

## A PRACTICAL SOLUTION FOR THE TRANSIENT RADIAL-VERTICAL HEAT CONDUCTION IN GEOTHERMAL WELLS

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### ABSTRACT

When drilling geothermal wells the fluid exchanges thermal energy to the surrounding rock during its vertical ascent toward reservoir colder areas. This transmission of heat from the casing toward the formation in low permeability reservoirs is essentially conductive. The importance of its study is based on the fact that this loss of heat could have influence in the natural fluid thermodynamic conditions and in the prediction of its natural original state. There is even another reason: during drilling, the first thermal data contain valuable information about rock thermal parameters, essentially effective thermal conductivity and specific heat. The analysis of the anomalies or discontinuities in the transmission of heat provide indirect information about the extension and abundance of fractured areas. In this document we present an exact semi-analytic model for the numerical analysis of the transient heat conduction when the thermal energy flows simultaneously in radial and vertical directions. This model provides a practical solution of the type  $T(r, z, t)$  resulting of coupling two classic solutions in two independent dimensions. We introduce some applications to a geothermal well in a Mexican field.

### NOMENCLATURE

$C_j$	Special constant of $U_0$ (eq. 14)
$C_r$	Rock specific heat [J/kg/°C]
$C_w$	Water specific heat [J/kg/°C]
Exp	Exponential function
$J_0$	0-Bessel function of the First kind
$J_1$	1-Bessel function of the First kind
$K_T$	Thermal conductivity [W/m/°C]
Ln	Natural logarithmic function
$Q$	Flow of Heat [W/m <sup>2</sup> ]
$Q_M$	Mass flow rate per unit area [kg/s/m <sup>2</sup> ]
$r$	Cylindrical coordinate and radial dimension [m]
$r_a$	Inner radius [m]
$r_b$	Outer radius [m]
$t$	Time [s]
$T$	Temperature [°C]
$T_a$	Temperature at the internal boundary $r_a$
$T_b$	Temperature at the external boundary $r_b$
$T_H$	Temperature at the bottom boundary $Z_H$

$T_i$	Initial temperature in the region $r_a < r < r_b$
$T_0$	Temperature at the upper boundary $Z_0$
$U_0$	0-Cross Product Bessel function (eq. 11)
$Y_0$	0-Bessel function of the Second kind
$Y_1$	1-Bessel function of the Second kind
$z$	Depth and vertical dimension [m]
$a_j$	Parameter in the j-root of $U_0$ ( $a_j r = 0$ )
$d_{jm}$	Unit tensor (1 if $j = m$ , 0 if $j \neq m$ )
$\partial$	Partial derivative
$\gamma$	Rock thermal diffusivity [m <sup>2</sup> /s]
$\nabla^2$	Laplacian operator
$\rho$	Cylindrical coordinate (negligible)

### INTRODUCTION

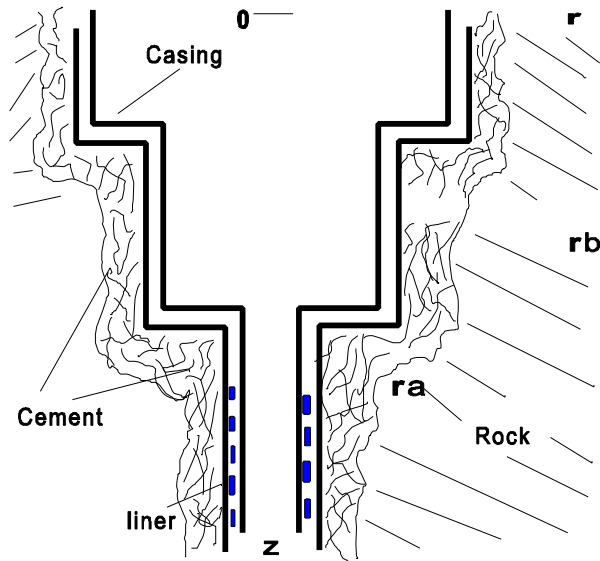
In spite of great advances obtained in our time concerning the study of heat flow in the terrestrial surface, the precise knowledge of the thermal behavior of the Earth's crust still presents enormous difficulties. For example, to estimate accurately the distribution of temperatures in function of depth, it is necessary to know the thermal conductivity and the position of the heat source or the distribution of heat flow in some deep boundary. The deep flow of heat coming from the planet's interior, together with the heterogeneous thermal diffusivity, are the main mechanisms that prevent the temperature to be uniform at least in the upper layers of the earth's crust. From a geothermal point of view, drilling wells causes irreversible local thermal alterations, forming punctual discontinuities in the crust. However, the study of these artificial discontinuities can allow the inference of some rock natural thermal parameters.

The borehole of any geothermal well loses its natural conditions during its construction. When drilling the well, the vertical distribution of temperature is unknown and altered at the same time, because of the mechanical perforation itself and because of the introduction of mud and water into the hole. In some media, for example in low permeability volcanic reservoirs, this thermal interference spreads approximately in radial form, affecting a local volume of the reservoir. The only available data when carrying out the drilling are: well's dimensions, thermal parameters measured in cores, drilling duration and the inlet and outlet mud temperatures.

In this paper, we present a practical radial-vertical conduction model that uses this information in flexible form to estimate the effect of the thermal disturbance in the neighboring formation of the well. The central objective of this model is the quantification of the amount of energy that is lost radially by conduction through the well, at any depth. If the natural heat flow outside the borehole is purely conductive, the estimation of those variables can be very accurate. If there are convective anomalies, the quantification of discrepancies will allow to locate areas of convection and consequently, of probable high permeability. Coupling our analytic model to a well simulator, could complement the study of this complex phenomenon. This is a current work in progress.

### **QUALITATIVE DESCRIPTION OF THE MODEL**

The conductive heat transfer from the well toward the formation is not always negligible (Ramey, 1962). Kanev and co-authors (1997) shown that calculated wellhead temperature could diverge 30 °C from measurements, if heat loss from the wellbore is not considered. This loss can have influence in the thermodynamics of certain type of wells. A summary of the scarce works relative to the heat transfer between geothermal wells and the reservoir was published recently by Kanev and others (1997). Maybe the most well known of those papers is that one written by Professor Ramey (1962). He developed an approximate solution to the transient conduction of heat in the movement of hot fluids through the wellbore. In this way fluid and casing temperature as a function of depth and time can be



estimated. At this moment several well simulators exist in the geothermal market, some reach great numerical sophistication (Garg & Combs, 1998), and include diverse correlations for

the calculation of friction, phase changes, two-phase feed zones, etc. The model we are introducing focuses on the problem of pure heat conduction between the cement in the annular column and the formation, up to a distance  $R_M$  inside the reservoir. This distance depends on depth.

Initially, in any region of the geothermal reservoir, temperature has an unknown vertical profile that, in time lapse of human life, can be considered stationary. Before well construction ( $t \leq 0$ ), at each point exists a natural temperature  $T_N(x,y,z)$ . At time  $t > 0$  the drilling action causes a complex vertical disturbance that spreads also in radial direction, inducing a second temperature profile  $T_D(r,t)$  at each depth. Both temperatures are coupled in a common boundary around the well. When drilling is completed, the perforated area lost heat. This is reflected in the temperature difference between the formation and the borehole. The movement of hot water inside the well establishes a convective heat flow, which creates a new thermal initial state in the hole, generating another temperature profile  $T_W(z, t)$  inside the well. Due to the high thermal conductivity of metal (25-50 W/m /°C), the vertical flow of heat inside the casing is very high. But at the same time the transient differential of temperatures between the fluid and the casing is insignificant. Therefore it is correct to suppose that water and casing are approximately at the same temperature. This does not hold for the cement and the reservoir rock, because of their smaller thermal conductivities.

### **THE TRANSIENT RADIAL-VERTICAL MODEL**

Figure 1 shows the region of the space included in the model. The radial transient conduction at each depth  $Z > 0$  is obtained by overlapping the solutions corresponding to several infinite hollow cylinders, embracing a region formed by the cement in contact with the formation from an inner radius  $r_a$ , until an external radius  $r_b$ , at respective temperatures  $T_a(z)$  and  $T_b(z)$ , both independent of the radius, but functions of depth. Initial temperature distribution in the region  $r_a < r < r_b$ , is given by  $T(r, z, 0) = F(r, z)$ , which can be assumed constant as a first approximation and recalculated later with this model, for a new initial state. The well internal temperatures can be measured at each time  $t$ . At distances larger than  $r_b$ , reservoir temperature is stationary and only function of depth.  $T_0$  is the boundary condition in an arbitrary plane  $Z_0$  and the bottom boundary condition is  $T_H$  at depth  $z=H$ . The mathematical form of the model that fulfills these requirements is the following one (see appendix & nomenclature):

$$T(r, z, t) = T_a + \frac{T_b - T_a}{\text{Ln}(r_b / r_a)} \text{Ln}\left(\frac{r}{r_a}\right) + \sum_{j=1}^{\infty} C_j \frac{U_0(a_j r)}{Y_0(a_j r_b)} e^{-h a_j^2 t};$$

$$\text{and } T_b = W(z)$$

$W(z)$  is any function of depth that satisfies the imposed boundary conditions. For example the function that we used in some calculations was developed by Donaldson in 1968 (see Appendix). Any piecewise linear function of  $z$ , ( $z_1 \leq z \leq z_2$ ), is also appropriate for this model. Both examples are given in equations 15-16. The internal temperature  $T_a$  is also a function of depth, but its values are always obtained by direct field measurements or calculated with a well simulator.

The development of equation (1) is described in the appendix. Different problems can be solved with this model: 1) - up to what radial distance well drilling perturbs the distribution of temperatures in the reservoir ? 2) - how is the distribution and evolution of  $T(r,z,t)$  in the region  $r_a < r < r_b$  ? 3) - how  $T(r,t)$  is affected by the vertical flow of heat ? 4) how much heat is exchanged radially between the wellbore and the reservoir ?

We call this model HEATW3; it was programmed in FORTRAN-77 language. The resulting code is applicable to a large variety of problems at different scales, including well completions. We illustrate the use of the model with two applications to a mexican geothermal well. First we study the effect of heat transfer between the casing and the formation, calculating the temperature distribution inside the cement. A second group of applications was focused on the distribution of temperature in the surrounding rock and the radial heat loss effects, from the bottom hole to the wellhead.

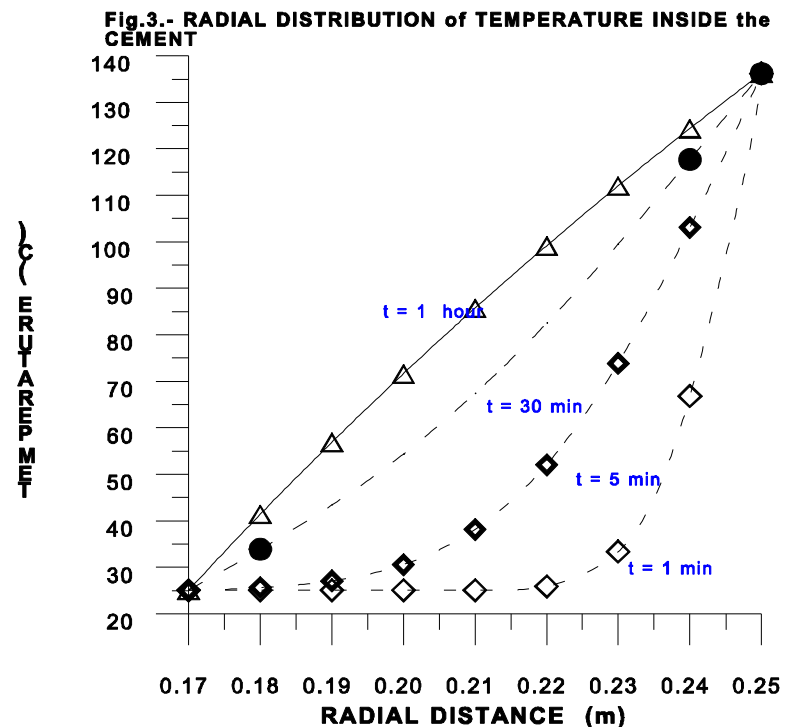
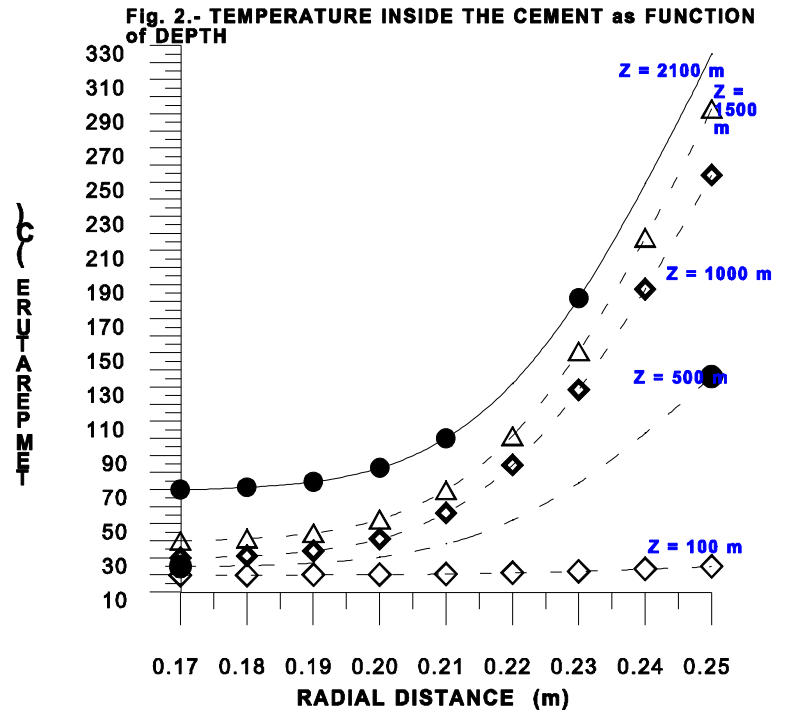
### CASING-FORMATION RADIAL HEAT TRANSFER

The cementing of a geothermal well, has as main purpose to fix and provide zonal isolation in the wellbore in order to exclude other non-geothermal fluids of the reservoir. Its primary objective is not the thermal isolation of the well. Cement thermal conductivity is not constant and depends almost exclusively on its density (Nelson, 1990). Usually light cement is used in hot deep areas, while more dense cement is used in cold superficial areas. Consequently, the radial heat loss is higher in shallow strata, because of a major conductivity gradient. We applied this model to the initial cementing conditions of well Az-01 of the Los Azufres, Mexico geothermal field with the following parameters for the cement: Thermal conductivity between 1.0 and 1.21 W/m/°C, density 1906 kg/m<sup>3</sup>, specific heat 879 J/kg/°C. Boundary temperatures are from several logs obtained in 1980. Figure 2 shows temperature distribution inside the cement at different depths, with water flowing for 30 minutes. Figure 3 illustrates temperature evolution inside the cement at 500 m depth.

A practical result obtained from this analysis is the estimation of thermo-mechanical stresses generated by the high thermal gradients inside the cement. This effect linked to the presence of air bubbles, could explain the origin of cement failure and wells'collapse observed in some mexican wells and in geothermal fields of other countries. For well Az-01, the thermal stress started to be important at around 500 m depth.

The highest calculated gradient is 260 °C at 2100 m depth.

### RADIAL HEAT TRANSFER IN THE WELL'S NEIGHBORING FORMATION



In the following application the model is used to calculate the evolution and distribution of temperature between the immediate vicinity of the well and the surrounding reservoir rock until a maximum distance is reached by the thermal disturbance. For the same well Az-01, this distance was estimated by trial and error. Its approximate value was found to be 11 meters for the initial temperatures measured in February 1980, after the well was completed. Figure 4 shows the temperature evolution in one year of this well in the interval [0.5, 10.5] m at 500 m depth.

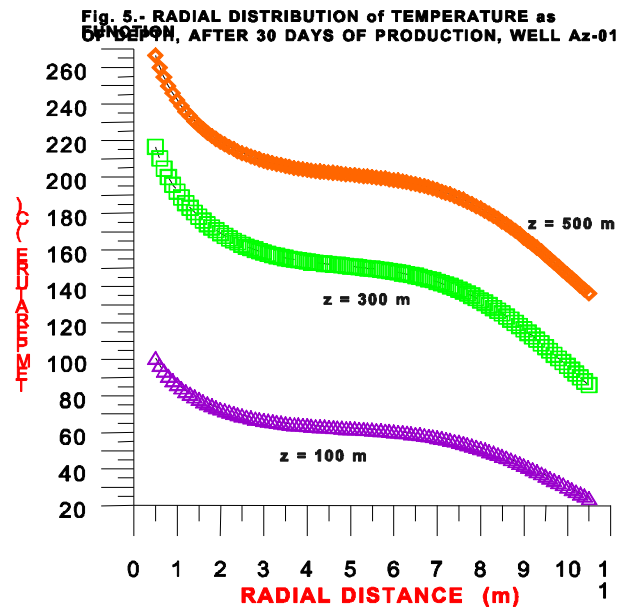
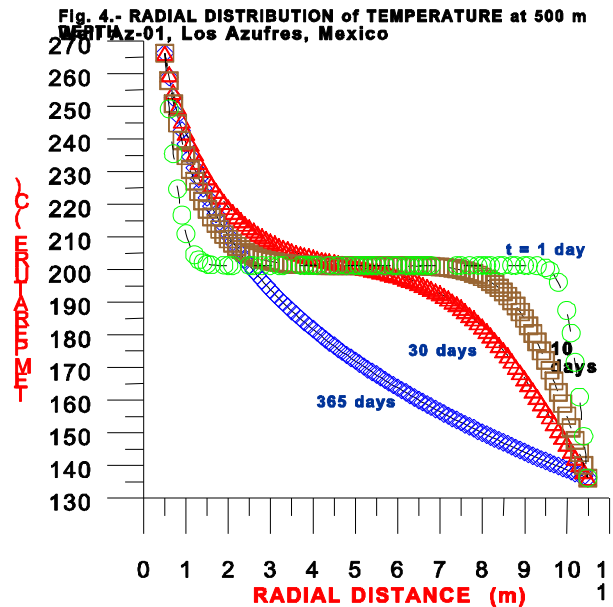
Figure 5 illustrates the same radial distribution, but at different depths, after well Az-01 was left open for production during 30 days. The following averaged parameters were used for the porous rock/water system: Thermal conductivity 2.5 W/m°C, density 2500 kg/m<sup>3</sup>, specific heat 1320 J/kg°C, porosity 0.1. The radial flow of heat in each interval  $\Delta z$  is calculated directly from equation 1 using the definition  $Q = -K_T \partial T / \partial r$ . For example, at 500 m depth The stationary portion of the heat flow is  $Q_s = 19.2 \text{ W/m}^2$ . Table 1 shows the evolution of the transient heat flow during one year of production for this well.

**Table 1.- Radial Component of Heat Flow at 500 m depth**

Time	$Q_s$ [W/m <sup>2</sup> ]	$Q_t(t)$ [W/m <sup>2</sup> ]
01 hour	19.160	-9.75
10 hours	19.160	119.96
24 “	19.160	387.83
240 “	19.160	546.35
30 days	19.160	392.19
100 “	19.160	201.28
200 “	19.160	103.03
365 “	19.160	37.44

**CONCLUSIONS**

We have developed and applied an exact semi-analytical conductive model to quantify the radial distribution and evolution of temperature in geothermal wells, at different depths. The decisive influence of depth is included directly in the external boundary condition. Our model was applied in a flexible way to estimate the thermal disturbance effect in the well’s neighboring formation, and to investigate the temperature behavior inside the annular column filled with cement, between the casing and the rock. These numerical results could be useful to quantify annulus internal thermal stresses. The model can also be helpful to estimate the maximum radial distance reached by thermal disturbances produced by drilling operations under different conditions.



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## APPENDIX

In cylindrical coordinates the model we are proposing has the following form:

$$\nabla^2 T = \frac{1}{h} \frac{\partial T}{\partial t} (r, z, t) \quad (2)$$

$$\nabla^2 T = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2}$$

With the following conditions in time and space:

$$T(r, z, 0) = F(r, z)$$

$$T(r_a, z, t) = T_a(z), T(r_b, z, t) = T_b(z) \quad (3)$$

$$T(r, z_0, t) = T_0(r); T(r, z_H, t) = T_H(r)$$

The dependence on  $z$  is assumed negligible and the unknown function  $T(r, z, t)$  breaks down in two parts, at every plane  $z =$  constant:

$$T(r, z, t) = U(r, z) + V(r, t) \quad (4)$$

Each one of the functions in (4) is in turn solution of different

simple models.  $U(r)$  is a stationary radial model describing temperature distribution between  $r_a < r < r_b$ :

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dU}{dr} \right) = 0; \quad (5)$$

$$U(r_a) = T_a(z), U(r_b) = T_b(z)$$

Its solution is elementary:

$$U(r) = T_a + \frac{T_b - T_a}{\ln(r_b / r_a)} \ln\left(\frac{r}{r_a}\right) \quad (6)$$

$V(r, t)$  corresponds to a transient radial flow of heat inside an infinite hollow cylinder, defined by an inner radius  $r_a$ , an outer radius  $r_b$ , initial temperature  $T_i(r)$  and boundary conditions at zero temperature:

$$\frac{1}{h} \frac{\partial V}{\partial t} (r, t) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) \quad (7)$$

$$V(r_a, t) = V(r_b, t) = 0; V(r, 0) = T_i(r)$$

This equation was proposed and solved by Carslaw & Jaeger (1959); unfortunately their result is incomplete. Perhaps due to a typographic omission, an important function is lacking in all the terms of the infinite series they obtained. For the solution of (7) it is necessary to use a similarity function integrating, in a single variable, the radial space and the time. The change of function:

$$t(r) = V(r, t) e^{h a^2 t} \quad (8)$$

$$r^2 \frac{d^2 t}{dr^2} + r \frac{dt}{dr} + (r a)^2 t = 0$$

transforms equation (7) in the first kind Bessel differential equation (8) of order 0. Its general solution is:

$$t(r) = C J_0(a r) + D Y_0(a r) \quad (9)$$

Where C and D are constants that can be deduced from the boundary conditions;  $a$  is a parameter related to the roots of the cross product Bessel functions defined in equation (11). The two conditions in (7) point out:

$$D = -C J_0(\mathbf{a} r_a) / Y_0(\mathbf{a} r_a) -$$

$$J_0(\mathbf{a} r_a) \bullet Y_0(\mathbf{a} r_b) - J_0(\mathbf{a} r_b) \bullet Y_0(\mathbf{a} r_a) = 0 \quad (10)$$

The last equation in (10) is valid for any constant C. Therefore, it exists a Cross Product Bessel function of order 0 (Carslaw & Jaeger, 1959 and Abramowitz & Segun, 1972), defined by:

$$U_0(\mathbf{a} r) = J_0(\mathbf{a} r) Y_0(\mathbf{a} r_b) - J_0(\mathbf{a} r_b) Y_0(\mathbf{a} r) \quad (11)$$

Notice that always  $U_0(\mathbf{a} r_b) = 0$  while  $U_0(\mathbf{a} r_a) = 0$  only if  $\mathbf{a}$  is a solution of (10). Given the nature of  $J_0$  and  $Y_0$  (Abramowitz & Segun, 1972), the roots of (10) are infinite in number. Then each solution of (9) has the same form, differing only in the value of  $\mathbf{a}_j$ . Applying the principle of superposition and replacing in (9):

$$\mathbf{t}_j(r) = -C_j \frac{U_0(\mathbf{a}_j r)}{Y_0(\mathbf{a}_j r_b)} ; j = 1, \infty \quad (12)$$

$$-V(r, t) = \sum_{j=1}^{\infty} C_j \frac{U_0(\mathbf{a}_j r)}{Y_0(\mathbf{a}_j r_b)} e^{-h \mathbf{a}_j^2 t}$$

The constants  $C_j$  are calculated using the orthogonality properties of  $U_0(\mathbf{a} r)$  (Carslaw & Jaeger, 1959) and the initial condition of (7). Multiplying by  $U_0$  and integrating both sides of the equation between  $r_a$  and  $r_b$ :

$$\begin{aligned} & \int_{r_a}^{r_b} r T_i(r) U_0(\mathbf{a}_m r) dr = \\ & \sum_{j=1}^{\infty} \frac{C_j}{Y_0(\mathbf{a}_j r_b)} \int_{r_a}^{r_b} r U_0(\mathbf{a}_j r) U_0(\mathbf{a}_m r) dr \quad (13) \\ & = \sum_{j=1}^{\infty} \frac{2 d_{jm} C_j}{Y_0(\mathbf{a}_j r_b)} \bullet \frac{J_0^2(\mathbf{a}_j r_a) - J_0^2(\mathbf{a}_j r_b)}{p^2 \mathbf{a}_j^2 J_0^2(\mathbf{a}_j r_a)} \end{aligned}$$

The final series is reduced to a single term (when  $j=m$ ) and from this expression all constants  $C_j$  can be deduced:

$$C_j = \frac{p^2 \mathbf{a}_j^2 J_0^2(\mathbf{a}_j r_a) Y_0(\mathbf{a}_j r_b)}{2 [J_0^2(\mathbf{a}_j r_a) - J_0^2(\mathbf{a}_j r_b)]} \int_{r_a}^{r_b} r T_j(r) U_0(\mathbf{a}_j r) dr \quad (14)$$

Equation (14) completes the solution of (7). It is important to notice that the zeros of  $U_0$  are infinite, real and positive. The model's numerical precision depends closely on the exact calculation of all of them. Their numeric evaluation must be performed carefully in double precision. Asymptotic approximations published, for example, by Abramowitz and Segun (1970) are not usable.

$W(z)$  is a simple, stationary model for the vertical distribution of temperature in the range  $z_0 < z < z_H$ . An appropriate solution for  $W(z)$  was proposed by Donaldson (1968), by means of a stationary vertical model of flow of mass and heat in a porous medium containing compressed water and with the same boundary conditions indicated in (3). For constant average parameters his analytic solution is:

$$W(z) = T_0 + (T_H - T_0) \frac{\Psi(z)}{\Psi(z_H)}, \quad (15)$$

$$\Psi(z) = 1 - e^{-(Q_M C_w z) / K_T}$$

This model contains the relative importance of convection compared with conduction. When convection dominates, the value of the last quotient is increased.

Another possible form of  $W(z)$  is a piecewise linear function defined in subintervals  $[z_1, z_2]$  of  $[z_0, z_H]$ :

$$W(z) = \frac{T_1 z_2 - T_2 z_1}{z_2 - z_1} + \frac{T_2 - T_1}{z_2 - z_1} z \quad (16)$$

Finally the radial component of the heat flow is calculated directly from its definition:

$$Q_r = -K_T \frac{\partial T}{\partial r} = -K_T \left( \frac{\partial U}{\partial r} + \frac{\partial V}{\partial r} \right) \quad (17)$$

Which from equation (1) becomes:

$$Q_r = -K_T \frac{\partial T}{\partial r} = -K_T \left( \frac{\partial U}{\partial r} + \frac{\partial V}{\partial r} \right) = \frac{T_a - T_b}{r \ln(r_b / r_a)} - \quad (18)$$

$$-p T_i K_T \sum_{j=1}^{\infty} C_j \frac{U_1(\mathbf{a}_j r)}{Y_0(\mathbf{a}_j r_b)} e^{-h \mathbf{a}_j^2 t}$$