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STABILITY OF HEAT PIPES IN VAPOR-DOMINATED SYSTEMS

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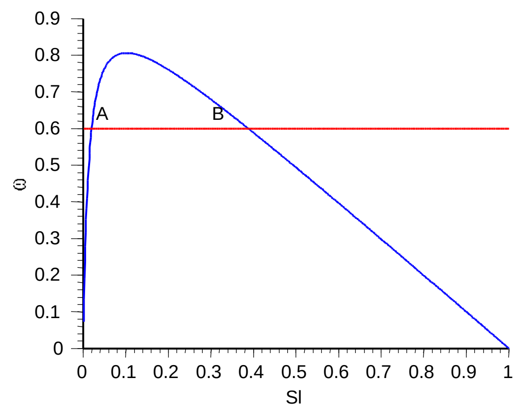
ABSTRACT

We study the linear stability of a two-phase heat pipe zone (vapor-liquid counterflow) in a porous medium, overlying a superheated vapor zone. The competing effects of gravity, condensation and heat transfer on the stability of a planar base state are analyzed in the linear stability limit. The rate of growth of unstable disturbances is expressed in terms of the wave number of the disturbance, and dimensionless numbers, such as the Rayleigh number, a dimensionless heat flux and other parameters. A critical Rayleigh number is identified and shown to be different than in natural convection under single phase conditions. The results find applications to geothermal systems, to enhanced oil recovery using steam injection, as well as to the conditions of the proposed Yucca Mountain nuclear waste repository. This study complements recent work of the stability of boiling by Ramesh and Torrance (1993).

INTRODUCTION

Heat pipes are steady-state, steam-water, countercurrent flow regimes in porous media driven by the application of a heat flux and gravity (White et al., 1971). Main characteristics of heat pipes are that their temperature is constant, and equal to the vapor saturation temperature at the prevailing pressure and that their liquid saturation is spatially constant (Udell, 1985). In theory, their spatial extent can be infinitely large, under the condition that the porous medium is homogeneous and that the temperature decrease due to the pressure drop is not significant (Satik et al., 1991, Stubos et al., 1993, Pestov, 1998). For a constant heat flux directed against the gravity vector, there are two possible steady-states, determined by the solution of the following equation, in the absence of heat conduction or capillary effects (Bau and Torrance, 1982, Udell, 1985, Stubos et al., 1993) (See schematic of Fig. 1).

$$\omega \equiv \frac{q\nu_v}{k\, g\, L_v\, (\rho_l - \rho_v)} = \frac{1}{[(\nu_l/\nu_v)\,(1/k_{rl}) + (1/k_{rv})]} \tag{1}$$



Here ω is a dimensionless parameter expressing the magnitude of the applied heat flux, and q, ν, k, g, L_v, ρ and k_r denote heat flux, kinematic viscosity, permeability, gravitional acceleration, latent heat of vaporization, density and the relative permeability, respectively. Subscript l and v denote liquid and vapor, respectively.

Fig. 1. The Saturation of the base state.

In the illustration of Fig. 1, straight-line relative permeabilities with zero residual saturations were taken. The straight line corresponding to a constant heat flux, ω, intercepts the heat flux-saturation curve at two points, A and B, provided that the flux is smaller than a critical value, ω_{max} (equal to about 0.8 in the figure). The two steady states correspond to a vapor-dominated or to a liquid-dominated heat pipe, depending on whether the liquid saturation is small (point A) or large (point B), respectively.

Above the critical value, a heat pipe per se, in the sense of a constant saturation region, does not form. Instead, a two-phase zone of considerably smaller extent forms, governed by the competition of capillary and gravity forces, where the saturation varies in the range (1,0) see Stubos et al. (1993).

The heat pipe regime can be connected to single-phase flow regimes above or below it. In applications when the liquid-dominated branch exists, a (subcooled) liquid layer overlies the heat pipe region. The point of transition between the heat pipe and the liquid depends on a variety of factors, including the heterogeneity of the medium (Stubos et al., 1993, Mc Guiness, 1996a, 1996b, Pestov, 1998). Typically, this application is encountered in boiling at low rates in porous media, where a liquid layer above the two-phase region is maintained, for example by keeping its temperature below boiling (Ramesh and Torrance, 1990, 1993). Conversely, a vapor-dominated heat pipe develops when a superheated vapor lies below the two-phase region. This situation, often referred to as "dryout", requires that superheated conditions exist below the two-phase regime. In either case, the transition between single-phase and two-phase flow regimes is also controlled by capillary forces (Stubos et al. 1993). While transitions between single-phase and two-phase flow regimes are possible, a transition between the two different heat pipe regimes, namely from liquid-dominated to vapor-dominated or vice-versa, is not possible (Mc Guiness et al., 1993, Stubos et al., 1993).

Regardless of the particular application, the existence of a heat pipe regime either below an overlying liquid or above an underlying vapor raises questions of stability. Consider, for example, the case of a liquid-dominated heat pipe. Given that the heat pipe is of a lower (although not by much) density than the overlying liquid, the possibility of a Rayleigh-Taylor type gravitational instability is apparent (Drazin and Reid, 1981). The onset of natural convection in the overlying liquid layer, due to its variable temperature, is also an important factor. In porous media, the onset of natural convection under single-phase flow conditions requires that the single-phase Rayleigh number, defined as:

$$Ra = (gkH\beta\Delta T) / (\alpha\nu) \qquad (2)$$

exceeds the critical value of $4\pi^2$ (Lapwood, 1948, Gebhart et al., 1988). Here, H is the thickness of the single-phase region, across which a temperature difference, ΔT, is applied, β is the thermal expansion coefficient of the liquid and α is the effective thermal diffusivity. Stabilizing factors, on the other hand, include conduction, the phase change at the liquid-heat pipe interface, and capillary effects.

The stability of liquid-dominated heat pipes was explored by Ramesh and Torrance (1990, 1993) in the context of boiling in porous media. They also reported the existence of a critical Rayleigh number above which the 1-D configuration is unstable to 2-D disturbances, and is a function of wave length and dimensionless heat flux. However the minimum critical value found was about half of that for the onset of natural convection in single-phase flow, suggesting that the underlying two-phase region is destabilizing the flow. Stability at large wavelengths is associated with viscous flow, while that at smaller wavelengths is due to conduction. A window of unstable wave numbers exists for Rayleigh numbers larger than the critical. Pestov (1998), examined the stability of the two-phase region overlying a vapor-dominated heat pipe, which she found to be stable. However the stability of the combined vapor-dominated heat pipes has not been explored at this time.

Vapor-dominated heat pipes find applications in similar contexts as liquid-dominated heat pipes. A most interesting visualization was provided recently by Kneafsy and Pruess (1999), who studied the flow mechanisms in heat pipes in a fracture, such that superheated conditions were maintained below the two-phase region. Although that study focused mainly on the mechanics of liquid flow, many issues related to flow instability and possibility that downwards-percolating liquid may "penetrate" the superheated region, were raised. At present, the stability features of this configuration are not known. Some of these features should be similar to the liquid-dominated case. For example, we should expect the onset of a natural convection mechanism for the vapor underlying the two-phase region, and a gravitional instability due to the two-phase region above being heavier (although only by a small amount) than the underlying vapor. The effect of the phase-transition at the interface is unclear, however, just as it has been unclear for the liquid-dominated heat pipes studied by Ramesh and Torrance (1990). In the context of other problems involving phase change in porous media, for example in steam injection processes for the recovery of heavy oil, we know that the condensation of steam at an advancing steam front is less destabilizing, than in non-condensing flows, due to the associated volume reduction. Conversely, the vaporization of liquid is more destabilizing, due to the associated volume expansion.

In this paper, we study the linear stability of vapor-dominated heat pipes by following a linear stability approach similar to Ramesh and Torrance (1990, 1993). In addition to the base-state configuration, however, other differences exist between the present approach and that of Ramesh and Torrance. We consider an infinitely long two-phase zone (heat pipe), as there are no compelling reason to restrict the two-phase region to a given length. The same difference applies also between our work and Pestov's (1998). Also our stability analysis is done using analytical methods, which allow for an asymptotic treatment of the problem (to be presented in a more extended version of this paper). On the other hand, in our analysis the compressibility of the vapor is not being considered, except for driving the natural convection (a Boussinesq type approximation) in contrast to Pestov (1998) where it is. The paper is organized as follows: First, we present a dimensionless formulation of the base state and discuss the properties of the vapor-dominated solution. Then, the linearized perturbation problem is presented based on normal modes. The stability analysis follows. Results and implications are discussed in the final section of the paper.

THE BASE-STATE

We consider the following base state. Due to the application of a heat flux, a dryout region of thickness H, consisting of a superheated vapor of almost constant pressure, P_v, underlies a two-phase region of infinite extent. The boundary between the two regions is a planar interface, with saturation temperature corresponding to P_v, denoted as t_{sat}. The two-phase region (heat pipe) corresponds to the vapor-dominated branch of the solution of equation (1). We assume that the heat flux is sufficiently small so that equation (1) has a solution (namely $\omega < \omega_{max}$). At base state conditions, the vapor is stagnant, and heat transfer is by conduction only. The two-phase zone, on the other hand, is a region of a constant temperature and constant saturation counter-current flow, where heat transfer is by convection. Under base-state conditions (denoted by subscript 0), the dimensionless equations governing the problem are:

a. Vapor Region

$$\partial P_0 / \partial y = Ra\, T_0 \qquad (3)$$

$$T_0 = 1 - y \qquad (4)$$

b. Two-phase Region:

$$\partial P_0 / \partial y = -(\omega M) / k_{rv} \qquad (5)$$

$$V_{l0} = M\, \omega \qquad (6)$$

$$T_0 = 0 \qquad (7)$$

$$S = 0 \qquad (8)$$

where P and T are dimensionless pressure and temperature, y is a dimensionless spatial coordinate directed upwards, V is velocity, the liquid saturation S is determined from equation (1) and we have made use of a Boussinesq type approximation. In the above we have also introduced the dimensionless two-phase Rayleigh number :

$$M = (C_{pv}\, H\, k\, g\, (\rho_l - \rho_v)) / (\nu_v\, k_e) \qquad (9)$$

where C_{pv} is the vapor heat capacity and k_e is the effective thermal conductivity of the porous medium. Note that our definition for the two-phase Rayleigh number is a factor of ρ_l/ρ_v larger than in Ramesh and Torrance (1990). In the above, we have neglected capillary effects.

STABILITY ANALYSIS

1. The Eigenvalue Problem

Subsequently, we carried out a linearized stability analysis of the problem by assuming that all dependent variables are perturbed in the transverse direction, x, and seeking the rates of growth of these disturbances in terms of normal modes. Thus, we take disturbances of the form

$$T = T_0 + \varepsilon\, \theta(y) \exp(i\kappa x + \sigma t) \qquad (10)$$

where ε is a small parameter, θ is the eigenfunction, κ is the wavenumber and σ is the rate of growth of the disturbance, and x and t denote transverse coordinate and time, all dimensionless. Similar expansions are taken for the pressure, the saturation and the interface position. These expansions are then substituted in the governing equations and the boundary conditions, and the system is linearized. The details of this process are considerable and will not be presented in this paper. The final results for the eigenvalue problem are shown below.

a. Vapor region (denoted by superscript – where appropriate)

$$\kappa^2 \pi^- + Ra\, d\theta/dy - d^2\pi^-/dy^2 = 0 \qquad (11)$$

$(\sigma^* - Ra + \kappa^2)\,\theta + d\pi/dy - d^2\theta/dy^2 = 0$ (12)

where $\sigma^*=\beta_1\,\sigma$, and
$\beta_1= [(1-\phi)\,\rho_r\,C_{pr} + \phi\,\rho_v\,C_{pv}]/\,(\rho_v\,C_{pv})$ is the ratio of the heat capacities for rock and vapor (and it is of the order of 1000).

b. Two-phase region (denoted by superscript + where appropriate)

$C1\,\pi^+ + C2\,S - C3\,dS/dy - d^2\pi^+/dy^2 = 0$ (13)

$C1\,\pi^+ - C4\,S + C5\,dS/dy - d^2\pi^+/dy^2 = 0$ (14)

Where the constants are defined as,

$C1=\kappa^2$,
$C2=(\phi\,\sigma\,\mu_l)/(k_{rl}\,\mu_v)$
$C3=(1/k_{rl})\,[(\partial P_0{}^+/\partial y) + M]$
$C4=(\phi\,\sigma\,)/k_{rv}$
$C5=(1/k_{rv})\,(\partial P_0{}^+/\partial y)$

where ϕ is porosity and μ denotes viscosity. The differential equations in these regions are to be solved subject to boundary conditions of constant temperature and zero vapor flux at y=0, no-flux conditions for vapor and liquid at $y=\infty$, and continuity of mass, energy, temperature and pressure at the interface ($y=\delta$).

The eigenvalue problem was then solved analytically. By incorporating the boundary conditions we obtained a fourth-order homogeneous linear system, the determinant of which must vanish for a non-trivial solution to exist. We note that because we have neglected the compressibility of the vapor, the saturation disturbance in our problem turns out to be zero (in contrast to Pestov, 1998). The vanishing of the determinant provides the solution for the rate of growth σ (or σ^*) as a function of the wavenumber κ and the various dimensionless parameters, among which key roles are played by the Rayleigh number and ω. Details of this calculation will not be presented here.

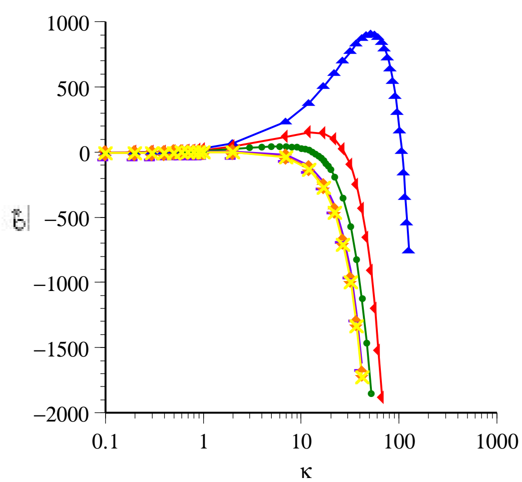
2. Results

Numerically accurate results were obtained by imposing the condition of vanishing of the fourth-order determinant. There are basically two parameters, that can vary independently, ω, which contains the dimensionless heat flux, and the two-phase Rayleigh number M. Note that the conventional Rayleigh number is related to these parameters via

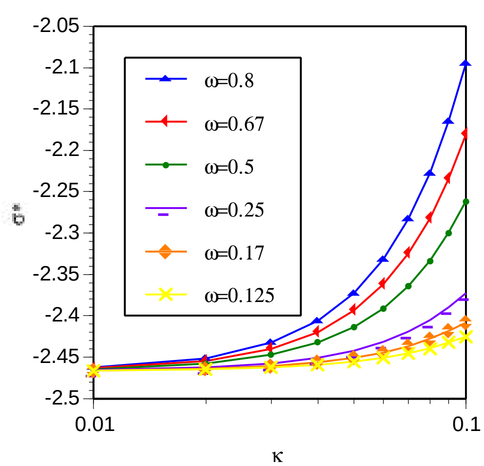
$Ra=\omega\,M^2\,N$ (15)

where we introduced the additional dimensionless parameter $N=(L_v\,\rho_v)/[C_{pv}\,(\rho_l - \rho_v)\,t_{sat}]$.

Typically, we find that the long waves are stable (see inset of Figs. 2 and 3 below). An asymptotic analysis of this regime can be performed, but will not be reported here. This behavior is similar to the boiling problem of Ramesh and Torrance (1993). Intermediate wavenumbers can be unstable, depending on whether or not the Rayleigh number is larger than a critical number, as discussed below. Sufficiently small wavelengths are stable, as in Ramesh and Torrance (1990).



(a)



(b)

Fig. 2. (a) The σ*-κ relation for different values of ω at constant M=50. (b) Inset of the plot at small κ.

Fig. 2 shows the rate of growth vs. the wavenumber curve for fixed M and variable ω. It is to be noted that ω cannot exceed a maximum value, for a heat pipe region to exist. It is shown that as ω decreases, the configuration is less unstable, reflecting the facts that the density of the heat pipe region diminishes at smaller ω and that the overall Rayleigh number is also smaller.

Fig. 3 shows corresponding result for the case of fixed ω and variable M. The configuration is shown to be more destabilized as the two-phase Rayleigh number M increases. In all these calculations the other parameters affecting the Rayleigh number definition above, for example the latent heat of vaporization, were held constant. Under this condition, it was found that the data collapsed on the same curve, if the variables ω and M were combined so that the Rayleigh number was constant. However, this should not be interpreted to imply that the Rayleigh number is the only relevant parameter of the problem (see below).

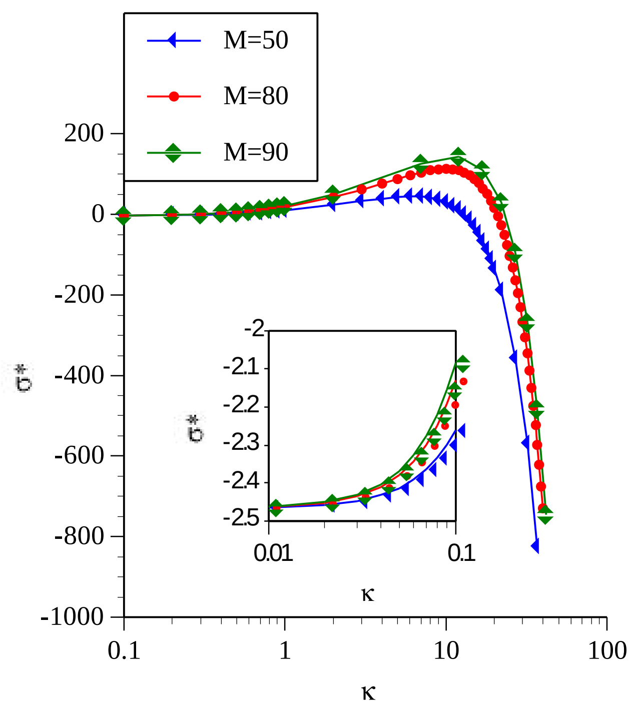


Fig. 3. The σ*-κ relation for different values of M at constant ω=0.5. and inset of the plot at small κ.

As in Ramesh and Torrance (1990), a critical Rayleigh number, Ra_{crit} exists which is a function of the other parameters of the problem, and particularly ω. Fig. 4 shows a plot of Ra_{crit} vs. ω obtained assuming a constant N. It is shown that Ra_{crit} is considerably smaller than the critical number corresponding to either single-phase natural convection or the liquid-dominated problem treated by Ramesh and Torrance
(1990). Furthermore, Ra_{crit} is found to increase as ω decreases. Calculations with variable N are currently in progress.

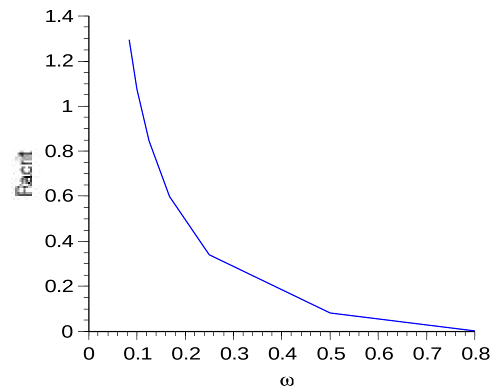


Fig. 4. The dependence of Ra_{crit} on ω.

To test the effect of the phase change process, we considered the sensitivity of the results to the latent heat, by keeping the Rayleigh number constant, namely we considered variable L_v but kept the product ω N (and M) constant. Results are shown in Fig. 5. It is shown that the problem becomes more unstable as the latent heat decreases, indicating the smaller energy requirements to sustain a destabilizing heat pipe above the vapor region as the latent heat is smaller.

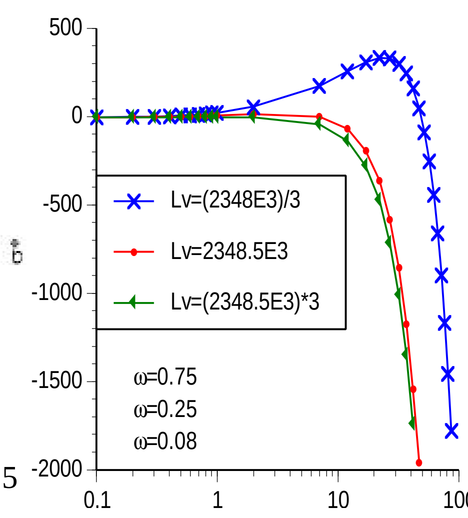


Fig. 5. The effect on stability of the change of latent heat.

CONCLUSIONS

In this paper, we studied the linear stability of a two-phase heat pipe zone (vapor-liquid counterflow) in a porous medium, overlying a superheated vapor zone. It was found that the problem has similarities with the liquid-dominated case, in that long and short waves are stable, but intermediate wavelengths can be unstable, depending on the parameter values. A critical Rayleigh number was identified and shown to be different than in natural convection under single-phase conditions in two respects: The critical value is significantly smaller (even smaller than liquid-dominated case), while the critical value is shown to also depend on the other parameters of the problem. In particular, we found that the latent heat affects the stability of the problem. The results find applications to geothermal systems, to enhanced oil recovery, as well as to the conditions of the proposed Yucca Mountain nuclear waste repository.

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