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METHOD OF CAPILLARY EFFECTS INCLUSION IN TWO-PHASE FLOW MODELLING OF A GEOTHERMAL RESERVOIR

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ABSTRACT

The present method of capillary effects inclusion in modelling of flow in porous and fractured media and the recent improvements are critically reviewed. The way of capillary pressure inclusion, based on volume averaging method, completed by some new formulae, is presented. A set of new equations and their benefits as compared with standard and improved models are discussed.

INTRODUCTION

In the last decade the problem of two-phase flow process inside geothermal reservoirs is attract big interest. This is connected with future plan of increase of reinjection in The Geysers, US, near start of commercial exploitation of Upper-Mutnovka and Mutnovka fields, Kamchatka, Russia (both with reinjection) and also with interest of Italian industry in exploitation of superheated steam regions (e.g., Monteverdi zone of the Larderello field).

Capillarity and adsorption are major processes controlled behavior of vapor-dominated fields (Home *et al.*, 1995; Sta.Maria and Home, 1996; Sta.Maria and Pingol, 1996). The above phenomena lead to effects of vapor-pressure lowering and heat pipes, which are important for accurate resource estimation, forecast of vapor-dominated field production performance and optimum production and reinjection program design. The importance of adsorption and capillarity forces has been determined also by direct experiments with Geysers rocks (Gruszkiewicz *et al.*, 1996) and Monteverdi cuttings (Bertani *et al.*, 1996).

Early geothermal reservoir simulators ignored the any capillary effects for two-phase flows. Obviously, at first Pruess (1991) pointed on the need of including of capillary effects in reservoir simulation and on theoretical and experimental development of capillary pressure relationships for real rocks. On the base of numerical simulation of reinjection problem, Pruess (1991) showed important role of capillary effects including for realistic vapor-dominated field's description.

Today most advanced simulators, e.g., TOUGH2 (Pruess, 1991, 1995, 1996), TETRAD, version 12 (also known as ASTRO), GSS and STAR (after Sta.Maria and Pingol, 1996; Sta.Maria and Home, 1996) include capillary and adsorption effects.

Presently many new laboratory experiments are in progress towards better understanding capillary and adsorption effects in porous and fractured media and in natural cores. The experiments are as follows: a study of relative permeability for steam-water flow (Ambusso *et al.*, 1996); capillary pressure curves (Persoff and Hulen, 1996) and adsorption measurements for real rocks (Bertani *et al.*, 1996; Gruszkiewicz *et al.*, 1996; Satik *et al.*, 1996); a study of boiling in porous media (Satik and Home, 1996).

Nevertheless, both in the above simulations (in governing equations' formulation), except in recent work of Pruess, 1995, and in experiments (in simulations for apparatus and methodology design and in formulae for experimental values' calculations) the standard Muskat-Leverett approach for two-phase flow modelling is exploited. This approach, adopted in petroleum reservoir engineering, consists in use of a standard multiphase extension of a Darcy's law and an empirical capillary pressure function in Leverett-like forms. Such approach may be advocated, as it is well realized in current literature (e.g., Barenblatt *et al.*, 1984), only when: (i) general flow is very slow; (ii) change of saturation goes in quasi-equilibrium manner; (iii) liquid and vapor does not hydraulically resist to each other and flow independently; (iv) distribution of phase is equal hydrostatic equilibrium one and therefore phase pressure's difference may be consider to be equal capillary pressure at macro scale; (v) gravity and capillary pressure drives dominate and therefore outer pressure drive is estimated to be a zero.

However standard approach is questionable at least by two reasons as follows.

- Presently there are not sound theoretical and experimental proofs of that approach and range of its validity. On the contrary, there is some contradiction experimental results (Barenblatt ***et** al.*, 1984), which conflict with condition (iii) of above paragraph. There is large theoretical diversity of treating the problem. The known mixing of scales (micro and macro) in definition of capillary pressure can lead even to paradoxes (Gray and Hassanizadeh, 1991a,b).

- Nature of petroleum reservoir engineering processes differ from geothermal one, where problems become more complicated because of boiling and condensation. Therefore it is rather difficulty to consider reinjection the immiscible displacement process but it is almost correct for flooding in petroleum case. Then errors concluded in standard approach may increase in the geothermal case. For instance Pruess (1991) notes that nearby the injection well pressure drive can be significant as compared with gravity and capillary drives. This fact conflicts with condition (v) from above paragraph.

Recently some attempts to intensify standard theory by means of explicit including capillary forces as drive forces have been appeared (Gray and Hassanizadeh, 1991, Pruess, 1995) and then to use such modified theory in the geothermal reservoir simulations (Pruess, 1995).

In current work the main aspect of the theory -- formulation of momentum equations for liquid and vapor phases -- is discussed. The lacks and advantages of standard and improved approaches are briefly reviewed. The new approach is suggested and preliminary set of the resulting equations is presented. The points of further needed theoretical works are sketched.

STANDARD APPROACH FOR CAPILLARY EFFECTS ACCOUNT

The standard extended Darcy's law for two-phase flow is formulated as (e.g., Barenblatt ***et** al.*, 1984):

$$\mathbf{u}_\beta = -\mathbf{K}\frac{k_{r\beta}}{\mu_\beta}\left(\nabla p_\beta - \rho_\beta \mathbf{g}\right), \qquad (1)$$

where $\mathbf{K}$ is absolute permeability tensor; k_β is relative permeability, which depend on saturation, $\beta = l, v$ (liquid and vapor), $\mathbf{u}_\beta$ and p_β are filtration velocity and pressure **of** the phase β.

The equation for capillary pressure (which typically is defined in the macro scale as $p_c = p_v - p_l$), as function of saturation, is usually used in Leverett-like forms as follows. (i) Leverett's form,

$$Pc = \sigma\sqrt{\phi/K}\,j(s_l), \qquad (2a)$$

where σ is the surface tension of water, ϕ is porosity, j is Leverett function (Collins, 1964), and (Pruess, 1991)

$$j = 1.417(1-s_l) - 2.120(1-s_l)^2 + 1.263(1-s_l)^3\,. \qquad (2b)$$

(ii) Van Genuchten form (Sta.Martia and Pingol, 1996),

$$Pc = p_o[s_{ef}^{-1/\lambda} - 1]^{-1/\lambda}, \qquad (3)$$

where effective liquid saturation is defined as $s_{ef} = (s_l - s_{lr})/(1 - s_{lr})$ and s_{lr} is the residual liquid saturation, λ is empirical coefficient. (iii) more general form (Sta.Maria and Pingol, 1996),

$$p_c = a[1 - s_l]^b, \qquad (4)$$

where a and b are fitting parameters, and (Persoff and Hulen, 1996)

$$p_c = a\,s_l^{\,b}\,. \qquad (5)$$

As it can be seen from Figure 1, different capillary pressure curves may be significantly different in forms and may give different absolute values.

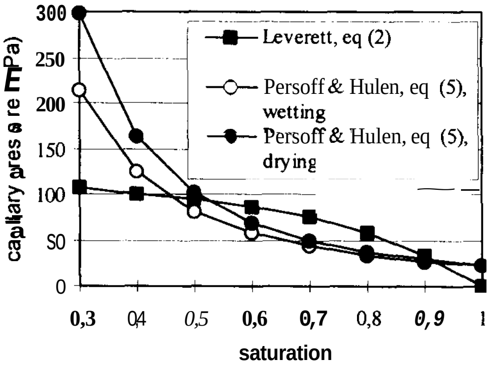

Fig. 1. Comparison of Leverett's capillary pressure curve with Persoff and Hulen's measurements (1996) for Geyser's core.

In Figure 1 Persoff and Hulen's equation (5), with experimental fitting parameters a=21.93, b=-1.89 for wetting and a=23.50, b=-2.11 for drying of Geyser's core from 319 m depth (core 26) with permeability 1.310^{-21} m^2 and porosity 1.9%, is compared with Leverett equation (2) for σ =0.073 N/m (20°C) and the same porosity and permeability. Perhaps the divergence of these curves (200% for saturation 0.3 to 0.5 and 30% for saturation greater 0.5) is caused by experimental method used by Persoff and Hulen (1996) – vapor pressure lowering, based on Kelvin equation.

As it has been mentioned in previous section the equation **(1)** does not satisfied to conditions of flow in geothermal reservoirs, especially in case of injection in vapor-dominated reservoir. To more clearly see that, let's subtract the equations (1) for liquid and steam phases, then we have:

$$\boldsymbol{u}_l \frac{\mu_l}{\boldsymbol{K}k_{rl}} - \boldsymbol{u}_v \frac{\mu_v}{\boldsymbol{K}k_{rv}} = \nabla p_c + (\rho_l - \rho_v)\boldsymbol{g} \tag{6}$$

From (6) it is follows that relative motion of phases depends on only gradient of macro capillary pressure and gravity forces. The gradient of capillary pressure (from any equations (2)-(5)) is defined by saturation gradient. So that in case of constant saturation (i.e., $\nabla s_l=0$) equation (5) *turns* into the equation followed from theory without capillary effect account. At the same time it is obviously clear that the relative motion must also be depended on gradients of phase pressure, even in case of constant saturation.

Moreover, it can be shown, that in equilibrium case standard equation (1) can lead to some paradoxes similar to negative absolute fluid pressure paradoxes in theory of unsaturated flow (Gray and Hassanizadeh, 1991a). Let consider vapor-water reservoir, consisted of equidistant vertical fractures in the hydrostatics state. Then (1) gives that phase pressure distribution is hydrostatic, same as in absence of capillarity. Nevertheless since the phase pressure in (1) means macro-scale pressure (average pressure in representative volume, as usually in filtration theory), from the physical point of view, phase's pressure gradients must explicitly depend on macro-scale capillary force besides gravity force.

Therefore it should be expected that for correct description of two-phase flow, equation (1) must has additional terms in right hand side, explicitly accounted capillarity, not simply by means of connection between phase's pressures in form of any equations (2)-(5).

RECENT IMPROVEMENTS OF STANDARD TWO-PHASE FLOW MODELLING WITH CAPILLARY EFFECT ACCOUNT

Obviously (after Bear ***et*** *al.*, 1968) at first Edlefsen and Anderson (1943) introduced capillary term in Darcy's law for unsaturated flow.

Later Gray and Hassanizadeh (1991b) formulated multiphase flow theory with Darcy's law including additional capillary terms. Their general momentum phase equation in the case of constant porosity, slow flow and negligible influence of one phase or interface movement on the adjacent interfaces reduces to equation as follows:

$$-\boldsymbol{R}_\beta^\beta \cdot \boldsymbol{v}_\beta = \nabla p_\beta - \rho_\beta \, \boldsymbol{g} + \\ + \rho_\beta \left[\frac{\partial A^\beta}{\partial a_{lv}} \nabla a_{lv} + \frac{\partial A^\beta}{\partial a_{\beta r}} \nabla a_{\beta r} + \frac{\partial A^\beta}{\partial s_l} \nabla s_l \right] \tag{7}$$

where v_β is velocity of β-phase, A^β is Helmholtz free energy per unit mass of the β phase, a_{lv} and $a_{\beta r}$ denote areas of the lv and βr interfaces per unit volume **of** medium, s_l is liquid saturation, subscript ***r*** denotes solid phase (i.e., rock). ***R*** is resistance tensor, connected with permeability tensor in such way, that for the unsaturated flow case, where solid matrix surface is totally wetted by the liquid (a_{vr}=0 and a_{lr} is constant), (7) reduced to

$$\boldsymbol{u}_l = -\boldsymbol{K} \frac{k_{rl}}{\mu_l} \left[\nabla p_l - \rho_l \boldsymbol{g} + \left(\Omega^l - \Omega^v \right) \frac{\nabla s_l}{s_l} \right], \tag{8}$$

where Ω^β is introduced by Gray and Hassanizadeh the wettability potential: $\Omega^\beta = \rho_\beta s_\beta \partial A^\beta / \partial s_\beta$.

Recently Pruess (1995) considered additional (that he called "dispersive") term in extended Darcy's law for liquid in form as follows:

$$\boldsymbol{u}_l = -\boldsymbol{K} \frac{k_{rl}}{\mu_l} \left(\nabla p_l - \rho_l \boldsymbol{g} \right) - \phi \, \boldsymbol{D}_{dis} \nabla s_l \, , \tag{9}$$

where $\boldsymbol{D}_{dis}$ is called the "dispersion" tensor. But the equation for steam velocity he keeps in form of (1), without any additional terms. Pruess (1995) also conducts numerical simulation by his TOUGH2 with included extension (9) for injection in vapor-dominated reservoir and shows that extra term strongly influences on injection plume shape (it increases lateral and decreases vertical plume dimensions) and on reservoir pressure distribution.

As one can see, equations (7) and (9) have similar extra capillary terms included liquid saturation gradient. Nevertheless both models have some lacks as follows. The Gray and Hassanizadeh's equations (7) and (8) were obtained by averaging theory and next use of thermodynamics of mixtures (Coleman and Noll method) for constitutive approximation of unknown terms, describing phase interaction on macro-scale level. As result, although their theory seems very strong and complicated, the equations contain unknown parameters demanding experimental measurement. Besides, the corollaries of their equations (7) and (8) (as well as of usual Darcy's law) can lead to paradoxes in full saturation hydrostatics case with spatially variable porosity (Pashkevich, 1996). The Pruess's equation (9) was obtained by simple extension of standard equation for liquid, based on Fickian diffusion analogy (Pruess, 1995). Besides, Pruess leaves equation for vapor in the standard form (1), without any capillary term, although it is clear that capillarity must influence on both liquid and vapor. Tensor $\boldsymbol{D}_{dis}$ presently is unknown and demands experimental measurement (of course, it is not equal standard dispersion tensor, since capillarity and diffusion are different processes). Therefore Pruess's approach for account capillary effects by adding (7) in standard theory seems rather incomplete.

PROPOSED APPROACH

The proposed approach for explicit account of capillary effects in two-phase flow modelling is concluded in use of new set of momentum equations for phases, derived by averaging of micro-scale steady first Cauchy law for each phase and phenomenological determination of unknown phase interaction terms. Derivation of the equations and connected formulae are presented in Appendixes A through **C.**

NEW SET OF THE GOVERNING EOUATIONS

The general momentum equation for liquid phase for quasi-equilibrium flow, where surface tension can be considered constant on vapor-liquid interface within averaging volume but varying in macro scale, is (see Appendix **A,** equation (A10)):

$$-\frac{\varepsilon_l\mu_l}{\boldsymbol{K}k_{rl}}\boldsymbol{v}_l-\frac{\varepsilon_l}{\boldsymbol{K}h_{lv}}\left(\boldsymbol{v}_l-\boldsymbol{v}_v\right)-\nabla\varepsilon_l p_l- \qquad (10)$$
$$-\frac{1}{V}\int_{A_{lv}}p_v{}'\boldsymbol{n}_{lv}dA+\sigma r\phi\nabla s_l+\varepsilon_l\rho_l\boldsymbol{g}=0.$$

This equation contains new unknowns: parameter $\boldsymbol{h}_{lv}$ which demands experimental measurements and the surface integral defining vapor pressure force on liquid-vapor interface which must be expressed in term of macro scale parameters. This is problem for further work. For the vapor phase a similar equation exists.

For slow flow, where drag force on vapor-liquid interface can be negligible the equation (10) becomes:

$$\varepsilon_l\boldsymbol{v}_l=\frac{\boldsymbol{K}k_{rl}}{}\quad\varepsilon_l p_l+\frac{1}{V}\int p_v{}'\boldsymbol{n}_{lv}dA- \qquad (11)$$
$$-\sigma r\phi\nabla s_l-\varepsilon_l\rho_l\boldsymbol{g}].$$

In the general case, for the mixture flow the equation exists as follows (see (A12)):

$$-\frac{\varepsilon_v\mu_v}{\boldsymbol{K}k_{rv}}\boldsymbol{v}_v-\frac{\varepsilon_l\mu_l}{\boldsymbol{K}k_{rl}}\boldsymbol{v}_l-\nabla\varepsilon_v p_v-\nabla\varepsilon_l p_l+ \qquad (12)$$
$$+\frac{1}{V}\int_{A_{lv}}p_c{}'\boldsymbol{n}_{lv}dA+\left(\varepsilon_v\rho_v+\varepsilon_l\rho_l\right)\boldsymbol{g}=0.$$

In quasi-equilibrium case mixture flow the equation (12) reduced to (see (A13)):

$$-\frac{\varepsilon_v\mu_v}{\boldsymbol{K}k_{rv}}\boldsymbol{v}_v-\frac{\varepsilon_l\mu_l}{\boldsymbol{K}k_{rl}}\boldsymbol{v}_l-\nabla\varepsilon_v p_v-\nabla\varepsilon_l p_l+ \qquad (13)$$
$$+\sigma r\nabla s_l+\left(\varepsilon_v\rho_v+\varepsilon_l\rho_l\right)\boldsymbol{g}=0.$$

Hydrostatics equation for liquid is (see (A11)):

$$\nabla\varepsilon_l p_l=\varepsilon_l\rho_l\boldsymbol{g}-\frac{1}{V}\int_{A_{lv}}p_v{}'\boldsymbol{n}_{lv}dA+\sigma r\phi\nabla s_l \qquad (14)$$

Hydrostatics equation for vapor-liquid mixture is (see (A14)):

$$\nabla\phi\left(1-s_l\right)p_v+\nabla\phi s_l p_l=\phi\left[\left(1-s_l\right)\rho_v+s_l\rho_l\right]\boldsymbol{g}+ \qquad (15)$$
$$+\sigma r\phi\nabla s_l,$$

or, with use of standard macro scale definition of capillary pressure $p_c=p_v-p_l$, in the other form:

$$\nabla\phi p_v-\nabla\phi s_l p_c=\phi\left[\left(1-s_l\right)\rho_v+s_l\rho_l\right]\boldsymbol{g}+ \qquad (16)$$
$$+\sigma r\phi\nabla s_l.$$

The hydrostatics equation, connecting the gradients of macro capillary pressure p_c and liquid saturation in the case of constant porosity and constant vapor pressure (e.g., for unsaturated porous media with constant air pressure), follows from (1***6):***

$$\nabla s_l p_c=-\left[\left(1-s_l\right)\rho_v+s_l\rho_l\right]\boldsymbol{g}-\sigma r\nabla s_l. \qquad (17)$$

The liquid pressure gradient from (17) becomes:

$$\nabla p_l=\left[\left(\frac{1}{s_l}-1\right)\rho_a+\rho_l\right]\boldsymbol{g}+\left[\frac{\sigma r+\left(p_a-p_l\right)}{s_l}\right]\nabla s_l,$$

or approximately for moderate air density:

$$\nabla p_l=\rho_l\boldsymbol{g}+\left[\sigma r+\left(p_a-p_l\right)\right]\frac{\nabla s_l}{s_l}. \qquad (18)$$

COMPARISON OF THE STANDARD. RECENT IMPROVED AND PROPOSED MODELS

Proposed here momentum equations for vapor and liquid phases differ from standard equations (1) by presence of additional terms explicitly accounting drag force on vapor-liquid interface and capillary force, expressed as term proportional liquid saturation gradient.

The momentum equation for vapor-liquid mixture has term expressed macro scale capillary force as integral over vapor-liquid interface of micro scale capillary pressure. This allows to differ macro and micro scale capillary pressure effects in explicitly manner, without previous mixing in standard approach.

Although derived here equations contain some terms demanding further reformulation through macro scale parameters, they have been obtained in clearly defined assumptions and explicitly account capillary pressure effects. As compared with advanced equations of Gray and Hassanizadeh (1991) and Pruess (1995), presented equations include less unclear parameters. Moreover, comparing (1 1) and (9) one can see that Pruess' rather unclear "dispersion" tensor $\boldsymbol{D}_{dis}$ is likely to be partly proportional to $\boldsymbol{K}k_{rl}r\sigma/\mu_l$. Besides, the capillary term must also exist in vapor momentum equation but not in only liquid one as it was suggested in (Pruess, 1995). And comparing corollary of Gray and Hassanizadeh's equation **(8)** for hydrostatics case with derived here equation (1**8)** (in same suggestions), one can obtain the explicit expression for equilibrium difference of wettability potentials: $(R'' - \Omega^l)_e = p_a - (p_l - \sigma r)$.

As one can show, the corollaries (15)-(16) of presented here momentum equations are free from known paradoxes (Gray and Hassanizadeh, 1991; Pashkevich, 1996) of standard approach in hydrostatics case.

CONCLUSION

Averaging method completed by new formulae for saturation gradient is strong basis for logical accounting of capillary pressure effects in models of two-phase flow in permeable media. Proposed set of momentum equations for liquid and vapor phases in different simplification levels provides consistent inclusion of capillary force in two-phase geothermal reservoir modelling as compared with standard approach. The derived here equations allow clearly understand influence of microscopic capillary forces in macro scale and avoid usual mixing of micro and macro scale capillarities in standard approach.

Presented equations contain fewer unknowns than equations in recent improvements of standard approach and even allow to clear up some unknown parameters of the improvements.

Hydrostatics corollaries of suggested momentum equations are free from paradoxes of standard model and lead to equation (17), connecting the gradients of macro scale capillary pressure and the saturation.

Nevertheless presented equations for each phase yet contain surface integral over vapor-liquid interface of opposite phase pressure, which demands presentation in terms of macro scale values. This is the problems for further work.

Presented result is another step for appropriate modelling of capillary effects in two-phase flow in geothermal reservoirs and can be useful for better understanding of capillarity role in specific reservoir engineering problems.

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APPENDIX A: *Derivation of governing equations.*

Let start from steady first Cauchy law for each phase in micro scale:

$$\nabla \mathbf{T}_\beta' - \nabla p_\beta' + \rho_\beta' \mathbf{g} = 0 , \qquad \text{(A1)}$$

where dash means micro scale value, $\mathbf{T}_\beta$ is viscous part of stress tensor.

In averaging theory (see, e.g., Slattery, **1972)** the phase average f_β of micro scale β phase value f_β', is defined as follows:

$$f_\beta = \frac{1}{V_\beta}\int_{V_\beta} f_\beta' \, dV = \frac{1}{\varepsilon_\beta V}\int_{V_\beta} f_\beta' \, dV , \qquad \text{(A2)}$$

where ε_β is phase fraction in average volume V:

$$\varepsilon_\beta = V_\beta / V . \qquad \text{(A3)}$$

In the two-phase flow phase fractions are connected with saturation and porosity as follows:

$$\varepsilon_l = V_l / V = \phi s_l , \; \varepsilon_v = V_v / V = \phi s_v = \phi(1 - s_l) \qquad \textbf{(A4)}$$

The Slattery-Whitaker's theorem says:

$$\frac{1}{V}\int_{V_\beta} \nabla f_\beta' \, dV = \nabla \varepsilon_\beta f_\beta + \frac{1}{V}\int_{A_{\beta\sigma}} f_\beta' \, \mathbf{n}_{\beta\sigma} dA , \qquad \text{(A5)}$$

where A, is a surface of β-σ interface, $\mathbf{n}_{\beta\sigma}$ is a outward unit normal vector to $A_{\beta\sigma}$.
Averaging of **(1)** over volume V gives:

$$\nabla \varepsilon_\beta \frac{1}{V}\int_{V_\beta} \mathbf{T}_\beta' \, dV + \frac{1}{V}\int_{A_{\beta\sigma}} \mathbf{T}_\beta' \, \mathbf{n}_{\beta\sigma} dA - \nabla \varepsilon_\beta p_\beta -$$

$$- \frac{1}{V}\int_{A_{\beta\sigma}} p_\beta' \, \mathbf{n}_{\beta\sigma} dA + \varepsilon_\beta \rho_\beta \mathbf{g} = 0.$$

The first term in above equation expresses internal viscosity forces in macro scale and often (see f.e Slattery, **1972)** suggested to be proportional to $\mu_\beta \Delta \mathbf{v}_\beta$ (so-called Brinkman's term for one-phase flow). Further consider slow flow so that this term will be neglected. Then it can write for liquid phase the equation as follows:

$$\frac{1}{V}\int_{A_{lr}} \mathbf{T}_l' \, \mathbf{n}_{lr} dA + \frac{1}{V}\int_{A_{lv}} \mathbf{T}_l' \, \mathbf{n}_{lv} dA - \nabla \varepsilon_l p_l -$$

$$- \frac{1}{V}\int_{A_{lr}} p_l' \, \mathbf{n}_{lr} dA - \frac{1}{V}\int_{A_{lv}} p_l' \, \mathbf{n}_{lv} dA + \varepsilon_l \rho_l \mathbf{g} = 0. \qquad \textbf{(A6)}$$

The first two terms in **(A6)** define resistance forces accordingly to motion of liquid through permeable rock (integral over liquid-rock surface) and to liquid motion with respect to vapor (integral over liquid-vapor interface). Fourth term in **(A6),** defining force of liquid pressure on pore or fracture wall, usually included in permeability tensor. Using approach analogues well known semi-empirical approach in numerous derivations of Darcy's law (see, e.g., Slattery, **1972)** one can write **(A6)** in form:

$$- \frac{\varepsilon_l \mu_l}{\mathbf{K} k_{rl}} \mathbf{v}_l - \frac{\varepsilon_l}{\mathbf{K} h_{lv}} (\mathbf{v}_l - \mathbf{v}_v) - \nabla \varepsilon_l p_l -$$

$$- \frac{1}{V}\int_{A_{lv}} p_l' \, \mathbf{n}_{lv} dA + \varepsilon_l \rho_l \mathbf{g} = 0, \qquad \textbf{(A7)}$$

where h_{lv} is some parameter determining liquid-and vapor flows mutual resistance such that $\varepsilon_l h_{lv} = \varepsilon_v h_{vl}$.

The equation for vapor is similar **(A7)**. Consider quasi-equilibrium flow such that a Young-Laplace law can be applicable on vapor-liquid interface:

$$Pc' = p_v' - p_l' = 2K_{Mlv}\sigma , \qquad \text{(A8)}$$

where K_M is mean curvature of the interface. Then substituting **(A8)** in **(A7)** we obtain:

$$- \frac{\varepsilon_l \mu_l}{\mathbf{K} k_{rl}} \mathbf{v}_l - \frac{\varepsilon_l}{\mathbf{K} h_{lv}} (\mathbf{v}_l - \mathbf{v}_v) - \nabla \varepsilon_l p_l -$$

$$\frac{1}{V}\int_{A_{lv}} p_v' \, \mathbf{n}_{lv} dA + \frac{1}{V}\int_{A_{lv}} 2K_{Mlv}\sigma \mathbf{n}_{lv} dA + \varepsilon_l \rho_l \mathbf{g} = 0. \qquad \text{(A9)}$$

Surface tension can be considered as constant on vapor-liquid interface within averaging volume (but varying in macro scale) and then using (B**17)** allows to write **(A9)** in form:

$$- \frac{\varepsilon_l \mu_l}{\mathbf{K} k_{rl}} v_l - \frac{\varepsilon_l}{\mathbf{K} h_{lv}} (v_l - \mathbf{v}_v) - \nabla \varepsilon_l p_l -$$

$$- \frac{1}{V}\int_{A_{lv}} p_v' \, \mathbf{n}_{lv} dA + \sigma r \phi \nabla s_l + \varepsilon_l \rho_l \mathbf{g} = 0 . \qquad \textbf{(A10)}$$

In the hydrostatics case, when velocities of phases are equal zero, **(A10)** is reduced to:

$$\nabla \varepsilon_l p_l = \varepsilon_l \rho_l \mathbf{g} - \frac{1}{V}\int_{A_{lv}} p_v' \mathbf{n}_{lv} dA + \sigma r \phi \nabla s_l \qquad \textbf{(A11)}$$

Sum of **(A7)** and the similar equation for vapor gives:

$$- \frac{\varepsilon_v \mu_v}{\mathbf{K} k_{rv}} \mathbf{v}_v - \frac{\varepsilon_l \mu_l}{\mathbf{K} k_{rl}} \mathbf{v}_l - \nabla \varepsilon_v p_v - \nabla \varepsilon_l p_l +$$

$$+ \frac{1}{V}\int_{A_{lv}} p_c' \, \mathbf{n}_{lv} dA + (\varepsilon_v \rho_v + \varepsilon_l \rho_l) \mathbf{g} = 0. \qquad \text{(A12)}$$

The surface integral in **(A 12)** explicitly defines macro scale capillary force as integral of micro scale capillary pressure over vapor-liquid interface. The **(A12)** differs from the equation followed from (1) in standard approach for capillary effects account. Substitution of **(A8)** and **(B17)** in **(A12)** yields:

$$- \frac{\varepsilon_v \mu_v}{\mathbf{K} k_{rv}} \mathbf{v}_v - \frac{\varepsilon_l \mu_l}{\mathbf{K} k_{rl}} \mathbf{v}_l - \nabla \varepsilon_v p_v - \nabla \varepsilon_l p_l +$$

$$+ \sigma r \nabla s_l + (\varepsilon_v \rho_v + \varepsilon_l \rho_l) \mathbf{g} = 0. \qquad \text{(A13)}$$

In the hydrostatics case, by using of **(A4)**, equation **(A** 13) reduced to:

$$\nabla \phi (1 - s_l) p_v + \nabla \phi s_l p_l = \phi [(1 - s_l)\rho_v + s_l \rho_l] \mathbf{g} + \qquad \text{(A14)}$$

$$+ \sigma r \phi \nabla s_l .$$

With use of standard macro scale definition of capillary pressure $p_c = p_v - p_l$, the (A14) becomes:

$$\nabla\phi p_v - \nabla\phi s_l p_c = \phi\left[(1-s_l)\rho_v + s_l\rho_l\right]\boldsymbol{g} + \\ + \sigma r\phi\nabla s_l . \tag{A15}$$

Finally, in the case of constant porosity and constant vapor pressure (e.g. case of unsaturated porous media with $p_v = p_a$, where p_a is constant air pressure) equation (A15) gives:

$$\nabla s_l p_c = -\left[(1-s_l)\rho_a + s_l\rho_l\right]\boldsymbol{g} - \sigma r\nabla s_l \tag{A16}$$

From (A16) the liquid pressure gradient is:

$$vp, = \left[\left(\frac{1}{s_l}-1\right)\rho_a + \rho_l\right]\boldsymbol{g} + \left[\frac{\sigma r + (p_a - p_l)}{s_l}\right]\nabla s_l$$

Neglecting air density as compared with liquid density in last equation we have:

$$\nabla p_l = \rho_l \boldsymbol{g} + \left[\sigma r + (p_a - p_l)\right]\frac{\nabla s_l}{s_l} . \tag{A17}$$

APPENDIX B: ***Derivation of formulae connecting interface surface integrals with porosity and saturation gradients.***

Consider averaging volume V in the multiphase system, consisted of a reference phase β and phase σ (which defined as a set of any number of other phases). The known corollaries of Slattery-Whitaker theorem (A5) are as follows: (i)

$$V\int_{V_\beta} f'_\beta dV = \int_{A_{\beta e}} f'_\beta \boldsymbol{n}_{\beta e} dA , \tag{B1}$$

where V_β is volume of β-phase, bounding by surface A_β, consisted of a surface of β-σ interface $A_{\beta\sigma}$ and a surface of β-phase's input and output in averaging volume $A_{\beta e}$ (i.e. β-phase's cross section of A): $A_\beta = A_{\beta e} + A_{\beta\sigma}$, and (ii)

$$V E_\beta = \frac{1}{V}\int_{A_{\beta e}} \boldsymbol{n}_{\beta e} dA = -\frac{1}{V}\int_{A_{\beta\sigma}} \boldsymbol{n}_{\beta\sigma} dA . \tag{B2}$$

Let f_β in (B1) will be the divergence of arbitrary vector $\boldsymbol{a}_\beta$ then, using divergence theorem, from (B1) one can obtain:

$$V\int_{A_{\beta e}} \boldsymbol{a}_\beta \boldsymbol{n}_{\beta e} dA + \nabla\int_{A_{\beta\sigma}} \boldsymbol{a}_\beta \boldsymbol{n}_{\beta\sigma} dA = \int_{A_{\beta e}} (\nabla\cdot\boldsymbol{a}_\beta)\boldsymbol{n}_{\beta\sigma} dA . \tag{B3}$$

Substitution of $\boldsymbol{a}_\beta = \boldsymbol{n}_\beta$ in (B3) with use of formula for surface's mean curvature $2K_M = -\nabla\cdot\boldsymbol{n}$ (Maugin, 1988), gives:

$$\nabla A_\beta = -\int_{A_{\beta e}} 2K_{M\beta e}\boldsymbol{n}_{\beta e} dA . \tag{B4a}$$

If $a_\beta = A_\beta / V$ means specific surface of β-phase, the last equation can be rewritten in followed form:

$$\nabla a_\beta = -\frac{1}{V}\int_{A_{\beta e}} 2K_{M\beta e}\boldsymbol{n}_{\beta e} dA . \tag{B4b}$$

Equations **(B4)** are additional useful corollaries of Slattery-Whitaker's theorem. **As** it is showed in Appendix C, the formulae **(B4)** can **also** be derived directly from transport theorem for closed surface, without addressing to Slattery-Whitaker's theorem, but following their approach.

From the other hand, remember variety of Stokes' theorem (Kom and Korn, 1968):

$$\int_A (\boldsymbol{n}\times\nabla)\times\boldsymbol{a}\, dA = \oint_C d\boldsymbol{r}\times\boldsymbol{a} , \tag{B5}$$

where C is bounding curve of the surface A. Substitution of $\boldsymbol{a} = \boldsymbol{n}$ in (B5), for the closed surface gives:

$$\oint_A (\boldsymbol{n}\times\nabla)\times\boldsymbol{n}\, dA = 0 . \tag{B6}$$

In the vector analysis for arbitrary vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ the followed equations exist (Korn and Kom, 1968):

$$2(\boldsymbol{a}\cdot\nabla)\boldsymbol{b} = \mathbf{V}\times(\boldsymbol{b}\times\boldsymbol{a}) + \nabla(\boldsymbol{b}\cdot\boldsymbol{a}) - \\ - \boldsymbol{b}(\nabla\cdot\boldsymbol{a}) + \boldsymbol{a}(\nabla\cdot\boldsymbol{b}) - \boldsymbol{b}\times(\nabla\times\boldsymbol{a}) - \boldsymbol{a}\times(\nabla\times\boldsymbol{b}) \tag{B7}$$

and (Kochine, 1951):

$$\times\nabla)\times\boldsymbol{b} - \boldsymbol{a}\times(\nabla\times\boldsymbol{b}) = (\boldsymbol{a}\nabla)\boldsymbol{b} - \boldsymbol{a}(\nabla\cdot\boldsymbol{b}) . \tag{B8}$$

Substitution of $\boldsymbol{a} = \boldsymbol{b} = \boldsymbol{n}$ in (B7) and (B8) gives:

$$(\boldsymbol{n}\nabla)\boldsymbol{n} = -\boldsymbol{n}\times(V\times\boldsymbol{n}) \tag{B9}$$

$$(\boldsymbol{n}\times V)\times\boldsymbol{n} - \boldsymbol{n}\times(V\times\boldsymbol{n}) = (\boldsymbol{n}\nabla)\boldsymbol{n} - \boldsymbol{n}(\nabla\cdot\boldsymbol{n}) . \tag{B10}$$

Siibstitution of (B9) in (B10) gives:

$$(\boldsymbol{n}\times V)\times\boldsymbol{n} = -\boldsymbol{n}(\nabla\cdot\boldsymbol{n}) = 2K_M\boldsymbol{n} . \tag{B11}$$

Substitution of (B11) in (B6) gives:

$$\oint_A 2K_M\boldsymbol{n}\, dA = 0 . \tag{B12}$$

Write (B12) for surface A_β which bounds β-phase volume V_β to obtain:

$$\int_{A_{\beta e}} 2K_{M\beta e}\boldsymbol{n}_{\beta e}\, dA = -\int_{A_{\beta\sigma}} 2K_{M\beta\sigma}\boldsymbol{n}_{\beta\sigma}\, dA . \tag{B13}$$

For the spherical averaging volume with constant radius $\boldsymbol{R}$ equation (B13) can be presented in the followed form (for sphere $K_M = -1/R$, Aris, 1962):

$$\frac{1}{V}\int_{A_{\beta\sigma}} 2K_{M\beta\sigma}\boldsymbol{n}_{\beta\sigma}\, dA = \frac{2}{VR}\int_{A_{\beta e}} \boldsymbol{n}_{\beta e}\, dA . \tag{B14}$$

Substitution of (B2) in (B14) yields:

$$\frac{1}{V}\int_{A_{\beta\sigma}} 2K_{M\beta\sigma}\boldsymbol{n}_{\beta\sigma}\, dA = \frac{2}{R}\nabla\varepsilon_\beta . \tag{B15}$$

The derived here formula (B15) is another useful component of volume averaging theory. In considered two-phase case (B15) yields (see (A4)):

$$\frac{1}{V}\int_{A_{vr}} 2K_{Mvr}\boldsymbol{n}_{vr}\, dA + \frac{1}{V}\int_{A_{vl}} 2K_{Mvl}\boldsymbol{n}_{vl}\, dA = \frac{2}{R}\nabla\phi(1-s_l),$$

$$V\int_{A_{lr}} 2K_{Mlr}\boldsymbol{n}_{lr}\, dA + \frac{1}{V}\int_{A_{lv}} 2K_{Mlv}\boldsymbol{n}_{lv}\, dA = \frac{2}{R}\nabla\phi s_l$$

and

$$\frac{1}{V}\int_{A_{rl}} 2K_{Mrl}\boldsymbol{n}_{rl}\,dA+\frac{1}{V}\int_{A_{rv}} 2K_{Mrv}\boldsymbol{n}_{rv}\,dA=-\frac{2}{R}\nabla\phi.$$

From the last tree equations we can conclude:

$$\frac{1}{V}\int_{A_{lv}} 2K_{Mlv}\boldsymbol{n}_{lv}\,dA=\frac{2}{R}\phi\nabla s_l\,,$$

$$\frac{1}{V}\int_{A_{rl}} 2K_{Mrl}\boldsymbol{n}_{rl}\,dA=-\frac{2}{R}s_l\nabla\phi, \qquad \text{(B16)}$$

$$\frac{1}{V}\int_{A_{rv}} 2K_{Mrv}\boldsymbol{n}_{rv}\,dA=-\frac{2}{R}(1-s_l)\nabla\phi.$$

The equations (B16) contain the radius of averaging volume *R*. In the averaging theory the dimension of average volume I must satisfy the inequality (see, e.g., Hassanizadeh and Gray, 1983): $l_o \ll I \ll L$, where l_o is micro scale dimension (effective pore radius, fracture aperture and so on), *L* is dimension of macro scale fluctuation. Then *R* must be at least one order greater pores' diameter or fractures' aperture. Introduce the notation $r=2/R$ then first equation of (B16) can be presented as follows:

$$\frac{1}{V}\int_{A_{lv}} 2K_{Mlv}\boldsymbol{n}_{lv}\,dA=r\phi\nabla s_l\,. \qquad \text{(B17)}$$

The equation (B17) is useful addition in averaging theory for two-phase flow and, as it is showed in the paper's main body, can help to express capillary force term in momentum equations by means of macro scale parameter – saturation gradient.

APPENDIX C: *Direct derivation offormula (B4).*

Use the approach analogues one of Slattery and Whitaker (1967,1972). Let any point on arbitrary curve in multiphase system is a centre of averaging volume *V* with bounding surface *A* (Fig. C1).

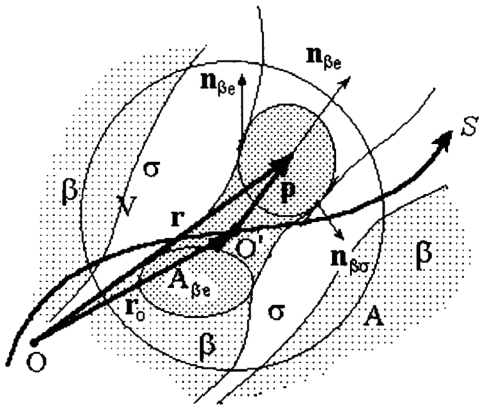

Fig. C1. Averaging volume with centre on arbitrary curve in multiphase system.

Then one can write: $V_\beta = V_\beta\,(S)$, $A_\beta = A_\beta\,(S)$, where *S* is arclength along the curve. Any point within averaging volume are defined in original coordinate system with centre **O** by means of vector ***r***, which also is function of arclength S: $\boldsymbol{r}=\boldsymbol{r}(S)$.

The transport theorem for moved 'material' closed surface ***A*** is as follows (Madelung, 1957, Maugin, 1988, Smirnov, 1931):

$$\frac{d}{dt}\oint_A \boldsymbol{an}dA = \quad \boldsymbol{d\,t} \quad \boldsymbol{w}\nabla\cdot\boldsymbol{a}\Big\}\boldsymbol{n}dA\,, \qquad \text{(C1)}$$

where ***a*** is arbitrary vector, defined on surface *A* with outward unit normal ***n***, ***a=a(r,t)***, *A=A(**r**,t)* ***n=n(r,t)*** and ***w*** is the velocity of fictitious particles consisted surface ***A***: ***w=dr/dt***. The theorem (C2) directly follows from well-known Reynolds transport theorem for volume.

In right side of (C1) time derivative exists with spatial, not surface, fixed coordinates of given point on surface *A*, i.e. with fixed vector **r** of points of surface *A*. In general case surface ***A*** and vector field ***a*** depend on time and spatial position. Nevertheless, following method of Slattery and Whitaker one can formally change scalar time ***t*** on scalar arclength S: *t*→*S*. Then, ***a=a(r(S))***, *A=A(**r**(S))*, ***w*** →***dr****/dS*, *d/dt*→ *d/dS*, ∂/∂*t*→ ∂/∂***S***. Let ***a=n***, then since ***n***·(∂ ***n***/∂ *S*)=0 and (C1) exists for any moment of time, from transport theorem (C1) one can obtain:

$$\frac{d}{dS}\int_{A(\boldsymbol{r},t)} dA=\int_{A(\boldsymbol{r},t)}\nabla\cdot\boldsymbol{n}\,\frac{d\boldsymbol{r}}{dS}\cdot\boldsymbol{n}dA\,. \qquad \text{(C2)}$$

Present vector ***r*** as sum of vector $\boldsymbol{r}_0$, which defines position of averaging volume centre O'and vector ***p***, which defines position of points on A_β relatively volume centre (Fig. C1). The vector ***dr/dS*** is tangent to the *β-σ* interface and the vector ***dp/dS*** is tangent to the surface $A_{\beta e}$ (Slattery, 1972, Howes and Whitaker, 1985). Then considering (C2) for surface A_β, since integral in left hand side of (C2) is function of averaging volume centre position, we have:

$$\frac{d\boldsymbol{r}_0}{dS}\nabla_0 A_\beta=-\int_{A_l} 2K_{M_{\beta e}}\frac{d\boldsymbol{r}_0}{dS}\cdot\boldsymbol{n}dA \qquad \text{(C3)}$$

where ∇_0 is defined in the macro coordinate system with origin **O. As** curve is arbitrary and therefore $\boldsymbol{dr_0/dS}$ is arbitrary too, from (C3) we finally have:

$$\nabla A_\beta=-\int_{A_{\beta e}} 2K_{M_{\beta e}}\boldsymbol{n}dA\,. \qquad \text{(C4)}$$

The formula (C4) coincides with (B4a).