.

5.3 Rate-Specified Injection Well

Injection rates are known in practice. The injected phase can be water, oil (possibly), or gas. With a total mass injection rate, Q_β (> 0), of phase β specified at the well, the well injection pressure is then evaluated by,

$$P_w = \left\{ Q_\beta + \sum_{j\in\eta_i} \left(\rho_\beta \lambda_\beta\right)_{ij+1/2} PI_{ij} \left[P_{\beta,j} - \rho_\beta g\left(D_j - D_w\right)\right] \right\} \Big/ \left\{ \sum_{j\in\eta_i} \left(\rho_\beta \lambda_\beta\right)_{ij+1/2} PI_{ij} \right\}, \quad (36)$$

with β = o, w, or g , and the well injection pressure is subject to the following constraint,

$$P_w \le P_{w,max}, \quad (37)$$

Table 10.6.1 Parameters for the three-dimensional, five-spot, well-flow problem.

Parameter	Value	Unit
Porosity	$\phi = 0.20$	
Water density	$\rho_w = 1{,}000$	kg/m^3
Oil density	$\rho_o = 800$	kg/m^3
Water phase viscosity	$\mu_w = 0.5\times10^{-3}$	Pa•s
Oil phase viscosity	$\mu_o = 2.17\times10^{-3}$	Pa•s
Permeability	$k = 1.579\times10^{-14}$	m^2
Water injection rate	$q = 1.9321 \times 10^{-2}$	kg/s
Irreducible water saturation	$S_{wir} = 0.30$	
Irreducible oil saturation	$S_{oir} = 0.067$	

where $P_{w,max}$ is the maximum well injection pressure allowed. Equation (37) is used when the specified injection rate is too high for the well condition. In this situation, the injection well is switched to a pressure-specified injection operation.

The actual injection rate for the specified phase is evaluated using Equation (29), with the well injection pressure determined by Equations (36) and (37), and then is added to Equation (25), (26) or (27) for Well Node i of injection.

5.4 Pressure-Specified Injection Well

If a fluid is injected at a specified well pressure, this pressure is directly substituted into Equation (29) for P_w of the injected phase. For a multilayered, pressure-specified injection well, the injection rate of the specified phase is determined and partitioned using Equation (29) for the phase and then added accordingly to its mass balance equation.

5.5 Special Considerations

The virtual node approach for treating well conditions, as discussed above, is a widely applicable, flexible, and efficient method. The major advantages of the virtual node scheme, as compared with the conventional potential or mobility allocation method, are that it naturally includes “back flow” and incorporates contributions from all the connections to a well into the Jacobian matrix. The full Jacobian matrix and the full implicit scheme make the method very robust and stable in solving a multilayered well problem. One potential problem, however, is that since the wellbore node has a very small volume and high flow rates, it may cause some numerical difficulties during a Newton iteration. This problem can be alleviated by increasing the volume of the wellbore nodes by a factor of 10^2 to 10^3. This has the effect of adding a pseudo wellbore storage effect and dampens out oscillations in the Newton iteration (Wu et al., 1996). Except for the case of very small-scale transient behavior of wells, numerical tests indicate that this pseudo wellbore storage has almost no effect on the converged solution. It is recommended that the pseudo wellbore

storage approach should always be used for rate-specified pumping or injection problems. For pressure-specified pumping or injection problems, however, an infinitely large volume of a wellbore can also be used to obtain a much better numerical performance, as is normally done in a TOUGH2 (Pruess, 1991) simulation.

In addition, when applying the virtual node method, special attention is needed in specifying the "rock properties" for the virtual node. Capillary pressure and relative permeability functions are needed for all the well nodes and should be specified differently for a particular phase of pumping or injection, since the well node is regarded as is a normal grid block in the solution.

6. Example

I present an example for three-dimension flow of oil and water in a five-spot, two-phase problem of injection and production to demonstrate the application of the proposed well treatment scheme. Comparisons between laboratory (Gaucher and Lindley, 1960) and simulation (Coats et al., 1967) results in the literature are also discussed. This sample problem is used to examine the numerical scheme described in this work for handling pumping and injection wells. The example is a well-known test case for which laboratory and numerical simulations results are known (Gaucher and Lindley, 1960; Wu et al., 1994). The model domain consists of a quarter of a five-spot well pattern, with constant-rate water injection and pumping wells, located diagonally, as shown in Figure 1.

A three-dimensional 10 x 10 x 5 brick-type grid was used for this problem, with $\Delta x = \Delta y = 14.23$ m and $\Delta z = 1.22$ m. The formation was treated as homogeneous and isotropic. Even though the simulation studies (Wu et al., 1994 and 1996) treated the wells as fully penetrating, it has been found that in terms of oil recovery rates there is little difference between using fully or partially penetrating wells for this problem. Therefore, partially penetrating wells were considered for this test.

Detailed input parameters were given in Coats et al. (1967) and Wu et al. (1994). Some input data used for the simulation are given in Table 2. In addition, a special liquid and gas table is specified for liquid and gas relative permeability and capillary pressures in this problem,

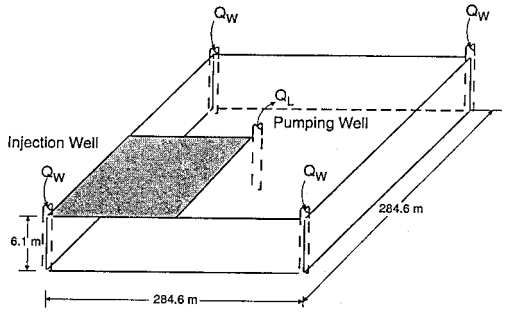

Figure 1. Schematic domain of five-spot, well-flow problem.

so that use of the Stone II function and three-phase capillary pressure correlation will give the exact forms of relative permeability and capillary pressure functions for this two-phase, water and oil flow problem. Capillary pressures are set to zero for well "rock" properties. The injection well is represented by an element with a constant water-injection rate and a volume-

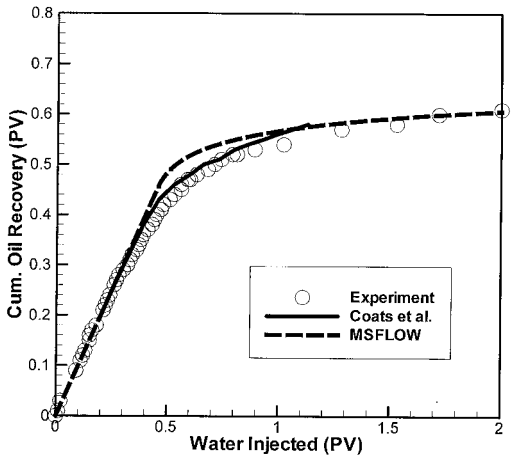

Figure 2. Cumulative oil recovery for the two-phase, five-spot, well-flow problem

increasing factor of 100. The well node is connected to only two elements, located at the two bottom layers of the grid. For this case, a uniform lumped well productivity parameter, PI = 24.85 (m^2), is used for all well block connections. Similarly, the and is also connected only to the two bottom layers of the grid.

The entire simulation was run to 178 years, and it took 233 time steps, 805 Newton iterations, and 760 seconds of CPU times on a 200 MH Pentium PC. A comparison of cumulative oil recovery versus injected water in terms of total pore volume (PV) is shown in Figure 2, indicating that the modeling results from the current work is in reasonable agreement with the results of the laboratory experiment and Coats et al. (1967). However, the current model predicts a little higher oil recovery over the range of 0.4 to 1.0 pore volumes (PV) of water injection. This is due to the differences in rock characteristic curves used in the present and the previous model (Coats et al., 1967).

7. Summary

This paper presents a virtual node method for handling well flow boundary conditions in reservoir simulations, including the formulation and numerical implementation of the methodology and discussions of applications. The virtual node scheme is a widely applicable, physically consistent approach that treats a wellbore as a single node or several computational nodes. The main advantages of this approach, compared with the conventional methods, are that the virtual node method can handle (a) back flow; (b) flow along wellbores for long, screened boreholes; and (3) different types of vertical, inclined, or horizontal wells. The numerical tests indicate that this method provides a natural and numerically efficient approach to handling well multiphase flow problems. It is also straightforward to implement the virtual node scheme in a thermal or compositional simulator.

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References

Aziz, K and A. Settari, *Petroleum Reservoir Simulation*, Applied Science Publishers LTD, London, 1979.

Coats, K. H., R. Nielson, M. Terhune, and A. Weber, "Simulation of Three-Dimensional, Two-Phase Flow in Oil and Gas reservoirs," *Soc. Pet. Eng. J.*, pp. 377-388, 1967.

Dikken, B. J., Pressure Drop in Horizontal Wells and its Effects on Production Performance, *JPT*, November, pp. 1426-1433, 1990.

Forsyth, P. A., Y. S. Wu and K. Pruess, "Robust Numerical Methods for Saturated-Unsaturated Flow with Dry Initial Conditions in Heterogeneous Media," Advances *in Water Resources*, Vol. 18, pp. 25-38, 1995.

Fung, L., A. Hiebert and L. Nghiem, "Reservoir Simulation with a Control-Volume Finite-Element Method," Paper SPE-21224, presented at the 11th SPE Symposium on Reservoir Simulation of the Society of Petroleum Engineers, Anaheim, California, 1991.

Gaucher, D. and C. Lindley, "Waterflood Performance in a Stratified Five Spot Reservoir, A Scaled Model Study," *Trans* AIME, Vol. 219, pp. 208-215, 1960.

Lee, S. H. and W. J. Milliken, "The Productivity Index of an Inclined Well in Finite-Difference Reservoir Simulation," Paper SPE-25247, presented at the 12th SPE Symposium on Reservoir Simulation of the Society of Petroleum Engineers, New Orleans, LA, February 28-March 3, 1993.

Peaceman, D.W., "A New Method for Representing Multiple Wells with Arbitrary Rates in Numerical Reservoir Simulation," Paper SPE-29120, presented at the 13th SPE Symposium on Reservoir Simulation of the Society of Petroleum Engineers, San Antonio, Texas, February 1995.

Peaceman, D.W., "Representation of a Horizontal Well in Numerical Reservoir Simulation," Paper SPE-21217, presented at the 11th SPE Symposium on Reservoir Simulation of the Society of Petroleum Engineers, Anaheim, California, February 1991.

Peaceman, D.W., "Interpretation of Well-Block Pressures in Numerical Reservoir Simulation with Nonsquare Grid Blocks and Anisotropic Permeability," Paper SPE-10528, presented at the Sixth SPE Symposium on Reservoir Simulation of the Society of Petroleum Engineers, New Orleans, LA, January 1982.

Peaceman, D.W., Interpretation of Well-Block Pressures in Numerical Reservoir Simulation, *Soc. Pet. Eng. J.*, *Trans.*, AIME, 253, pp. 183-194, 1978.

Pruess K., *TOUGH2—A General Purpose Numerical Simulator for Multiphase Fluid and Heat Flow.* Report LBL-29400, UC-251. Berkeley, California: Lawrence Berkeley National Laboratory, 1991.

Thomas, G. W., *Principles of Hydrocarbon Reservoir Simulation*, International Human Resources Development Corporation, Boston, 1982.

Wu, Y. S., *MSFLOW: Multiphase Subsurface Flow Model of Oil, Gas and Water in Porous and Fractured Media with Water Shut-off Capability*, Documentation and User's Guide, Walnut Creek, California 1998

Wu, Y. S., P. A. Forsyth, and H. Jiang, "A Consistent Approach for Applying Numerical Boundary Conditions for Multiphase Subsurface Flow," *Journal of Contaminant Hydrology*, 23, pp. 157-184, 1996.

Wu, Y. S., P. S. Huyakorn, and N. S. Park, "A Vertical Equilibrium Model for Assessing Nonaqueous Phase Liquid Contamination and Remediation of Groundwater Systems," *Water Resources Research*, Vol. 30, No. 4, pp. 903-912, 1994.