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THREE-DIMENSIONAL MODELING OF SP DATA

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ABSTRACT

Since the SP method is sensitive to primary flows, such as heat flow or fluid flow, it has been used often to detect or monitor geothermal systems. Forward and inverse modeling, hopefully in three-dimensions, is important to fully assess the information content of these data. Three-dimensional (3D) forward modeling is a straightforward generalization of a technique advanced by Sill (1983) and illustrated by him for two-dimensional (2D) structures. The technique is illustrated using an analytic simulation of a convection cell.

The inverse problem is an instance of the source identification problem familiar from other fields such as cardiology. It reduces to a Fredholm Integral Equation of the First Kind, and as such is ill-posed and ill-conditioned. Regularization of the equation using some means can give a source distribution, which must then be interpreted in terms of the primary flow parameters.

Interpretation of data gathered over the Newcastle geothermal system in terms of a radially symmetric fluid flow source distribution illustrates these concepts in a particularly nice manner.

INTRODUCTION

The spontaneous potential (SP) method measures the electrical manifestation of other primary flows, such as heat flow or fluid flow. As such, it has been often mentioned as a method of detecting or monitoring geothermal systems (Zohdy et al., 1973; Corwin and Hoover, 1979). Since the SP method is sensitive to so many different phenomena it has many possible applications, but the interpretation of the data can be exceedingly difficult due to the number of degrees of freedom in the inverse problem.

In the past twenty years, a number of remarkable works addressing this problem in various manners have been published.

Somasundaran and Kulkarni (1973).

FORWARD MODELING OF SP

Numerically modeling the SP response using the Sill approach involves several steps. In the first step, the primary flows, be they heat or mass, are predicted using some algorithm and a model of physical properties such as permeability or thermal conductivities. The second step involves calculating equivalent electrical sources from model cross-coupling coefficients. The third step introduces the equivalent sources into an algorithm which calculates the voltages on the earth's surface or in a borehole due to the equivalent electrical sources distributed throughout the primary flow regime.

In this work, the first step uses analytic or numerical techniques which calculate the heat and mass flow in convecting systems. The particular technique which we will use will depend on the application and hence will be discussed at the time the application is introduced. For the second step, we will depend on the cross-coupling compilations mentioned above. The third step uses either analytic expressions for the electric potential of sources, if we assume that the variation of electrical conductivity is at most one dimensional, or uses an integral equation formulation based on the work of Hohmann (1975), if the electrical conductivity is permitted to vary spatially. The process is summarized in the step by step procedure below.

Step 1 - Calculation of primary flows

This step depends on the application. Sometimes definitive information concerning the **SP** anomaly can be deduced from the theory of the primary flow.

Step 2 - Calculation of eauivalent sources

Consider a geological system possibly containing electric, thermal, hydrologic, and chemical fluxes, denoted individually by Ji, and the associated forces Xi. In general, a particular flux Jj is linearly related to all of the forces via the matrix equation

$$J_j = L_{ij} X_i. \qquad (1)$$

The entries L_{ij} **are** the cross-coupling coefficients. In this equation, we have assumed that the geological system **is** isotropic at the scale of the physical property calculations. If anisotropic material **is** desired, then the cross-coupling coefficients become cross-coupling matrices. If the off-diagonal entries of the cross-coupling matrix are assumed to be zero, then the scalar relationships expressed by the diagonal terms will be Ohm's law, Fourier's law, Darcies law, and Fick's law. Further details concerning these matrix and scalar relationships can be found in Onsager (1931), Kittel (1958), Freeze and Cherry (1979), and Waldram (1985).

Under geological conditions of interest to us, some assumptions concerning the Lij terms are defensible. First, since we will concentrate on pressure and temperature sources, the 4x4 matrix becomes a 3x3 matrix. Then since the temperature and pressure sources are impressed and are not influenced by the value of the electric potential, the matrix decouples into the three equations

$$J_{heat} = -K_{therm}\nabla\tau - L_{therm,\,mass}\nabla h, \qquad (2)$$

$$J_{mass} = -L_{mass,therm}\nabla\tau - K_{mass}\nabla h, \qquad (3)$$

and

$$J_{elec} = -L_{elec,therm}\nabla\tau - L_{elec,mass}\nabla h - \sigma\nabla\phi, \qquad (4)$$

where h is the hydraulic head, τ is the temperature, ϕ is the electric potential, σ is the electrical conductivity, K_{mass} is the hydraulic conductivity, and K_{therm} is the thermal conductivity.

If there are no external electric sources, then $\nabla . J_{elec} = 0$, and substituting from equation (4) gives

$$\nabla . (\sigma\nabla\phi) = \nabla . (-L_{elec,therm}\nabla\tau - L_{elec,mass}\nabla h). \qquad (5)$$

Since the cross-coupling coefficients enter into the definition of the source for the **SP** equation (5), the SP for a given pressure and temperature source can be calculated only if the cross-coupling coefficients are known - hence the importance of understanding the possible range of these coefficients (Sill, 1983). If we assume that electrokinetic effects are predominant, equation (5) becomes

$$\nabla . (\sigma\nabla\phi) = \nabla . (-L_{elec,mass}\nabla h). \qquad (6)$$

This equation can be given an alternate form if we define the streaming potential as (Sill, 1982) $CP = L_{elec,mass} / \sigma$.

Equation *(6)* assumes that the fluid flow is caused by a pressure gradient resulting from a pressure source or sink. Sill (1982) notes that a similar equation in terms of the fluid velocity can be derived, which is the appropriate equation to use when the flow is thermally driven. The analogous equation is then

$$\nabla\cdot(\sigma\nabla\phi) = -\nabla\cdot(L_v V). \quad (7)$$

In (7), $L_v = L_{elec,mass} / k$, where k is the fluid permeability, and V is the fluid velocity. If divergence-free flow is assumed, (7) becomes

$$\nabla\cdot(\sigma\nabla\phi) = -\nabla L_v \cdot v, \quad (8)$$

or in terms of C_p,

$$\nabla\cdot(\sigma\nabla\phi) = -\nabla(\sigma C_p / k)\cdot V;$$
$$\nabla\cdot V = 0. \quad (9)$$

Step 3- Analytic and Integral Equations Modeling of Voltage Response

The equivalent sources of equations (6) or **(9)** can now be substituted as an electrical source into any algorithm which calculates the DC voltage response of impressed electrical current sources. Sill **(1983)** used a finite difference algorithm, which dealt with a two-dimensional model. While the two-dimensional modeling algorithm is fast and should be appropriate in many cases, true three-dimensional modeling capability is always desirable, both to discern when three-dimensional effects are important and to interpret data so affected.

Three dimensional effects can be estimated in many ways. In general we use an integral equations code developed and discussed by Hohmann **(1975).**

The forward modeling technique can be illustrated using an analytic simulation of a two-dimensional plume in a convecting slab given by Sill **(1982)**. In our work, we use the plume with a limited strike extent, thus making a three-dimensional flow field. Sill's original model for the plume was a parametric divergence free velocity field designed to exemplify the geometric characteristics of a convecting system. The components V_x and V_z for this model are

$$V_x = v_0\, a^2\, \beta x\, (1-\beta z)\, (\exp(1-\beta z))\, (a^2+x^2)^{-1} \quad (10)$$

and

$$V_z = -v_0\, a^2\, (a^2-x^2)\, \beta z\, (\exp(1-\beta z))\, (a^2+x^2)^{-2}. \quad (11)$$

In this model, the free parameters a and β scale the horizontal and vertical extent of the convective cell. Figure 1 shows a contour plot for the velocity field for the cell. The analysis of Lapwood (**1948)** gives the perturbation about the reference temperature T_0 as

$$T - T_0 = -2V_z / \alpha g k, \quad (12)$$

where

g= acceleration of gravity,

k = permeability,

and

α = derivative of water density with temperature.

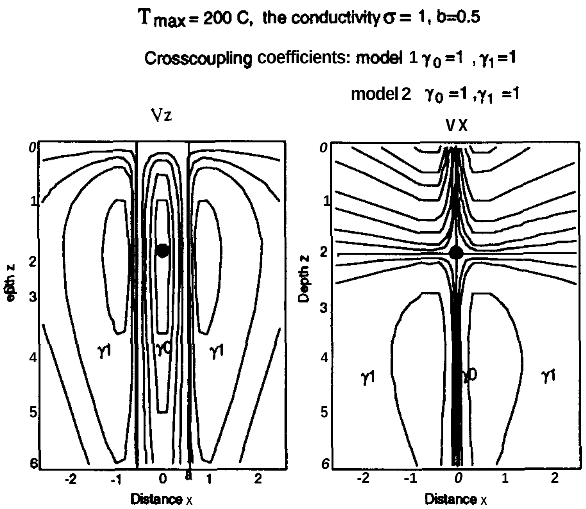

Fig. 1. Contour plot for velocity field.

Combination of the temperature and velocity models gives the temperature distribution

$$\Delta T(x,z) = T(x,z) - T_0$$
$$= -T_m V_z(x,z) / V_0, \quad (13)$$

where

$T_m = 200°$ C

and

$V_0 = 1.$

Taking experimental data from Ishido and Mizutani **(1981)**, Sill develops the temperature models

$$L_v(T) = L_v(T_0)\,(1. + 10^{-2}\, \Delta T(x,z)) \quad (14)$$

and

$$\sigma(T) = \sigma(T_0)\, \eta(T_0) / \eta(T). \quad (15)$$

Figure **1** shows the geometry of the cross-coupling models. Two sets of cross-coupling parameters were used - one in which $L_\gamma^0(T_0) = L_\gamma^1(T_0) = 1$, denoted Model 1, and Model 2, where $L_\gamma^0(T_0) = 1$ and $L_\gamma^1(T_0) = 0$. In both cases, $\sigma(T_0) = 1$S/m, while $\alpha = \beta = .5$.

Figure 2 shows the SP anomalies associated with Models **1** and 2.

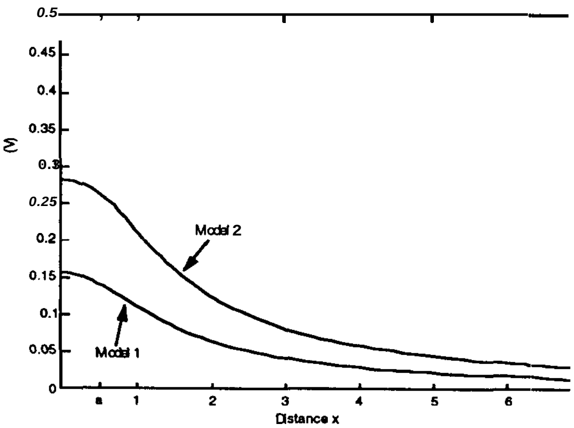

*Fig. 2. **SP anomalies associated with models 1 & 2.***

INVERSE MODELING

In forward modeling, we use primary flow patterns and cross-coupling coefficients to calculate equivalent sources, which can then be used to calculate electric potentials at any point in the model earth. This process is well-posed in that any flow and cross-coupling model has a unique electrical response. It is well-conditioned in that a small amount of noise in the flow and cross-coupling parameters is not amplified in the electrical response.

In the inverse problem, this sequence of steps is reversed. First, measured electric potentials are used to determine equivalent electric sources. These sources are then used to determine the flow parameters.

The first stage of this process is equivalent to solving the integral equation (Hohmann, **1987**),

$$\Phi(\mathbf{r}) = \int S(\mathbf{r'})\, G_0(\mathbf{r}, \mathbf{r'})\, dv', \qquad (16)$$

for the electric source distribution $S(\mathbf{r'})$. In this equation, $\Phi(\mathbf{r})$ is the distribution of measured electric potentials at the observation sites $\mathbf{r}$, $G_0(\mathbf{r}, \mathbf{r'})$ is the Green's function for the background earth, and the integration is over the support for the source distribution. The source distribution $S(\mathbf{r'})$ also has the representation $S(\mathbf{r'}) = \nabla'\sigma(\mathbf{r'}) \cdot \nabla'\Phi(\mathbf{r'}) / \sigma(\mathbf{r'})$ where $\sigma(\mathbf{r'})$ is the conductivity distribution.

The second step of the process is to use the distribution $S(\mathbf{r'})$ in the equation

$$\nabla \cdot (L_{e,h} \nabla \zeta_{Heat} + L_{e,m} \nabla \zeta_{Mass}) = \nabla'\sigma(\mathbf{r'}) \cdot \nabla'\Phi(\mathbf{r'}) / \sigma(\mathbf{r'}), \qquad (17)$$

in the case of a flow arising from a gradient or the equation

$$\nabla \cdot (L_{e,m} V / k) = \nabla'\sigma(\mathbf{r'}) \cdot \nabla'\Phi(\mathbf{r'}) / \sigma(\mathbf{r'}), \qquad (18)$$

for a non-conservative flow.

Following this solution stream in the case of real field data is a non-trivial task. The source identification problem is equivalent to solving a Fredholm Integral Equation of the First Kind, and as such is an archetypal ill-posed and ill-conditioned problem (Delves and Walsh, **1974;** Zabreyko et al., **1975;** Tripp, **1982**). This type of problem has been encountered in many fields, including cardiology, antenna analysis and design, and signal processing. The classic "fix" for the ill-posedness and ill-conditioning is to impose some a-priori information in the form of a "regularization condition". In our application, the a-priori information must be geological.

Newcastle Data Set

Some of the inversion concepts can be illustrated by an examination of a SP data set gathered over the Newcastle geothermal area in southwest Utah, as discussed by Ross et al. **(1990).**

The Newcastle system is a "blind" hydrothermal system located on the southeast edge of the Escalante Valley. It was discovered in **1975** by water well drilling and has since been the subject of extensive geological and geophysical investigations. The interested reader is referred to Ross et al. for a general discussion of geology and geophysics.

An SP survey was completed over the heat flow anomaly. Figure 3 shows the SP data, while Figure **4**

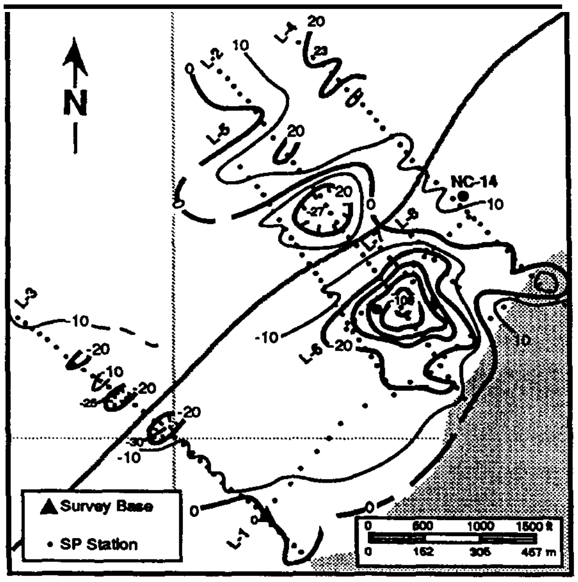

Fig. *3. **SP data.***

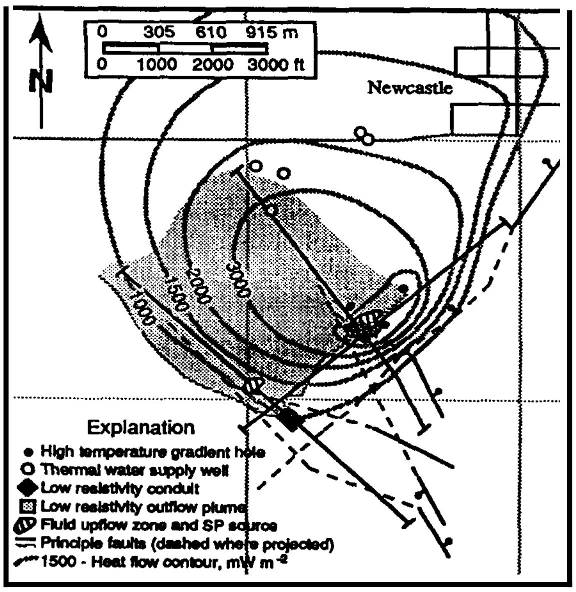

***Fig.* 4.** *Heat flow* **data.**

illustrates the heat flow data. The heat flow data shows a radially symmetric pattern "flattened" on one side by the range front faulting in the area. The SP data has a major circularly-symmetric anomaly with a peak value of - 108 mV. A smaller anomaly with a **peak** value of - 27 mV is located towards the valley fill. Some minor trending along the fault is noticable. Dipole-dipole resistivity data were also gathered in the area.

Ross et al. interpret the SP data as the signature of an upwelling fluid source along fault intersections. This model can be represented as

$$\phi(r) = -\int (\rho(z)\,\mu(z)C_0(z)Q(z)) \,/\, 2\pi k(z)\, \| (r,z) \|_2 \, dz, \quad (19)$$

which is the response of a line source of varying physical properties. In this equation, $\rho(z)$ is the resistivity profile, $\mu(z)$ is the viscosity, $C_0(z)$ is the cross-coupling coefficient, $Q(z)$ is the flow, and $k(z)$ is the permeability. Thus, a radially symmetric SP response can support only a line source interpretation, and the data only give an estimate of the line integral of the source term $S(z) = (\rho(z)\,\mu(z)C_0(z)Q(z)) \,/\, 2\pi k(z)$. Estimation of the source term is an ill - posed and ill-conditioned problem and requires some geological regularization condition. Even given an accurate estimate of $S(z)$, deconvolution of the source into physical parameters requires a great deal of a-priori information to begin. An elaborate **flow** parameter interpretation of this SP data set, by itself, clearly would be misleading.

PLANS AND CONCLUSIONS

Although these conclusions might be viewed as disparaging the use of SP, they actually support its use by defining its role. Some salient points are:

1) Evaluating an integral is not a bad application. It is, after all, an invariant which is proportional to flow magnitude and as such might be viewed as an ideal surface indicator of a flow source.

2) Subsequent flow inverse modeling might well use the SP defined invariant as a constraint on the interpretation.

3) The ill-posed and ill-conditioned nature of the SP inverse problem is a consequence of the fact that there are many different source mechanisms for SP and hence many different applications. However, an application must be carefully defined and constrained so that the exact nature of the information to be derived from the SP is understood.

The approach suggested by Sill for SP forward modeling is applicable to 3D, and **as** such is available for examining the nature of the SP response. Inverse modeling is dependent on hard constraints, and should be done carefully.

It seems to the authors that limited work has been done determining the exact information content in **SP** data vis-à-vis primary flows.

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