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SP INTERPRETATION FOR FORCED CONVECTION ALONG A VERTICAL FRACTURE ZONE

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ABSTRACT

SP interpretation can be ambiguous - giving highly non-unique physical models. The ambiguity can be limited by constraining the interpretation. If the SP anomaly arises from forced convection along a vertical fracture zone, an integral equation can be derived from which the SP anomaly may be calculated or inverted to a source. The source arises from divergence of a product of flow parameters, divided by the permeability. The formulation permits an assessment of the non-uniqueness present in SP interpretations for forced convection flow.

INTRODUCTION

The SP method has been used extensively and successfully to locate geothermally prospective zones (see, for example, Ross et al., 1990). Since the method is responsive to geothermal parameters such as fluid and heat flow and chemical environment, it has been thought to be a natural candidate for geothermal reservoir description and monitoring. With this motivation, the SP response of reservoir models has been numerically simulated by Ishido and Tosha (1998), and Nishi et al. (1998). Yet, as Tripp et al. (1998) observe, great care must be taken to constrain the inversion as much as possible, because of the level of modeling ambiguity present in SP interpretation. In fact, even with a half-space geometry, the flow parameters cannot be uniquely determined from SP data without additional information. From this beginning, determining the parameters for more realistic situations seems problematic. However, some appreciation of the possibilities and limitations of the SP method can be found by examination of severely constrained models. Such a model class will be examined in this paper. First, a brief recapitulation of the mathematical basis of SP interpretation is motivational.

The SP interpretation problem has two parts. The first part consists of determining equivalent electrical sources from the SP data. The second part is determining the physical processes and parameters, which give rise to the equivalent electric source. Both of these steps can be ill-conditioned and ill-posed in the absence of independent geological information. An examination of the pertinent equations will illustrate this point.

Determination of equivalent sources for a SP response can be formulated as the solution of a volume integral equation for the DC resistivity problem with the primary field being equal to zero. In this case the relevant equation is given by the equation

$$\Phi(\mathbf{x}_r) = ?\ G_0\,(\mathbf{x}_r,\mathbf{x}_s)S(\mathbf{x}_s)\,d\mathbf{x}_s, \qquad (1)$$

where $\Phi(\mathbf{x}_r)$ is the measured potential and $G_0\,(\mathbf{x}_r,\mathbf{x}_s)$ is the Green's function for the assumed background electrical conductivity distribution. Determining the distribution of $S(\mathbf{x}_s)$ from $\Phi(\mathbf{x}_r)$ via equation (1) constitutes the first part of the SP interpretation problem.

Decomposition of $S(\mathbf{x}_S)$ into physically significant terms constitutes the second part of the SP interpretation problem.

If the source of the SP anomaly is a flow with a potential ζ, then the source term is (Sill, 1983, eq. (7)),

$$S(\mathbf{x}_S) = - – (C/\rho). –\zeta - (C/\rho) –^{2} \zeta. \qquad (2)$$

where C is a cross-coupling term.

For the case when a gradient formulation is not possible and the flow is divergence - free, the source is (Sill, 1982, eq. (5)),

$$S(\mathbf{x}_S) = - –. (L_{\mathbf{v}}\mathbf{v}), \qquad (3)$$

where $L_{\mathbf{v}} = (L_{e,m} / k) = (\sigma C_{e,m}/k)$, k is the matrix permeability, **v** is the fluid velocity, $L_{e,m}$ is the Onsager cross-coupling coefficient linking fluid flow and the electrical potential, and $C_{e,m}$ is the streaming potential.

Whatever the form of the source, it is apparent that inverting to hydrologic parameters is an under-determined problem. Hence severe constraints are necessary for any meaningful solution to the problem.

One geological situation, which is often encountered, is a SP anomaly arising from forced convection along a near vertical fault. This problem, with its constrained geometry, gives a nice test case for examination.

Hence, we suppose that forced convection occurs along a fracture zone. The flow nears the surface over a limited area, and then forms a plume, which is dispersed away from the anomaly. We wish to measure the spontaneous potential $\Phi(\mathbf{x}_r)$ on the earth's surface and invert the measurements to give information concerning flow parameters along the fracture zone.

The strategy for this problem is to derive an appropriate integral equation by constraining the source term in (1). For any form of the source, we assume that physical properties and measurables vary only with respect to the vertical dimension. The resulting equation in velocity flow parameters is

$$\Phi(\mathbf{x}_r) =$$

$$? \vartheta (\mathbf{x}_r, z_S) [\quad (L_{e,m} v_{zS} / k)/ \quad z_S] dz_S. \qquad (4)$$

where

$$\vartheta (\mathbf{x}_r, z_S) = ? ? G_0 (\mathbf{x}_r, \mathbf{x}_S) dx_S dy_S \qquad (5)$$

is a function of the geometry of the fault cross-section and the electrical background.

If we assume that the fracture zone extends from z = a to z = b, then we can integrate (4) by parts. The resulting equation is

$$\Phi(\mathbf{x}_r) = [\vartheta (\mathbf{x}_r, z_S)] [L_{e,m} v_{zS} / k]|_a^b$$

$$- ?_a^b [\quad (\mathbf{x}_r, z_S)/ \quad c] [L_{e,m} v_{zS} / k] dz_S \qquad (6)$$

These equations give an appreciation of the level of ambiguity of SP interpretation when the background electrical conductivity distribution and the fault zone geometry are both known. In this case,

1) Equation (4) has the form of a Fredholm Integral Equation of the First Kind in terms of the fault parameters and is ill-posed and ill-conditioned. The null space contains the functions $[L_{e,m} v_{zS} / k]$ = constant. Hence only the vertical derivative of the quantity $P = L_{e,m} v_{zS} / k$ can be derived;

2) If P is known at the ends of the fault zone, then (6) may be solved for P along the fault zone;

3) If $P/ \quad z_S$ is determined from (4), then the values can arise from variations in $L_{e,m}$, v_{zS}, or k;

4) Variations in $L_{e,m}$ can arise due to phase transitions or changes in chemistry;

5) Variations in vertical velocity and permeability are coupled and are related to lithology variations;

(6) If we assume that discharge from the fault occurs only at a discrete point or along a line, and that the physical properties and the flow

velocity are constant along the fault up to the point of discharge, then (6) becomes

$$\Phi(\mathbf{x}_r) = \vartheta(\mathbf{x}_r,\mathbf{x}_s)\,[\,\delta P(z_s)], \qquad (7)$$

where $\delta P(z_s)$ is the step magnitude of $P(z_s)$ at z_s.

We will now demonstrate the importance of the initial assumption of background physical properties on the solution of equations (4) and (6).

INFLUENCE OF BACKGROUND PHYSICAL STRUCTURE

We will first remind the reader of some general relationships between the background physical property structure and the SP response and then discuss relationships which hold true for our specialized geometry.

Recall the equation linking gradient driven flows and the SP response:

$$-.\,(\sigma-\phi) = - -\,(C/\rho)\,.\,-\zeta - (C/\rho)\,-^{2}\,\zeta\,.$$

From this equation, we can conclude:

1) The source terms are inversely proportional to the electrical resistivity ρ. This means that sources are pronounced in conductive areas. However, the effect of the source may be attenuated by the conductive material.

2) Sources may be induced by gradients in either C or ρ which parallel pressure gradients.

3) The SP response ϕ is invariant to linear scaling of the ρ distribution. In the special case of a resistivity half-space, the SP response is invariant to the resistivity.

4) The SP response ϕ is invariant with respect to the ratio C/ρ.

5) The SP response $\phi \propto (C\,\Gamma_1 / a\,L_{e,m})$, where Γ_1 is the hydrologic flow source, $L_{e,m}$ is the hydrologic conductivity, and a is a scale factor (Sill and Killpack, 1982).

In the case of a fluid velocity source, the equation is

$$-.\,(\sigma-\phi) = - -\,(C_{e,m}/\rho)\,.\,\mathbf{v},$$

and similar conclusions hold.

In equation (6) the electrical structure of the earth determines the kernel $\vartheta(\mathbf{x}_r,z_s)$. Thus it determines the surface expression of an SP source. For example, a very conductive surface layer will mask an SP source at depth. Evaluating the effect of the electrical structure is best done numerically, for each individual case. However, some representative calculations illustrating some of the major issues encountered are in order.

For our calculations, we will use the 2D earth – 3D pressure source algorithm discussed by Sill (1983). Besides being convenient to use, this algorithm is appropriate for situations where convection is occuring along a major fault at a location of increased permeability, say at the point of intersection of a minor fault. This situation is encountered often.

Model 1:

This model is the baseline model and represents a shallow fluid discharge into an alluvial layer. We assume that there is no marked conductivity contrast between the basement units.

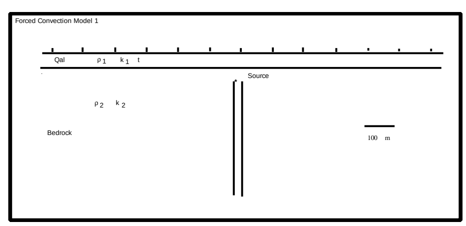

Figure 1

Figure 1 shows the structure and source geometry used for the simulations. Since we assume that the flow parameters of the fault are constant with depth, the only part of the flow which will give rise to a potential is the flow from the fault zone, which is the source denoted. Further, the fault zone itself is modeled by a low permeability zone. This has the effect of directing fluid flow away from the zone, which is what our model dictates. A cap zone can be permeable or not, and has the effect of varying the extent to which the flow can access the top layer. The cross-coupling coefficients of the various structural units are functions of the physico-chemical environment and their gradients parallel to the hydrologic flow will introduce electric source terms.

In all the simulations we will assume a pressure source strength of 1. Although a large number of parameter combinations are possible, for this study, we will confine our attentions to a small number of models which illustrate major themes. Tripp et al. (1999) contains a complete parametric study based on the same model family.

The parameters for this case are:

$\rho_1 = 50\ \Omega-$m; $k_1 = 1$ darcy ; $C_1 = 20$mv / atm
$\rho_2 = 500\ \Omega-$m; $k_2 = .001$ milli-darcies; $C_2 = C_1$

The permeability values are taken from Freeze and Cherry (1979) and are representative of the permeability of alluvium and granite. Although the actual permeability of a fault zone should be greater than that of the model, the model fault permeability was chosen to force flow out from the fault zone. The values of the cross-coupling coefficients are consistent with data contained in Morgan et al. (1989) for atmospheric temperatures. The location of the source corresponds with the intersection of the fault zone with the alluvial layer. The source is 125 m deep, while the fault zone is 25m wide.

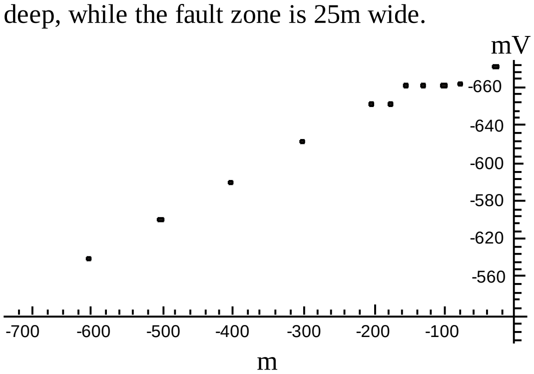

Figure 2

Figure 2 illustrates the response for a source of strength 1 L/s as a profile which is centered on the source. The anomaly is moderate and broad.

Since the source is located on the interface of the alluvium and the bedrock, we should expect little sensitivity to the relative values of the cross-coupling coefficients, which is confirmed by a numerical perturbation of the upper-layer coeffient to -20mv/atm, which would correspond to a slight increase in temperature.

Model 2:

This model is Model 1 with a low permeability cap, as is illustrated in Figure 3. Here the permeability of the cap has been changed to .001 millidarcies and its resistivity is 50 Ω–m. All cross-coupling coefficients are maintained at 20 mv/atm. The extent of the cap greatly affects the response, as is shown in Figure 4. An extended cap greatly accentuates the anomaly. This result is to be understood as an increase in source pressure, since the source strength has been maintained for the increased cap permeability.

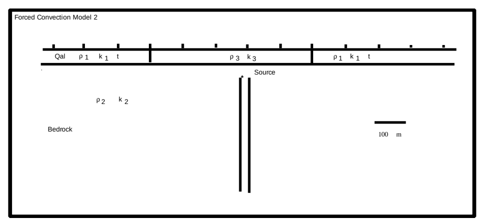

Figure 3

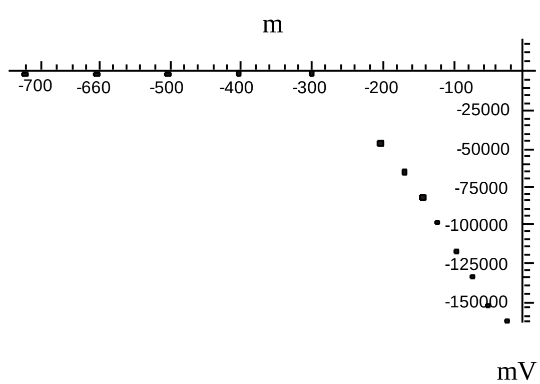

Figure 4

Model 3

Model 3 simulates a situation in which conductive cover is present, possibly due to rising steam, with subsequent condensation, gravity settling, and low temperature conductive alteration. The geometry of the units is the same as that of Model 1. The conductivity of the top layer was changed to 5 Ω–m, which gave a nearly constant response of -170 mV. To test the sensitivity to the cross-coupling coefficient, we then increased the coefficient of the top layer to 2000 mV/atm, an increase of a factor of 100. Figure 5 shows the subsequent negative anomaly, which has a mimimum value of -310 mV over the source. Again, as the geometry of the source suggests, the overburden cross-coupling coefficient has little effect on the subsequent magnitude of the anomaly.

Model 4

Model 4, shown in Figure 6, has a resistive contact which is reminiscent of a Range Front fault. The response, shown in Figure 7, is another "monster" negative anomaly, centered on the contact. When a conductive layer is added, as shown in Figure 8, the response becomes much attenuated, broadened, and is centered over the source, as shown in Figure 9.

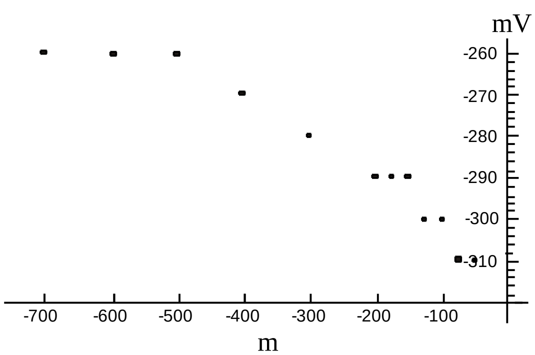

Figure 5

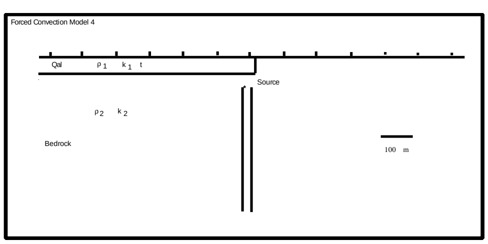

Figure 6

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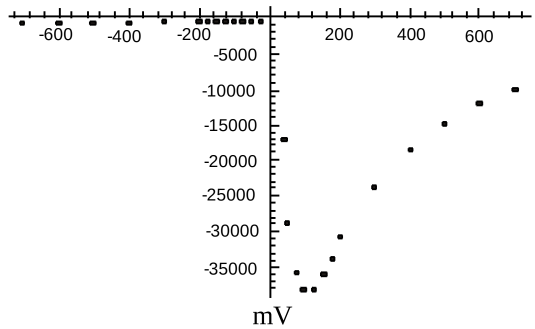

Figure 7

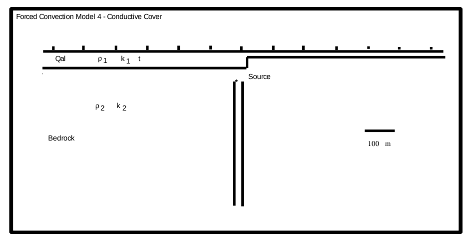

Figure 8

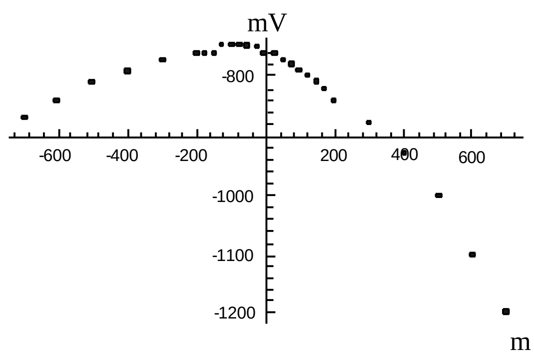

Figure 9

CONCLUSIONS

The basic SP equations permit specialization to particular geological structure - which permits an assessment of the structure of candidate solutions. This has been demonstrated for the case of forced convection along a fracture zone. Even when such radical constraints on an interpretation are made, the solution is highly non-unique, which suggests that great care must be exercised incorporating SP interpretations into an integrated reservoir or exploration model.

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