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CONVECTIVE STABILITY OF FLUID IN TWO-LAYER GEOTHERMAL STRATUM.

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ABSTRACT

Thermal regime of a geothermal system considerably depend on availability or absence of free fluid convection in it. Other side geothermal systems are characterized by significant heterogeneity and, specifically, permeability and other parameters may change from layer to layer . So, estimations of permeability coefficients of geothermal systems, according to reference data, differ by several orders [3] . Usually for estimation of convection onset in these systems the value of the critical Rayleigh number $4\pi^2$ is used, talang place for uniform layer [2]. In case of significantly heterogene layers the values of the critical Raileigh number, calculated by mean permeability, as will show below, may considerable differ fiom $4\pi^2$. In connection to indicated and other factors the problem about onset of fluid convection in a geothermal stratum, consisting of two horizontal sublayers with different permeabilities is considered. The parametrical investigation of the probem in dependance on permeability and thickness of layers is carried out. It is necessary to mark, that permeabilities of real geothermal strata may have and essential anisotropy. Influence of uniform anisotropy in the simplest case was investigated by Epherre [1]. Understandmg of quntitative and qualitative influence of indicated and other factors on origin and structure of a stream in a geothermal system will assist to interpretate the geophysical experimental data, specifically, by heat flow. As it has been shown in [3], convection in geothermal systems is not exclusion, but a d e and induces a positive anomaly of heat flow on the Earths surface above a geothermal system.

FORMULATION OF THE PROBLEM

The porous region (stratum) consists of two horizontal layers with permeabilities and widths k_1, h_1, k_2, h_2 accordingly. It is supposed for simplicity, that *all* effective thermophysical parameters of both layers, specifically thermal conductivities coinsides and are stable. Let us direct the axis OZ up along vertical line and OX along a horizontal are. The coordinate Z is counted off from common boundary. The upper boundary of the upper layer and the lower boundary of the lower layer are impermeable for liquid and on them the constant temperature is maintained (the lower temperature is more). On the common boundary the normal component of velocity, pressure, temperature and heat flow are continuous. Let us write the nondimensional set of equations for free convection in porous medium in the Darcy- Boussinesq approximation [2]. Counting *off* the pressure and the temperature from their distributions for mechanical equilibrium, we have

$$\mathbf{u}_j = -w_j R(\nabla p_j - T_j \mathbf{e}_z),\ j=1,2$$

$$\mathrm{div}\,\mathbf{u}_j = 0 \qquad (1.1)$$

$$b\frac{\partial T_j}{\partial t} + \mathbf{u}_j . \nabla T_j - \mathbf{u}_j . \mathbf{e}_z = AT_j$$

$$R = \frac{k\rho g\beta A h^2}{\chi\mu}, \quad b = \frac{m(\rho c_p)_c}{(\rho c_p)_f}, \quad \chi = \frac{\lambda}{(\rho c_p)_f}$$

$w_1=k_1/k_s$, $w_2=k_2/k_s$, $k_s=k_1h+k_2h_2$, $h_1+h_2=1$

Boundary conditions

$$\mathbf{u}_1\mathbf{e}_z = 0,\ T_1=0,\quad (z=h_1)$$

$$T_1=T_2,\ \frac{\partial T_1}{\partial z} = \frac{\partial T_2}{\partial z},\ p_1=p_2,\ (\mathbf{u}_1-\mathbf{u}_2)\mathbf{e}_z=0, \quad (A) \qquad (1.2)$$

$$\mathbf{u}_2\mathbf{e}_z=0,\ T_2=0,\ (z=-h_2)$$

where the indices 1,2 concern the upper and the lower layers accordingly; $\mathbf{u}$ is the velocity of liquid filtration, $\mathbf{e}$ is the single vector along the axis OZ; p is the pressure; T is the temperature ; A is the gradient of equilibrium distubution of the temperature; ρ is the liquid density ; β is the coefficient of heat expansion, λ is the coefficient conductivity; χ is the effective temperature conductivity, μ is the dynamic viscosity; C_P is the heat capacity; m is the porosity. k_1, k_2 are the permeabilities in the first and the second layers correspondingly; k_S is the mean permeability; R is the filter Rayleigh number, delined accordmg to integrated width of layers and mean permeability, indices f and c concern liquid and porous medium correspondingly. In (1.1) the following charakteristic scales are selected, h- of lengths; Ah- of temperatures; $\lambda / h(\rho c_p)_f$ - of velocity; $h^2 (\rho c_p)_f/\lambda$ - of time; ;$\beta\rho g A h^2$ - of pressure.

From the mathematical point of view the problem is come to find of nontrivial solution of the problem (1.1)- (1.2). Condition of existence of this solution permits to find the critical value of the Rayleigh number, depending on ratio of permeabilities and widths of layers at exceeding of whch mechanical equilibrium of fluid is impossible.

SOLUTION OF THE LINEARIZED SET OF EQUATIONS

For the problem in question , accorging to general methods [2], it is possible to show the principle of monotony of disturbances for heating from below. There fore for our aims it is enough *to* consider the stationary problem. By standart method after linearization let us reduce equations (1) to equations relatively temperature [2].

$$\Delta^2 T_j = -w_j R \Delta_{xy} T_j \qquad j=1,2 \quad , \qquad (2.1)$$

where Δ and Δ_{XY} are Laplasians in a space and the plane OXY accordingly.

Let us seek the solution in the form

$$T_j = \Theta_j(z)e^{ikr} \qquad , j=1,2 \quad . \qquad (2.2)$$

Substituting (2.2) in (2.1) and taking into account the boundary conditions (1.2), for $\Theta_j(z)$ we obtain

$$\left(\frac{d^2}{dz^2} - k^2\right)^2 \Theta_j = w_j k^2 R \Theta_j \, ,$$

$$k^2 = k_x^2 + k_y^2 \qquad , j=1,2 \qquad (2.3)$$

$$\Theta_1 = 0, \; \Theta_1'' = 0, \; (z=h_1)$$

$$\Theta_1 = \Theta_2 \qquad , \Theta_1' = \Theta_2', \; \Theta_1'' = \Theta_2''$$

$$\Theta_1^{(3)} - k^2\Theta_1' = w(\Theta_2^{(3)} - k^2\Theta_2') \; (z=0)$$

$$\Theta_2 = 0, \Theta_2'' = 0, \; (Z = -h_2)$$

where $\Theta_1' = d\Theta_1 / dz$, $\Theta_1'' = d^2\Theta_1 / dz^2$ etc.

The solution of the problem (2.3) may be obtained in the form

$$\Theta_1 = a_1 sh\lambda_1(z-h_1) + a_2 sh\lambda_2(z-h_1)$$

$$(2.4)$$

$$\Theta_2 = a_3 sh\lambda_3(z+h_2) + a_4 sh\lambda_4(z+h_2)$$

$$\lambda_1 = \sqrt{k^2 + \sqrt{w_1 k^2 R}}$$

$$\lambda_2 = \sqrt{k^2 - \sqrt{w_1 k^2 R}}$$

$$\lambda_3 = \sqrt{k^2 + \sqrt{w_2 k^2 R}}$$

$$\lambda_4 = \sqrt{k^2 - \sqrt{w_2 k^2 R}} \, .$$

Substituting (2.4) in (2.3) fiom the boundary conditions we obtain the set of four algebraic equations relatively a_i ;because of conditions on the upper and lower boundaries of the stratum are fulfilled beforehand. Conditions of solutability of this set permit to find the neutral curve sought. Solving after this the set of equations, we will find a_i; with exactness to constant multiple.

DISCUSSION OF THE RESULTS

Computations show, that the neutral curve in the problem in question unlike the case of an uniform layer, may have one or two minimums of the wave number in dependance on parameters correlation. Availability of two minimums has place for significantly different widths of layers and characterises conditions of stability loss of liquid of the whole system and in the layer with lesser width ,but with greater permeability. The considerably different wave numbers correspond to these mimimums. In Fig.1 and 2 we have plotted the minimal

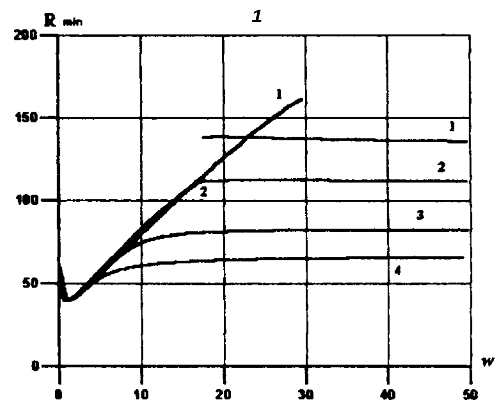

Fig. 1. The minimal Rayleigh number against *w*.

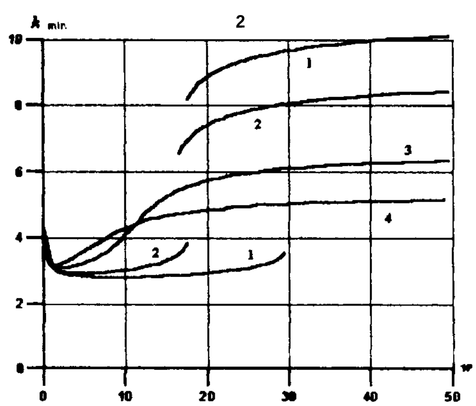

Fig.2. The wave number against *w*

.Rayleigh number and the wave number against $w = k1/k2$ sor h_1 = 0,25; 0,3; 0,4; 0,5 . As it can be seen in Fig.1, at h_1= 0,25 and w >20 the critical Rayleigh number is approximately 4 times more than whole system in dependance on , which of minimums is less.

REFERENCES

1. Gebkhart B., Jelurija J., Makhogan R., Summakija B. (1991). "Free convective currents, heat and mass exchange". M.: Mir, V.2, 528p.

3. Gershuni G.Z., Zhukhovitski E.M.,(1972), " Convective stability of noncompressible liquid ". M. ;Nauka, 392 p.

3.Karakhin A.V., Lobkovski L.I., Mjasnikov V.P.,(1984), "Hydrothermal convection in the upper layer of the Earthcrust and its influence to the geotermal gradient. Theoretical and experimental investigations on thermics of the seas and oceans." M Nauka,128 p

4 Magomedbekov Kh.G and Ramazanov M.M.,(1996) "Linear analysis of convective instability of a fluid in a horizontal annular cavity occupied by a porous medium," Fluid Dynamics, Vol.31, No 3