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Transient Pressure Analysis of Pressure Sensitive Naturally Fractured Reservoirs

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ABSTRACT

This paper presents the results of a study with the main aim of investigating the effect of considering the pressure sensitivity of the natural fractures of a fissured formation on the transient pressure analysis of well tests. Two motivations for this study were as follows. First, many (or most in some cases) of the producing reservoirs (geothermal and petroleum) are naturally fractured, showing a stress sensitivity behavior of the fractures. Second, some reservoir simulators consider the option of varying the fractured media properties with the exploitation time. This work discusses a mathematical model for the radial flow of a fluid through a pressure sensitive naturally fractured formation, with pseudosteady state matrix-fracture flow as described by Warren and Root. This model includes the effects of wellbore storage, a finite skin (damage) around the wellbore and high-velocity flow. A finite difference solution for this model is also briefly described. Simulated examples of drawdown tests are analized.

INTRODUCTION

It has been recognized that porous media are not always, rigid and non deformable. This problem is ussually handled by means of properly chosen "average" properties. This method only reduces the errors involved and generally and does not totally eliminate them.

This is especially true for naturally fractured reservoirs. A review of the well testing literature indicates that this variable rock properties problem for naturally fractured systems has not been addressed. In other words, the published treatments solve the diffusivity type coupled equations for the fractured and matrix systems, for homogeneous flow, assuming that the diffusivity is a constant independent of pressure. When both pressure and property changes of both systems are small, the constant property assumption is justified. If instead, rock and fluid properties changes are important over the pressure range of interest, these changes cannot be neglected and a variable property solution should be obtained.

The motivations for this work were basically two. The first was based on the fact that many hydrocarbon reservoirs and most geothermal reservoirs are naturally fractured, and it has been shown through petrophysical studies (Jones, 1975) and well testing (Adams, 1983), that the permeability of the fractures is a function of the effective stress (in a simplified manner defined as the difference between the external or overburden stress and the pore pressure). Whenever a formation presents this problem, it is necessary to quantitatively assess the impact of its stress sensitivity characteristics on the producing conditions of the wells.

The second motivation was based on the option of some commercial reservoir simulators, that consider the variation of the naturally fractured systems during its exploitation (Peng et al., 1983; GeoQuest, 1998).

The main aim of this paper is to present the results of a study that considers the pressure dependency of the fracture properties. Implicit in this work is the assumption of a linear-elastic medium with no hysterisis. Results are presented for drawdow tests, in terms of a psudopressure p_{pfD}. The investigation also includes the effects of wellbore damage, wellbore storage and high-velocity flow.

MATHEMATICAL FORMULATION

To formulate the mathematical model, the assumptions ussually made in well testing theory are applied. We assume horizontal flow with no gravity effects, a fully penetrating well, isothermal single-phase fluid obeying Forchheimer equation, an isotropic and homogeneous formation, and purely elastic fracture properties.

The assumption of horizontal flow is not quite valid because of changes in porosity and height with pressure. However, it is easily shown (Samaniego,

1974) that the vertical component of flow is negligible and can properly be neglected. Gavalas and Seinfeld also made this assumption (1973).

The fracture properties needed in the flow equation-porosity, permeability and fracture compressibility-as functions of pressure- are found scarcely in the literature; thus, data available for a carbonate formation was used (Jones, 1975).

The continuity equation for single-phase radial flow in the fractured system, when stress-sensitive and pressure-dependent fluid properties are considered is

$$\frac{1}{r}\frac{\partial}{\partial r}[r\, r(p_f)h(p_f)u] - u^* =$$

$$-\frac{\partial}{\partial t}[h(p_f)f_{fb}(p_f)\, r(p_f)] \qquad (1)$$

where u^* represents the rate of fluid transfer from the matrix to the fractures:

$$u^* = [q^* h(p_f)\, r]_{ma-f} \qquad (2)$$

where the mass rate of fluid transfer per unit area $q^*\, r$ can be expressed as

$$q^*\, r = \frac{r\, a\, k_{ma}}{m(p_{ma})}(p_{ma} - p_f) \qquad (3)$$

Forchheimer´s equation for horizontal flow can be written:

$$-\frac{\partial p_f}{\partial r} = \frac{m(p_f)}{k_{fb}(p_f)}u_r + b(k_{fb}(p_f))\, r(p_f)u_r^2 \qquad (4)$$

The matrix pseudosteady state flow equation can be expressed as

$$a\frac{k_m}{m(p_{ma})}(p_{ma} - p_f) = -(fc)_{ma}\frac{\partial p_{ma}}{\partial t} \qquad (5)$$

If it is assumed that matrix bulk volume is not a function of pressure, that the volume occupied by solid material in the fractures is a constant and that the reservoir is areally constrained (Jacob, 1940), then

$$h(p_{fi})[1 - f(p_{fi})] = h(p_f)[1 - f(p_f)] \qquad (6)$$

The necessary equation of state is the expression for the isothermal coefficient of compressibility for the liquid:

$$c(p_f) = \left[\frac{1}{r(p_f)}\frac{\partial\, r(p_f)}{\partial p_f}\right]_T \qquad (7)$$

If compression or expansion of the porous medium is limited to the vertical direction, the compressibility of the fractures is given by

$$c_f(p_f) = \frac{1}{V_{pf}} - \frac{\partial V_{pf}}{\partial p}$$

$$= \frac{1}{h(p_f)f_{fb}(p_f)}\frac{\partial\,[h(p_f)f_{fb}(p_f)]}{\partial p_f} \qquad (8)$$

For a specific reservoir, the fracture and fluid properties depend only on pressure for a given fluid. A pressure dependent pseudopressure has been defined by Raghavan et al. (1972), which adapted for this naturally fractured pressure sensitive flow problem is as follows

$$p_{pf}(p_f) = \int_{p_o}^{p}\frac{k_{fb}(p_f)\, r_f}{[1 - f_{fb}(p_f)]\, m(p_f)}dp_f \qquad (9)$$

Combining Eqs. 1 through 9 a and writing the resulting equation in dimensionless form, we get

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\, dk_D\frac{\partial p_{pfD}}{\partial r_D}\right)$$

$$-\frac{a\, r(p_{ma})k_{ma}}{}\frac{(p_{ma} - p_f)^2\, r_w\, h(p_i)[1 - f_{fb}(p_i)]}{m(p_{ma})q_i\, r(p_i)[1 - f_{fb}(p)]}$$

$$= w(p_i)h_D\frac{\partial p_{pfD}}{\partial t_D} \qquad (10)$$

where the dimensionless diffusivity h_D and the storage capacity evaluated at initial conditions are given by Eqs. 11 and 12.

$$h_D = \frac{h(p_i)}{h(p_f)} = \frac{\dfrac{f_{fb}(p_f)m(p_f)c_{ft}(p_f)}{k_{fb}(p_f)}}{\dfrac{f_{fb}(p_i)m(p_i)c_{ft}(p_i)}{k_{fb}(p_i)}} \tag{11}$$

$$w(p_i) = \frac{f_{fb}(p_i)c_{ft}(p_i)}{[f(p_i)c_t(p_i)]_t} \tag{12}$$

Other dimensionless groups used in Eq. 10 to make the results of this study generally applicable, are as follows:

Dimensionless time:

$$t_D = \frac{b_t k_{fb}(p_i)t}{[f(p_i)c_t(p_i)]_t m(p_i) r_w^2} \tag{13}$$

Dimensionless pseudopressure:

$$p_{pfD} = \frac{h(p_i)[1 - f_{fb}(p_i)][p_{pf}(p)_i - p_p(p_f)]}{a_o q_i r(p_i)} \tag{14}$$

Dimensionless radial distance:

$$r_D = \frac{r}{r_w} \tag{15}$$

For the flow problem to be specified completely, the differential equations should satisfy suitably prescribed boundary and initial conditions. In this problem, a constant mass-rate production is specified at the wellbore boundary and no-flow is specified at the outer boundary.

Since the fluid is withdrawn at the surface of a well rather than directly from the producing formation, there is a time lag before the withdrawal from the formation reaches a constant mass rate. This time lag is caused by the compressibility of the liquid and the storage capacity of the wellbore. This boundary condition, including high-velocity flow can be written as

$$-\left(dk_D r_D \frac{\partial p_{pfD}}{\partial r_D} \right)_{r_D=1} = 1 - \frac{c(p_{pf})[1 - f_{fb}(p_{wf})]m(p_{wf})k_{fb}(p_i)}{c(p_i)[1 - f_{fb}(p_i)]m(p_i)k_{fb}(p_{wf})}$$

$$C_D\left(\frac{\partial p_{pfD}}{\partial t_D} \right)_{r_D} \tag{16}$$

where the dimensionless wellbore storage C_D is defined as

$$C_D = \frac{V_w c(p_i)}{2ph(p_i) f_{fb}(p_i) c_{ft}(p_i) r_w^2} \tag{17}$$

The formation damage is represented by a zone of reduced permeability around the wellbore. The "damaged" zone has a fracture permeability that it is also stress dependent. It is assumed that the reduction in permeability with pressure in the damaged zone is the same ratio as in the rest of the formation:

$$\frac{k_{fbs}(p)}{k_{fbs}(p_i)} = \frac{k_{fb}(p)}{k_{fb}(p_i)} \tag{18}$$

where the subscript s means permeability of the damaged (or improved-permeability) zone

The external boundary condition for the closed reservoir is

$$\frac{\partial p_{pfD}(r_{De}, t_D)}{\partial r_D} = 0 \tag{19}$$

This completes the flow problem

The dimensionless groups used in this study are completely analogous to their counterparts in the constant-property Warren and Root liquid flow theory. If fracture and fluid properties were considered to be constants, the $p_{pfD}(r_D, t_D)$ solutions would be identical to the $p_{fD}(r_D, t_D)$ solutions for the constant-property liquid flow theory.

An implicit finite difference solution was obtained for the non-linear flow problem discussed in the previous section. The validation of the model was carefully taken care of.

RESULTS AND DISCUSSION

Table 1 presents the basic data regarding simulations carried out in this work. Each of them considered a drawdown test followed by a buildup test. Typical values for a naturally fractured formation of the storage capacity w and of the

TABLA 1. SIMULATIONS USED IN THIS STUDY

Simulation	Skin Factor	Dimensionless Wellbore Storage, C_D	Dimensionless Highy-Velocity flow Coefficient b_D	Warren and Root Constant Property Solution	Variable Properties
1	0	0	0	x	
2	0	0	0		x
3	5	0	0	x	
4	10	0	0	x	
5	5	0	0		x
6	10	0	0		x
7	0	10	0	x	
8	0	10^2	0	x	
9	0	10	0		x
10	0	10^2	0		x
11	5	10	0	x	
12	5	10	0		x
13	0	0	0.83	x	
14	0	0	0.83		x
15	5	10	0.83	x	
16	5	10	0.83		x

interporosity flow coefficient l of $5x10^{-2}$ and $6x10^{-6}$ were used.

Fig. 1 presents drawdown results for simulations 1 and 2 graphed, for this and the remaining cases, in terms of the dimensionless pseudopressure $p_{pfD}(1,t_D)$ versus dimensionless time t_D in a semilogarithmic scale. In all figures in this paper, continuous lines indicate the $p_{fD}(1,t_D)$ Warren and Root solutions, while individual symbols represent the $p_{pfD}(1,t_D)$ of this study. Note that the computed $p_{pfD}(1,t_D)$ solutions fit the Warren and Root $p_{fD}(1,t_D)$ results extremely well for transient flow conditions.

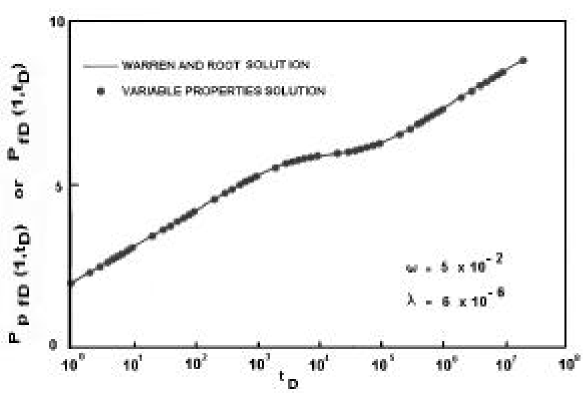

Fig. 1. Drawdown tests, simulations 1 and 2

In view of the foregoing discussion, the transient well behavior for flow in a pressure dependent naturally fractured reservoir can be expressed for all practical purposes by

$$p_{pfD}(1,t_D) = p_{fD}(1,t_D) \tag{20}$$

Thus, it has been concluded that $p_{pf}(p)$ pseudopressure linearizes this problem well for the case of constant sand-face mass flow.

It has been found convenient (Samaniego, 1974; Samaniego et al., 1977) to express Eq. 20 in terms of a normalized pseudopressure $p^1_{pf}(p)$ defined by:

$$p^1_{pf}(p) = \frac{[1 - f_{fb}(p_i)]m(p_i)}{k_{fb}(p_i)r(p_i)} p_{pf}(p) \tag{21}$$

Thus, Eq. 20 can be written in terms of $p^1_{pf}(p)$, and the flow conductivity, $k_{fb}(p_i)h(p_i)$, can be determined accurately from a drawdown test.

Fig. 2 shows the results for a damaged reservoir. This indicates that variable property liquid flow in a composite naturally fractured reservoir is no more rate-sensitive than in a non-damaged reservoir. Also plotted in this figure are the results in accordance to the model of Warren and Root. The results when expressed in terms of $p_{pfD}(1,t_D)$, are essentially the same as the $p_{fD}(1,t_D)$ results of conventional liquid flow. This indicates that for times long enough that the wellbore response is under the influence of the matrix-fracture systems, the straight line of the drawdown graph is parallel to the $p_{fD}(1,t_D)$ solution for $s = o$, indicating that the flow conductivity at initial conditions, $k_{fb}(p_i)h(p_i)$, can be determined accurately from the slope of a drawdown test in terms of $p^1_{pf}(p)$.

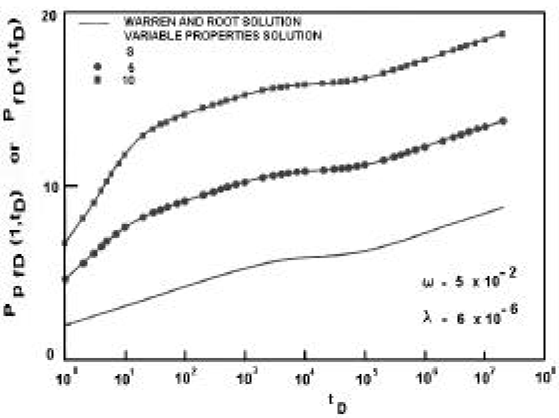

Fig. 2 . Drawdown tests, simulations 1 and 3-6

In view of the foregoing discussion, Eq. 20 can now be written for transient flow conditions as

$$p_{pfD}(1,t_D) = p_{fD}(1,t_D) + s \tag{22}$$

Immediately after production begins, wellbore storage controls the transient behavior of a well test. This causes a delay of pressure response at the wellbore.

Fig. 3 shows drawdown results for simulations 1 and 7 through 10. For all practical purposes, the computed points fall on the $p_{fD}(1,t_D)$ simulated Warren and Root solutions. After the wellbore storage dies out, the $p_{fD}(1,t_D)$ solution for $C_D = o$ is approached and, thus, the proper semilog straight line is reached. This indicates that, in the presence of wellbore storage, when this pressure dependent flow is expressed in terms of $p_{pfD}(1,t_D)$, the result is essentially the same as the $p_{fD}(1,t_D)$ solutions.

Fig. 4 presents drawdown results of simulations 1, 3, 5 and 11 through 18. Once again, for all practical purposes, the computed points fall on the $p_{fD}(1,t_D)$ simulated Warren and Root solutions. This indicates that, even in the presence of wellbore

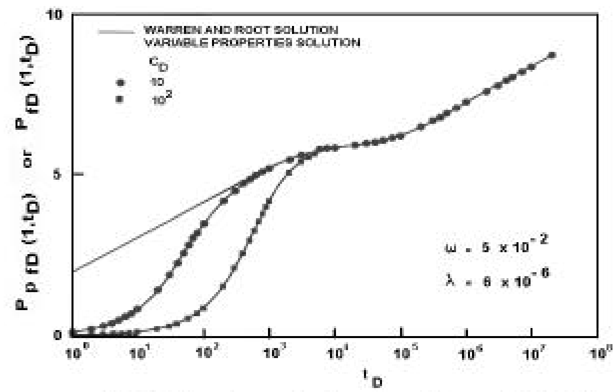

Fig. 3 . Drawdown tests, simulations 1, 7-10

storage and skin effect, when this pressure. dependent flow is expressed in terms of $p_{pfD}(1,t_D)$, the result is essentially the same as the $p_{fD}(1,t_D)$ solutions. Again, this is only true before the outer boundary affects the pressure.

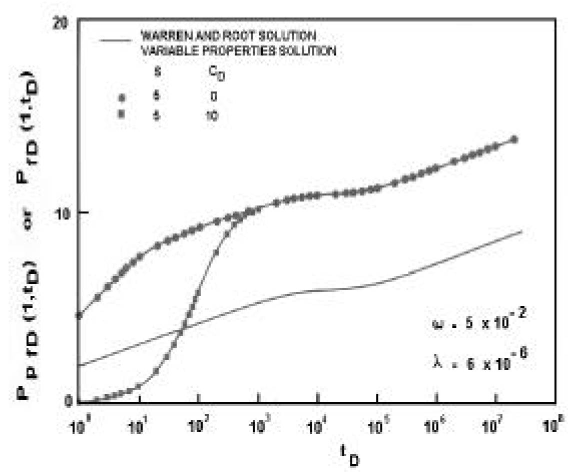

Fig. 4 . Drawdown tests, simulations 1,3, 5, 11 and 18

It has been concluded that flow towards a well completed in a naturally fractured reservoir occurs in most cases under high-velocity flow conditions. The constant properties pressure transient analysis of naturally fractured reservoirs considering the effect of high-velocity flow has been studied by Villalobos et al. (1989). Fig. 5 shows results of the present work, considering high-velocity flow. Analysis of results show that the pressure behavior of a well undergoing unsteady state high-velocity flow, when expressed in terms of $p_{pfD}(1,t_D)$, are, at least for this level of high-velocity, essentially parallel to the $p_{fD}(1,t_D)$ solutions. The late times semilogarithmic straight line is displaced vertically by an amount which corresponds to the extra pressure drop due to high-velocity flow, s_{hv}. Thus, for this cases of negligible skin factor, Eq. 20 can be expressed as

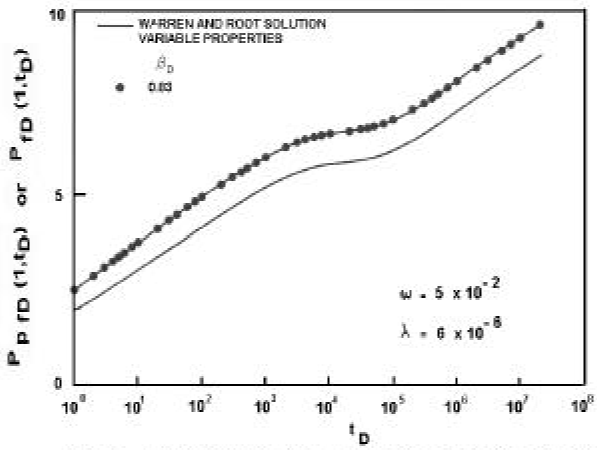

Fig. 5 . Drawdown tests, simulations 1,3, and 14

$$p_{pfD}(1,t_D) = p_{fD}(1,t_D) + s_{hv} \qquad (23)$$

Last, Fig. 6 presents the most general drawdown results for simulations 1, 3, 5, 15 and 16, that include the pressure dependency of the properties, wellbore damage, wellbore storage and high-velocity flow. It can be observed as already discussed with regard to the results of Fig. 5, that for this case of non zero wellbore damage, the pseudopressure difference at late times is equal to the pseudoskin factor due to high-velocity flow, s_{hv}. Thus, the late time pressure behavior of a damaged well in a pressure dependent naturally fractured reservoir, under the effect of high-velocity flow, is again approximately described by a semilog-straight line with the proper slope, but displaced with respect to the no skin case, by an amount equal to the total skin factor, s_t, defined as

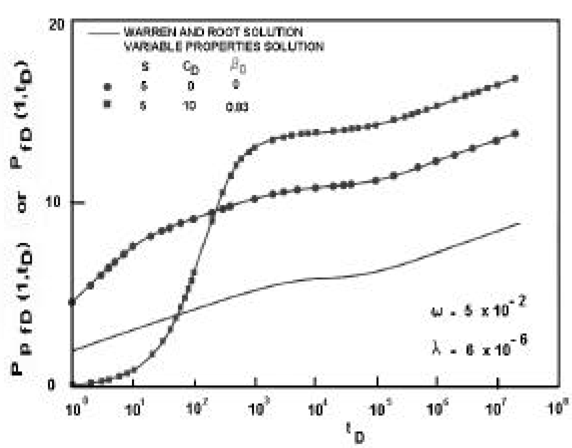

Fig. 6 . Drawdown tests, simulations 1.3.5.15 and 16

$$s_t = s + s_{hv} = s + D(p_f)q_i \qquad (24)$$

Summarizing, $k_{fb}(p_i)h(p_i)$ can be obtained from a drawdown test; the skin factor s_t and the turbulent term coefficient $D(p_f)$ may be obtained by means of the Ramey´s method (1965). An alternative analysis procedure can be that of Camacho et al. (1996).

CONCLUSIONS

The main purpose of this work has been to present the results of a systematic study of the transient pressure analysis of liquid dominated wells, producing from naturally fractured pressure dependent wells. The motivations for the research subject of this study came from the facts that many reservoirs (geothermal and petroleum) are naturally fractured, showing a stress sensitivity behavior of the fractures, and some reservoir simulators consider the variation of the fractured media properties with the exploitation time.

Based on the material presented in this paper the following conclusions are pertinent:

1. A mathematical model was derived for the radial flow of liquids through a pressure sensitive naturally fractured reservoir, with pseudosteady-state matrix-fracture flow as described by Warren and Root.
2. The model includes the effects of a finite skin (damage) around the wellbore, wellbore storage and high-velocity flow.
3. The $p^1_{pf}(p)$ pseudopressure is an excellent “linearizing” tool for constant mass-rate transient tests in naturally fractured pressure dependent reservoirs.
4. The $p^1_{pf}(p)$ is a unique function for each set of fluid and fracture properties, so it is necessary to have data on these properties to take advantage of this linearization.
5. For transient flow conditions, all $p_{pfD}(1,t_D)$ solutions were virtually identical to the classical constant-property $p_{pfD}(1,t_D)$ solutions.

NOMENCLATURE

$c(p)$	=	pressure-dependent fluid compressibility
C_D	=	dimensionless wellbore storage constant, Eq.17.
$h(p_f)$	=	pressure dependent thickness, Eq. 6
k_D	=	ratio of the initial permeability of the formation at any radial distance to the undamaged initial permeability, $k_{fb}(p_i,r)/k_{fb}(p_i)$
$c_t(p_i)$	=	total compressibility of the system
$k_{fb}(p)$	=	pressure dependent permeability
p	=	pressure
$p_{pf}(p)$	=	pseudopressure, Eq. 9
$p^1_{pf}(p)$	=	normalized pseudopressure, Eq. 21
$p_{pfD}(r_D,t_D)$	=	dimensionless pseudopressure, Eq. 14
q_i	=	initial well flow rate
q^*	=	matrix to fracture rate of fluid transfer, Eq. 3
r	=	radial distance
s	=	van Everdingen and Hurst skin factor
s_{hv}	=	high-velocity pseudo-skin factor
s_t	=	total skin factor, Eq. 24
t	=	time
t_D	=	dimensionless time, Eq. 13
u	=	velocity
u^*	=	rate of fluid transfer from the matrix to the fractures
v_w	=	wellbore volume
a_o	=	pressure unit conversion factor (141.2 for the English system)
$D(p_f)$	=	turbulent term coefficient
b	=	velocity coefficient
b_t	=	time unit conversion factor ($2.637\text{x}10^{-4}$ for the English system)
d	=	L.I.T (laminar, intertial and turbulent) correction factor for Darcy´s law.
l	=	interporosity flow coefficient
$m(p_f)$	=	pressure-dependent viscosity
$r(p_f)$	=	pressure-dependent density
s	=	external (overburden) stress
s'	=	net effective stress
$f(p_f)$	=	pressure dependent porosity
w	=	dimensionless fracture storativity

Subscripts and Superscripts

b	=	bulk
D	=	dimensionless
f	=	fracture
fb	=	fracture referred to bulk volume
hv	=	high-velocity
i	=	initial
ma	=	matrix
w	=	wellbore
s	=	skin
t	=	total

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