

Dispersion of a liquid plume injected to the base of a geothermal reservoir

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Abstract

We develop a model to describe the spreading of a plume of liquid after it has been injected into a geothermal reservoir. We examine the morphology of the plume as it spreads along the base of the reservoir under the force of gravity. We present an exact analytic solution for the long time flow of the injectate along an inclined impermeable boundary, such as the base of the reservoir. We illustrate how this may be used to track the motion of a discrete mass of tracer injected into the reservoir with the injectate. We show that for a continuous source of fluid, after a time t , a pulse of tracer added to the injectate will have advanced downslope a distance proportional to t while spreading cross-slope a distance proportional to $t^{1/3}$.

Introduction

In order to interpret the effectiveness of liquid reinjection and the associated tracer injection studies in geothermal systems, it is useful to have an understanding of motion of the injectate through the reservoir. In general this is a complex problem owing to the heterogeneous structure of the fractured rock, and sophisticated numerical schemes exist for specific examples. However, it is useful to develop some of the broad principles which govern the motion and distribution of injectate through a system. Since highly detailed, spatially accurate, descriptions of reservoir properties are difficult to obtain, simple analytical models which predict the main flow properties in terms of the bulk reservoir properties are especially useful.

Here we examine the motion of liquid injectate as it spreads along the inclined impermeable base of a reservoir after issuing from an injection well which provides a constant source of liquid. Before reaching the base of the aquifer, the liquid sinks as a plume from the injection well. In equilibrium, the vertical descent speed will be uniform and a cylindrical plume descends to the base of the reservoir. Once the plume has reached the reservoir floor it then spreads out.

We focus on the situation in which the reservoir floor is inclined to the horizontal, so that fluid drains down-slope, while also spreading downslope (figure 1). In the model, we assume the reservoir is vapour dominated, and we examine the long-time steady flow regime in which the liquid has established a flow path through the reservoir, cooling that part of the reservoir en route. In this case, any vaporisation that does occur will be at the leading edge of the injectate; therefore if one wishes to track the motion of a tracer after injection, and before it vents at an extraction well, it is necessary to model the steady liquid flow.

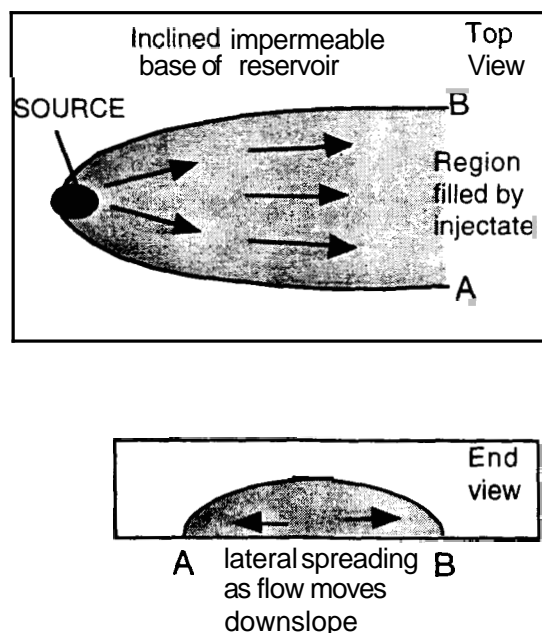


Figure 1 - Schematic of Injection Geometry

The model

To model the motion of injected liquid as it spreads along the base of an aquifer, we note that for motion primarily along the slope, the pressure in the liquid satisfies the hydrostatic balance

$$p = \rho g((h-z)\cos \theta - x \sin \theta)$$

where θ is the angle of inclination of the slope and h is the thickness of the current, with x , y and z the coordinates downslope, cross-slope and normal to the slope. We assume that the remaining rock is saturated with vapour so that the density difference between the vapour and the liquid is close to that of the liquid.

We use Darcy's law for the vertically averaged flow,

$$(u,v) = -K\rho g(h_x \cos \theta - \sin \theta, h_y \cos \theta) / \mu$$

where K is the permeability, ρ the fluid density, μ the fluid viscosity, and where we are assuming that for large scale flow the reservoir resembles a permeable porous matrix. Subscripts denote partial derivatives with respect to that variable. The local equation for the conservation of mass in the current satisfies

$$h_t + (u h_x)_x + (v h_y)_y = 0$$

Combining these equations, we find that the current satisfies the nonlinear equation

$$(\mu / K\rho g)h_t + 0.5(h^2_{xx} + h^2_{yy})\cos \theta = h_x \sin \theta$$

Although this is a non-linear diffusion equation, the system admits a rather simple steady asymptotic solution in the case of a steady point source issuing from a point on the slope, once the current has spread sufficiently far that $h^2_{xx} \ll h^2_{yy}$. The solution for the motion along the plane, then has the form

$$h = \tan \theta (a^2 - y^2 / x^{2/3}) / 12 x^{1/3}$$

for the thickness of the current, where, in this solution, at any position x downslope of the source, the cross-slope extent of the

current is

$$-ax^{1/3} < y < ax^{1/3}$$

The value of a is given by the condition of mass conservation, that the flux of liquid passing through any cross-slope section downslope of the source is Q . Since the speed of the current downslope is $k\rho g \sin \theta / \mu$, this gives the condition that

$$Q = a^3 k\rho g \sin^2 \theta / 9\mu \cos \theta$$

Fig 2a shows calculations of the cross-slope shape of the plume as a function of the distance downstream in dimensionless form.

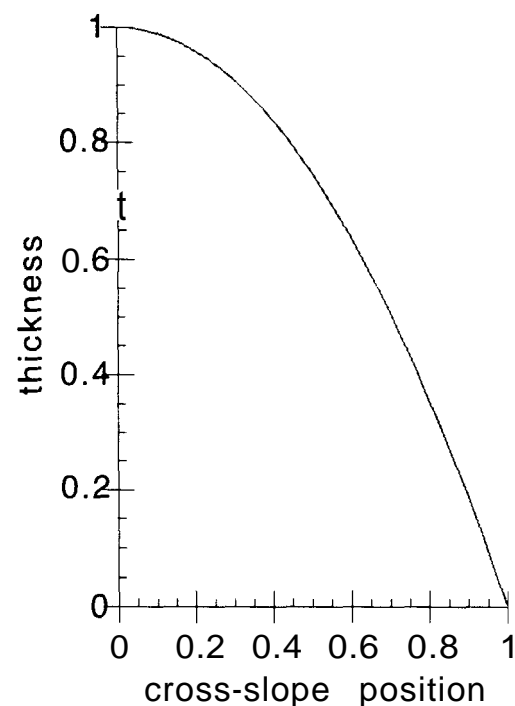


Fig 2b and 2c show the width and thickness of the current as a function of downslope position. The figures illustrate the important control of the spreading of the current on the reservoir permeability, (taken to be $k = 10^{-14} \text{m}^2$ except for the case $k = 10^{-12} \text{m}^2$ shown on the figures), the flow rate, taken to be $0.001 \text{m}^3/\text{s}$ except for the case 0.0001 as shown, and the angle of slope, taken to be 10 degrees, except for

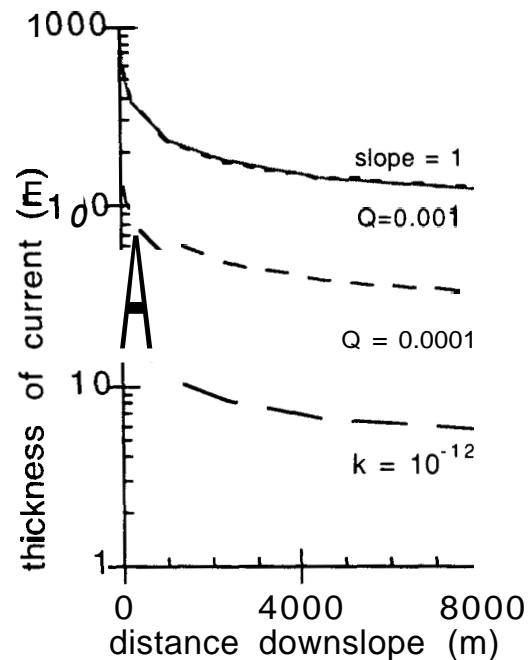
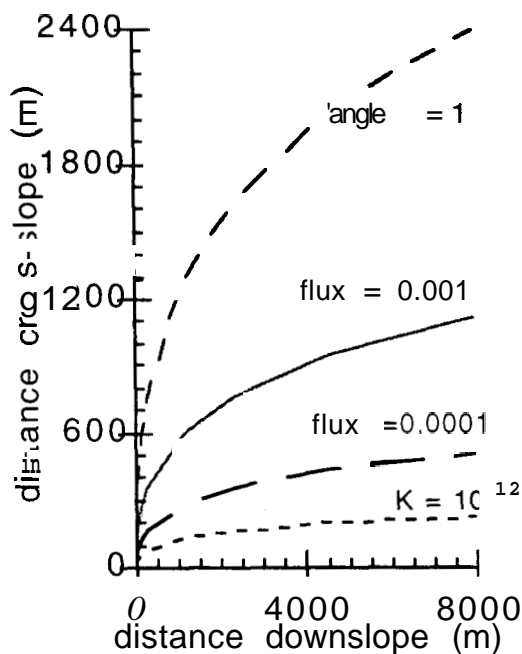
the case 1 degree as shown. With higher permeability, the current moves downstream more rapidly and hence is thinner and of smaller cross-slope extent; with lower flow rate, the current is again thinner and narrower, but with a smaller angle of slope, the downslope flow is slower, hence leading to greater cross-slope spreading and to a deeper current. Note that in all these calculations, the flow speed of the liquid is only of order $1e-8-1e-9$ m/s, since the flow is driven by gravity through very impermeable rock; as a result of such low flow speeds, the current becomes very dispersed.

constant speed, $k\rho g \sin\theta/\mu$ to leading order, so that after a time t , we expect tracer to be distributed along the line ahead of the source

$$x = t k\rho g \sin\theta / \mu ,$$

$$- a(t k\rho g \sin\theta / \mu)^{1/3} < y < a(t k\rho g \sin\theta / \mu)^{1/3}$$

However, this model neglects the effects of small scale dispersivity owing to the tortuosity of the flow through the permeable layer. This will cause the tracer to mix into neighbouring fluid as it advances through the reservoir. If the reservoir is vapour dominated, then most of this dispersion will



Small-scale dispersion of tracer

The solution presented here for the downslope spreading of a plume of injected fluid provides a powerful means of predicting the motion of liquid through a reservoir. This may then be used to track the advance of a tracer injected with the input fluid at some time. The model identifies that the tracer will propagate downslope with

occurs along the current, causing the pulse of tracer to disperse in the along flow direction over a thickness $(Dt)^{1/2}$ after time t .

Liquid into liquid injection

A similar approach may be used to model the flow of cold liquid injected into a warm reservoir; after the flow has reached a

steady state and cooled the host rock along the path of the flow (cf Phillips 1991), so that effects of thermal inertia do not cause heating of the injectate, the injected fluid will migrate along the lower boundary of the domain, in a cooled layer of rock, driven by the density contrast between this fluid and the host reservoir fluid (assuming the mineral load of the host fluid is sufficiently small that the density of the warm host fluid remains less than the cold injectate. However, in all above expressions, $\delta\rho$ should be used rather than ρ , where $\delta\rho$ is the difference in density between the host fluid and the injectate.

Conclusions

A simple analytical model has been developed to track injected liquid as it spreads along

the inclined boundary of a geothermal reservoir. The model provides useful scalings for the dispersal of liquid through a reservoir and an important reference for checking more sophisticated numerical predictions.

Future work will address the initial stages of the flow, accounting for the vaporisation and cooling of the rock.

Acknowledgements

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References

Phillips, O, 1991, Flow and reactions in permeable rocks, CUP.