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MANAGEMENT OF DRY STEAM PRODUCING GEOTHERMAL FIELDS AND VOLCANIC EXPLOSION FORECAST: (T,P) SOLITARY WAVES, A NEW POTENTIAL SOURCE OF SIGNIFICANT INFORMATION

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ABSTRACT

Phreatic explosion craters such as those recognized in the Larderello area, or the recently reconstructed history of repeated paroxysmal events in The Geysers, plus as yet unconfirmed evidence of phreatic explosion crater (oral comm. J. Hulen), point out a continuous set of states of nature ranging from stable steady to changing states and beyond, to paroxysmal events up to mechanical destabilization of the crust (large explosive eruptions) in natural systems from geothermal (hydrothermal) to strictly speaking volcanic ones.

A class of conceptual models that appear to aptly describe states of nature intervening between mechanically stable steady states and fully unstable states is presented in this paper.

Attention is focused on analytical rather than numerical models, i.e., on representing the basic features under as general as possible conditions rather than defining details of particular cases, considering at once as broad as possible classes of models, rather than going after detailed modelling of some particular system. This choice fits in with the requirements of the Bayesian approach in the presence of no or limited information on the specific system under consideration, re *principle of simplicity*.

Transfer of mass and/or energy (as work and thermal energy) from deep in the Earth to its surface requires some build up of energy at depth, which can bring the considered natural system to instability and to sustain changes by internal forces.

The theory shows that energy build up in the subsurface (in the form of thermal energy, fluid pressure, volume increase and related stress etc build up) can give place to heat-stress (and for that matter, porosity-permeability etc) solitary waves that travel to the surface. In particular, the pressure gradient generated by the wave is as more destabilizing as closer to the surface and to gaseous state is the involved fluid. Waves and related pressure gradient profiles range from peak to step-like (from "shock waves" to "solitons") depending on whether the energy build up is peak or step-like, and so carry with other information on the state of the system at depth, hence about subsurface conditions and their change.

To any extent that solitary waves are forerunners of change towards unstable mechanical conditions their theory is relevant to forecasting volcanic explosions. With regards to geothermal systems, solitary waves may signal underground energy build up, e.g., shed light on energy transfer to the system from its thermodynamic Universe. More generally concerning observable information, it hints to some of many ways that monitoring both apparent and subtle mass and energy surface emission may contribute to the understanding of what goes on in the subsurface. Closer discussion of such a subject is premature as long as the practice remains limited to considering only well data and systematic investigation of the distribution of surface emissions is hardly carried on.

INTRODUCTION

The focus of the present paper is on the avenue of thought pointed to by its title, inasmuch as it generates possible conceptual models of generation of solitary (P,T, ...) waves travelling from deep in the Earth to its surface. Inasmuch as 1st, it is likely to occur on transition from mechanically stable to unstable states, and 2nd, it is expected to be recognizable based on surface observations, it is also a significant process. Explosive volcanic eruptions forecast appears to be its major prospective application, however, its potential to afford inferring subsurface conditions based on surface observations is possibly significant with a view at the management of DSP (Dry Steam Producing) geothermal fields.

Many a similarity exist between central volcanoes and DSP geothermal fields. They both feature broadly centripetal water circulation with associated lower fluid pressure and anomalous geothermal (high) and pressure (low) gradient at center, associated with local transfer of thermal energy and mass to the Earth's surface under mechanically steady conditions in time, and both undergo upsurges of fluid emission and even explosions. Even more significant with a view to the subject matters, on the one hand DSP geothermal fields undergo paroxysmal emission of fluids unloading energy from the system to the Atmosphere (see geologic section, Figure 1a, and block diagram, Figure 1b), on the other hand materials taken to the Earth's surface by volcanic explosions range from all non volcanic (e.g. in many phreatic explosions) through older volcanic and to fresh magmatic materials. With reference to the two major and longest known geothermal fields worldwide, Larderello (Tuscany, Europe) and The Geysers (California, USA), the evidence comes respectively from a phreatic explosion crater (Marinelli, G., verbal comm.n) and from fluid inclusions investigation (e.g., Moore et al., 1995 and 1998) as well as fields observations suggestive of phreatic explosion (Hulen, J.B., verbal comm.n, to be confirmed, Hulen et al., 1997).

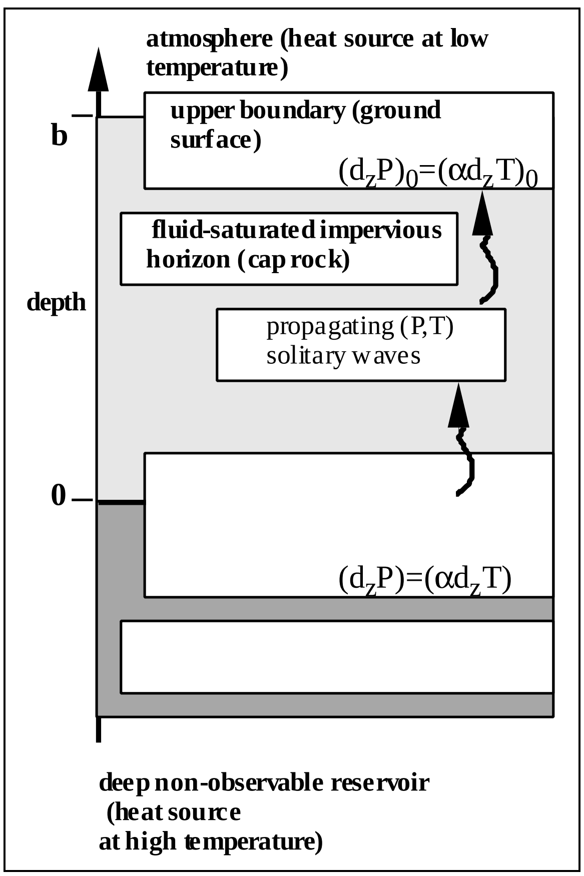

Figure 1a. Conceptual geological section of a stratified system for a hydrothermal domain.Westerly granite is assumed as a rock type representative of the horizon overlying the aquifer. Rock deformation-fracturing is supposed to start on the boundary aquifer-caprock as a buried thermo-mechanical source arises in terms of fluid-rock temperature and pore fluid pressure changes over a steady state regime characterised by initial values $T_0 = 400^0 C$ *and* $P_0 = 2.5 \cdot 10^7\ Pa$. *This steady state corresponds to a supercritical state of water under a vertical geothermal gradient of about* $1^0 C / 7.5 m$ *at a depth of* $2.5 - 3\ Km$.

Close up study of specific geothermal systems, as seen in terms of varying (e.g., increasing) energy in the subsurface, is likely to require, respectively: 1st level, the approximation describing them as a mosaic of sub-systems -- specifically: geothermal phases -- at stable steady conditions, see, e.g., Figure 1b; 2nd, solitary waves supply a class of analytical models that aptly describe the relationship between subsurface thermodynamic conditions and observable processes at the ground surface, as required to infer the formers from the latters. Eventually, when destabilization takes place, 3rd, level of subsurface energy, the most general (thermodynamic) model says performed work shall be lesser or equal to the change of the system internal energy between initial and final state. More detailed fluid-dynamic models will be feasible on a case by case basis, subject to available data describing the considered system.
Figure 1b points out possible interactions between the system and its Universe. Possible complications are symbolized by the "AQUIFER R'" box, and can be disregarded for the purpose of this paper, that is, assessing the impact of solitary waves theory on possible study of the above said second level process transitional to destabilization. The simple model of Figure 1a is the same as used by Facca and Tonani (1961).

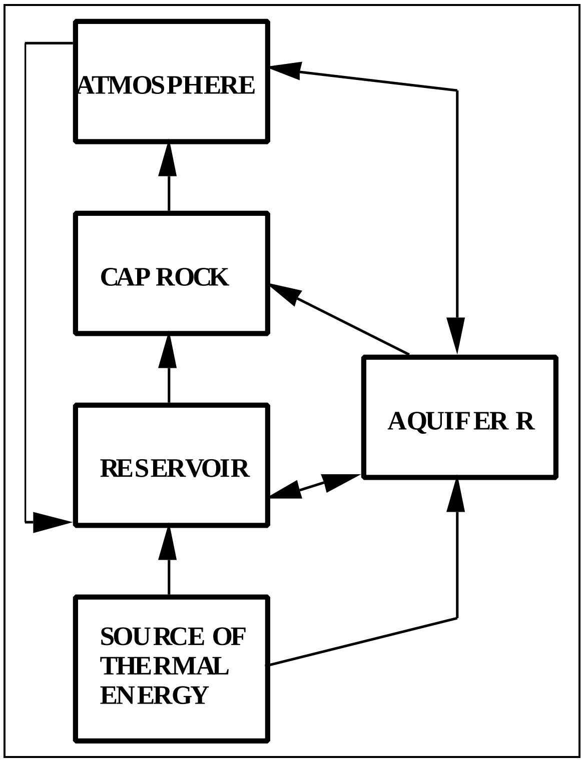

Figure 1b. Block diagram of the basic (simplest) model of geothermal field.

The set of possible models of **a n y** DSP geothermal system fall into a domain defined by the possible combinations of two idealised elements, the *impervious homogeneous cap rock* and the *underlying porous-permeable homogeneous "reservoir"* of thermal energy.
The total thickness of the considered system can be arbitrarily defined for each individual possible case. Say it is about the same as the maximum drillable depth under the specific case, and let us take it as our unit thickness. In principle, the model may be fully defined by fixing the ratio between cap rock and "reservoir" thickness, which in the light of the former condition will be represented by the thickness of either the cap rock or the reservoir expressed as a fraction of the total considered thickness, i.e., of unity. This also will rarely be relevant to such general argument as the subject of the present paper, a fact depicted by showing only the top of the reservoir (see, e.g., Figure 1a).
Experience of use in previous papers by Facca, Tonani or both authors suggests that the above approach to modelling geothermal fields is an effective way of defining the set of possible models.

As long as the maximum commercially drillable depth is small compared to total depth to the heat source feeding thermal energy to the system, the heat flow through the system is an externally fixed condition.

The block diagram of Figure 1b points out the interactions of CAP ROCK and RESERVOIR among themselves and with their Universe, i.e., of cap rock with the Atmosphere and of reservoir with the source of thermal energy. Other interaction symbolised by AQUIFER R' in Figure 1b can be ignored for our purposes. In the two examples taken as reference here, Larderello and The Geysers, cap rock and reservoir are defined differently: as a geologic formation (flysch) other than the "reservoir" formation (mesozoic limestones and dolomites) in the former case, as two different states of the same formation (Franciscan sandstones respectively sealed, not sealed by hydrothermal deposits, a process termed self-sealing) in the latter case.
In this simple case the set of possible models represented by the **open** set (0,1) can be related to the possible values of one variable which is a fraction of unity depending on the thickness of the cap rock and the reservoir.

THEORY OF HYDRAFRAC AND RELATED PAROXYSMAL FLUID EMISSION

On the basis of the block diagram of Fig. 1b, a class of theoretical models which aptly decribes cyclic episodes of rock deformation-fracturing through the cap rock of Fig. 1a and related catastrophic emission of fluids may be obtained by:

1. considering Burgers' equation, which is derived as a combination of the fluid-rock energy and stress-diffusion equations describing subsurface coupled fluid-rock thermo-dynamics and related hot and pressurized fluid movement, and

2. hypothesizing the physical nature of the buried thermo-mechanical energy source which is supposed to exist within a layer in the proximity of the aquifer roof of Figure 1a.

Derivation of Burgers' equation and buried (T,P) source

As a hydrothermal system is perturbed from its steady state and is characterized by episodes of hydrothermal brecciations and paroxysmal emission of fluids which may generate phreatic explosion craters, the associated thermo-dynamical behaviour and transition from stable steady to unstable states can be described by adopting modern thermo-poro-elasticity theory and its two heat-like equations (Rice and Cleary, 1976; McTigue, 1986; Natale and Salusti, 1996; Natale et al., 1998):

$$(1) \qquad \frac{\partial P}{\partial t} - h\frac{\partial^2 P}{\partial z^2} - a\frac{\partial T}{\partial t} = 0$$

(2) $$\frac{\partial T}{\partial t} - k\frac{\partial^2 T}{\partial z^2} - b\frac{\partial P}{\partial z}\frac{\partial T}{\partial z} - c\left(\frac{\partial P}{\partial z}\right)^2 = 0$$

Equation (1) has been derived by McTigue (1986) and corresponds to the stress-diffusion equation. $P[Pa]$ and $T[{}^0C]$ are the pore fluid pressure and the fluid-rock temperature variables respectively, $h[m^2/s]$ the fluid diffusivity and $a[Pa/{}^0C]$ the source term due to differential fluid-rock thermal expansivity.

Equation (2) has been formulated by Natale and Salusti (1996) and Natale et al. (1998) after Rice and Cleary (1976) and Bejan (1984) and corresponds to the fluid-rock energy equation under the assumption of a local fluid-rock thermal equilibrium. $k[m^2/s]$ is the diffusive thermal diffusivity, $b[m^2/Pa \cdot s]$ is the convective thermal diffusivity and $c[m^2/Pa^2 \cdot s \cdot {}^0C]$ is the dissipative diffusivity due to fluid-rock friction (Table I, Appendix). Derivation of equation (2) can be found in Natale (1998).

Let us now assume that over a pre-existing steady state regime, characterised by P_0 and T_0 values, a buried thermo-mechanical source corresponding to pore fluid pressure P_I and associated fluid-rock temperature T_I changes is built up within a layer in the proximity of the roof aquifer of Figure 1a, yieding as boundary and initial conditions:

(3) $$\begin{array}{lll} T = T_0 + T_I & P = P_0 + P_I & z = 0 \\ T = T_0 & P = P_0 & 0 < z < b \end{array}$$

As in equation (1) fluid diffusivity h is negligible compared to the term a (see Table I), Natale and Salusti (1996) have shown that, for rock types with medium-low permeability values as it must be the case for the cap-rock overlying the aquifer of Figure 1a, the stress-diffusion equation can be simplified as follows:

(4a) $$\frac{\partial P}{\partial t} \cong a\frac{\partial T}{\partial t}$$

entailing, with the initial conditions (3), the solution:

(4b) $$P - P_0 = a\left(T - T_0\right)$$

Equations (1) and (4b) can be combined, obtaining:

(5a) $$\frac{\partial T}{\partial t} = a b^I \left(\frac{\partial T}{\partial z}\right)^2 + k\frac{\partial^2 T}{\partial z^2}$$

with $b^I = a c + b$

which, by z-deriving, gives:

(5b) $$\frac{\partial}{\partial t}\frac{\partial T}{\partial z} = 2 a b^I \frac{\partial T}{\partial z}\frac{\partial^2 T}{\partial z^2} + k\frac{\partial}{\partial z}\frac{\partial^2 T}{\partial z^2}$$

corresponding to Burgers' equation for $c(z,t) = -2ab^I \partial_z T \approx -2b^I \partial_z P$ (Whitham, 1974). Thus, by adopting the change of variable $c(z,t) = -2ab^I \partial_z T \approx -2b^I \partial_z P$, (5b) can be rewritten as:

(5c) $$\frac{\partial c}{\partial t} - k\frac{\partial^2 c}{\partial z^2} + c\frac{\partial c}{\partial z} = 0$$

Equation (5b), or alternatively (5c), describes rock deformation-fracturing and concurrent hot and pressurised fluid movement through a thermo-elastically reactive rock on the basis of the balance of:

1. a nonlinear propagation factor $2ab^I \partial_z T \partial_{zz} T$ (or $2b^I \partial_z P \partial_{zz} T$ in the light of expression (4b)) related to convective thermal energy migration, dissipative fluid diffusivity due to fluid-rock friction and thermo-elastic response of the overbuden rock and which can be rewritten as $-2b^I m V_D \partial_{zz} T / K_f$ by introducing Darcy's law $V_D = -K_f \partial_z P / m$, and

2. a diffusive factor $k \partial_z \partial_{zz} T$ related to diffusive thermal energy migration.

The above factors balance each other in such a way that some analytical solution can be obtained, corresponding to the propagation of different sorts of thermo-mechanical or (T,P) solitary waves, depending on both the role played by the dissipative term, present in the convective propagation factor, and on the assumption of the physical feature which controls fluid-rock temperature and pore fluid pressure and related local (T,P) gradient changes at the buried thermo-mechanical source.

Upsurge of (T,P) solitary shock waves and related surface signals

Let the buried thermo-mechanical source be controlled by a Dirac function, which instantaneously damps out fluid-rock temperature and pore fluid pressure gradient changes, yielding as initial condition to be associated with Burgers' equation (5c):

(6) $c_{SW}(z,0) = -2 a\, b^I T_I\, d(z) \cong -2 b^I P_I\, d(z)$

Natale and Salusti (1996) have found that in this case the solution of Burgers' equation (5c) is:

(7) $c_{SW}(z,t) = \begin{cases} z/t & 0 \le z \le \sqrt{4 a b^I T_I t} \\ 0 & otherwise \end{cases}$

Solution (7) above has the dimensions of a velocity and corresponds to the propagation of a thermo-mechanical solitary shock wave along the positive z direction, travelling upwards with a rapid weakening of its velocity and amplitude. Darcy's velocity front $V_{D_{SW}} = -K_f \P_z P / m = K_f c_{SW}(z,t) / 2b^I m$ is formed at $z_{f_{SW}} = \sqrt{4 a b^I T_I t}$, the velocity value across the wave front jumps from zero to $c_{f_{SW}} = \sqrt{4 a b^I T_I / t}$ and the wave front moves at speed $V_{SW} = \sqrt{a b^I T_I / t}$ (Table II).

In Figure 2a-b the vertical profile of the T,P wave $c_{SW}(z,t)$ vs z and the related surface signal $c_{SW}(z = b,t)$ vs t are reported, respectively, for the Westerly granite.

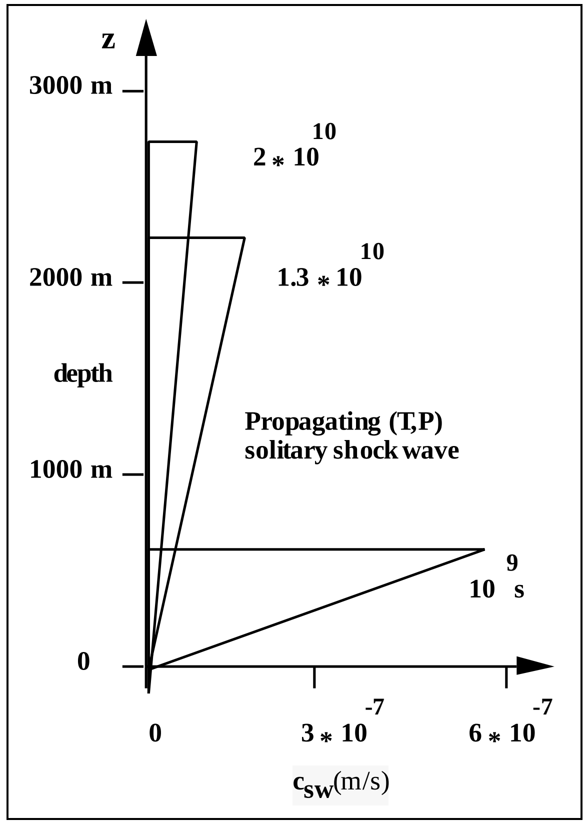

Figure 2a. Vertical profile of the (T,P) solitary shock wave for the Westerly granite.

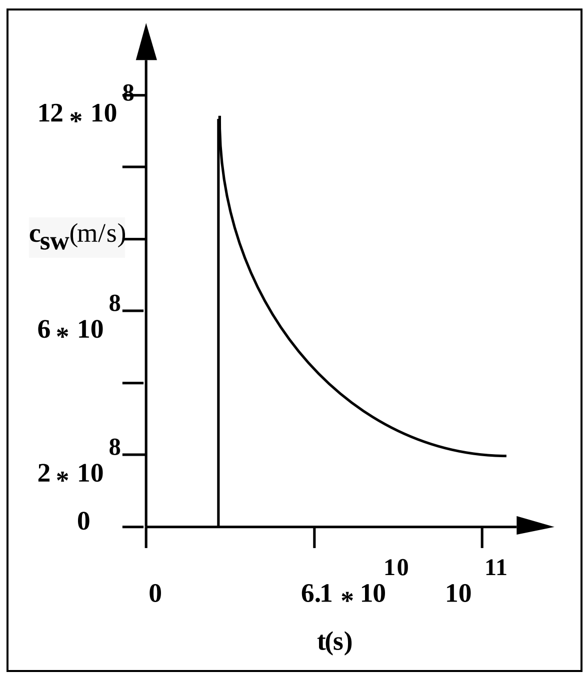

Figure 2b. Related (T,P) solitary shock wave surface signal.

Upsurge of (T,P) solitons and related surface signals

Let the buried thermo-mechanical source be controlled by a Heaviside step function, which maintains through time the fluid-rock temperature and pore fluid pressure gradient changes, yielding as initial condition to be associated with Burgers' equation (5c):

$$c_S(z,0) = -2\alpha\beta^I \partial_z T = -2\beta^I \partial_z P = \psi(z)$$

(8)
$$\text{with } \psi(z) = \begin{cases} 0 & \text{for} \quad z > 0 \\ c_{0_S} > 0 & \text{for} \quad z \le 0 \end{cases}$$

Garcìa and Natale (1998) have found that at the wave front, i.e. for $0 < z/t < c_{0_S}$, a solution of Burgers' equation (5c) is:

(9)
$$c_S(z,t) = \frac{c_{0_S}}{2}\left\{1 - \tanh\left[\frac{c_{0_S}}{4k}\left(z - \frac{c_{0_S} t}{2}\right)\right]\right\}$$

Solution (9) has the dimensions of a velocity and corresponds to the propagation of a thermo-mechanical soliton along the positive z direction, travelling upwards at constant wave velocity and amplitude. The wave velocity is constant and given by $V_S = c_{0_S}/2$, whilst Darcy's velocity can be obtained through the pore fluid pressure gradient carried up by the wave, namely $V_{D_S} = -K_f \partial_z P/m = K_f c_S(z,t)/2\beta^I m$. Finally, the amplitude of the soliton is c_{0_S} which remains constant through time (Table III and Figures 3a-b).

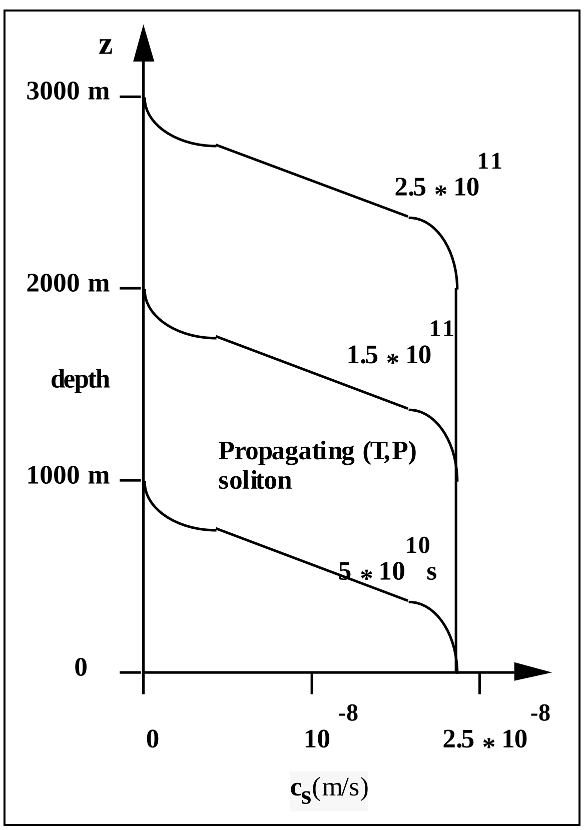

Figure 3a. Vertical profile of the (T,P) soliton for the Westerly granite. c_S is proportional to the pressure gradient.

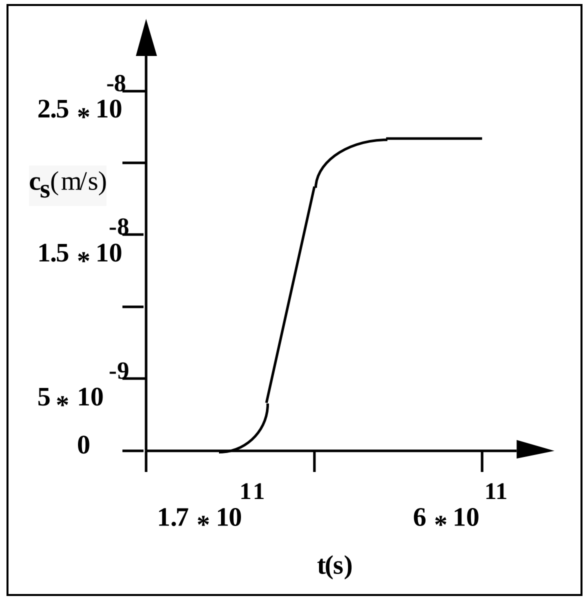

Figure 3b. Related (T,P) soliton surface signal.

CONCLUSIONS

The focus of the present paper as pointed out by its title is on the avenue thought opened by solitary waves as likely forerunners of mechanical destabilization of the Earth's crust. This adds up to general analogies between central volcanoes and DSP geothermal fields and makes them both good subjects for application of apt indicators of thermal energy build up and approaching mechanical destabilization. Conceptual models of solitary waves are instrumental to developing such indicators.

The potential information from surface geothermal gradients, the ideal data in principle to infer the distribution of thermal energy at depth, in actuality is heavily impoverished by such complications as atmospheric-upper crust water circulation and the fact that identifying heat with thermal gradients is not as obvious whilst observing natural systems as it is in laboratory experiments.

As the result, geochemical indicators of temperature (thermometers) and of on going steam-water separation process have been more successful and cost effective ways than geothermal gradients in geothermal exploration. The solitary waves theory holds promise for similarly inferring (P,T) and mechanical instability conditions at depth, based on diffuse fluid flow surface data.

We submit the idea that such data collected over large areas are complementary to downhole data in investigating subsurface conditions with a view at geothermal field management, as well as at volcanic forecast.

Material property	Westerly granite	Units
G	$1.5\cdot 10^{10}$	Pa
B_S	0.85	
n	0.25	
n_u	0.34	
a_f	10^{-3}	$^0C^{-1}$
a_m	$2.4\cdot 10^{-5}$	$^0C^{-1}$
K_f	$4\cdot 10^{-19}$	m^2
f	10^{-2}	
K_T	3	$J/m\cdot^0 C\cdot s$
m	$8\cdot 10^{-5}$	$Pa\cdot s$
r_m	$2.7\cdot 10^3$	Kg/m^3
r_f	$3.6\cdot 10^2$	Kg/m^3
c_m	10^3	$J/Kg\cdot^0 C$
c_{v_f}	$2.1\cdot 10^3$	$J/Kg\cdot^0 C$
k	$1.1\cdot 10^{-6}$	m^2/s
$b \cong b^I$	$5\cdot 10^{-13}$	$m^2/Pa\cdot s$
c	$1.87\cdot 10^{-21}$	$m^2/Pa^2\cdot^0 C\cdot s$
a	$1.7\cdot 10^6$	$Pa/^0 C$
h	$2.7\cdot 10^{-4}$	m^2/s

G: shear modulus, B_S: Skempton parameter, $n(n_u)$: drained (undrained) Poisson ratio, $a_f\,(a_m)$: fluid (medium) thermal expansibility, K_f: permeability, f: porosity, K_T: average thermal conductivity, m: viscosity, r_m: medium density, r_f: fluid density, c_m: medium thermal capacity, c_{v_f} : fluid thermal capacity at constant volume, k: average thermal diffusivity due to diffusion, b: average thermal diffusivity due to convection, c: average dissipative diffusivity due to fluid-rock friction, a: differential fluid-rock thermal expansivity , h: fluid diffusivity

Table I. Definition and values of the quantities for the Westerly and Charcoal granites, the Weber sandstone and the Tennessee marble saturated with water at $P_0 = 3\cdot 10^7\,Pa$ and $T_0 = 400^0 C$ and with temperature and pressure perturbations in the order of $T_I = 50^0 C$ and $P_I = 2-2.5\cdot 10^7\,Pa$.

APPENDIX

In equations (1) and (2) the source term due to the differential fluid-rock thermal expansivity is (Table I):

(A1)

$$a = \left(G a_m \frac{4(1+n)}{3(1-n)} + G B_S f\left(a_f - a_m\right)\frac{2(1+n)(1+n_u)}{3(n_u - n)} \right) \frac{B_S(1-n_u)(1-n)}{3(1-n)(1+n_u) - 6(n_u - n)}$$

the average thermal diffusivity due to diffusion is:

(A2) $$k = \frac{K_T}{f r_f c_f + (1-f) r_m c_m}$$

the fluid diffusivity is:

(A3) $$h = \frac{K_f}{m}\left(\frac{2 G B_S^2 (1+v_u)^2 (1-v)}{9(1-v_u)(v_u - v)} \right)$$

the thermal diffusivity due to convection is:

(A4) $$b = \frac{K_f}{f m}$$

and the dissipative diffusivity due to fluid-rock friction is:

(A5) $$c = \frac{K_f}{m\left[f r_f c_f + (1-f) r_m c_m\right]}$$

Material property	Westerly granite	Units
b	$3\cdot 10^3$	m
$P_I\vert_{z=0}$	$2\cdot 10^7$	Pa
$T_I\vert_{z=0}$	50	^0C
$V_{D sw}$	10^{-10}	m/s

b :thickness of the fluid-saturated porous horizon, P_I: fluid pore pressure (associated fluid-rock temperature) change at the buried thermo-mechanical source,

$V_{D_{SW}} = -K_f \, \partial_z P / m = K_f c_{SW}(z,t) / 2b^I m$: initial Darcy's velocity associated to thermo-mechanical solitary shock wave propagation

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Table II. Values of the propagating thermo-mechanical shock xave for the considered rock types.

Material property	Westerly granite	Units
b	$3 \cdot 10^3$	m
$\partial_z P\|_{z=0}$	$2 \cdot 10^4$	Pa/m
$\partial_z T\|_{z=0}$	$5 \cdot 10^{-2}$	$^0C/m$
V_{D_S}	10^{-10}	m/s

b : thickness of the fluid-saturated porous horizon, $\partial_z P|_{z=0}$ ($\partial_z T|_{z=0}$): pore fluid pressure (associated fluid-rock temperature) gradient at the buried thermo-mechanical source, $V_{D_S} = -K_f \, \partial_z P / m = K_f c_S(z,t) / 2b^I m$: initial Darcy's velocity associated to thermo-mechanical soliton

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Table III. Values of the propagating thermo-mechanical solitons for the considered rock types.

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REFERENCES

Bejan, A. (1984), "Convection heat transfer," J. Wiley & Sons, NY.

Facca, G. and Tonani, F. (1961), "Natural steam geology and geochemistry," Porceedings U.N. Rome Conference on New Sources of Energy, **vol. 2**, 271.

Garcìa, R. and Natale, G. (1998), "Subsurface propagation of thermo-mechanical kinks through hydrothermal fluid-saturated porous horozons," In press in J. Geophys. Res..

Hulen, J.B., Heizler, M.T., Stimac, J.A., Moore, J.N., and Quick, J.C. (1997), "New constraints on the timing of magmatism, volcanism, and the onset of vapor-dominated conditions at The Geysers steam field, California," Proceedings, 22nd Workshop on Geothermal Reservoir Engineering, Stanford University, Stanford, California, Jan. 27-29, 75-82.

McTigue, D. F. (1986), "Thermoelastic responce of fluid-saturated rock," J. Geophys. Res., **91/89**, 9533-9542.

Moore, J.N., Anderson, A.J., Adams, M.C., Aines, R.D., Norman, D.I. and Walters, M.A. (1998), "The fluid inclusion and mineralogic record on the transition from liquid- to Vapor-dominated conditions in The Geysers geothermal system," Proceedings, 23rd Workshop on Geothermal Reservoir Engineering, Stanford University, Stanford, California, Jan. 26-28.

Moore, J.N., and Gunderson, R.P. (1995), "Fluid inclusion and isotopic systematics of an evolving magmatic-hydrothermal system," Geochimica et Cosmochimica Acta, **59/19**, 3887-3907.

Natale, G. (1998), "The effect of fluid thermal expansivity on thermo-mechaical solitary shock waves in the underground of volcanic domain," Pure and Appl. Geophys., **152**, 193-211.

Natale, G. and Salusti, E. (1996), "Transient solutions for temperature and pressure waves in fluid-saturated porous rocks.," Geophys. J. Intern., **124/3**, 649-656.

Natale, G., Salusti, E. and Troisi, A. (1998), "Rock deformation and fracturing processes due to nonlinear shock waves propagating in hyperthermal fluid-pressurised domains," J. Geophys. Res., **103/B7**, 15,325-15,338.

Rice, J.R. and Cleary, M.P. (1976), " Some basic stress diffusion solutions for fluid-saturated elastic porous media with compressible constituents," Rev. Geophys. Space Phys., **14**, 227-241.

Whitham, G.B. (1974), "Linear and non-linear waves," Wiley Interscience, New York, NY.