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SUBSURFACE PROPAGATION OF THERMO-MECHANICAL FRACTURE SHOCK WAVES IN HYDROTHERMAL REGIMES

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ABSTRACT

In this paper a one-dimensional analytical model for the mechanism of rock fracturing in hydrothermal regimes is described. The model is based upon the modern thermo-poro-elasticity theory and its two non-linear heat-like equations. On the boundary aquifer-caprock a buried thermo-mechanical source is supposed to be built up in terms of subsurface fluid-rock coupling dynamics. With this considerations it is assumed that fluid pore pressure approaches the breakdown pressure of the caprock such that this starts failing. In order to study rock fracturing the variability of fluid-rock thermal diffusivity and fluid diffusivity at the onset of rock failure is taken into account in the two non-linear heat-like equations. A series of fluid-rock temperature and fluid pore pressure shock wave fronts is obtained as a solution of Burgers' equation. These fronts, moving through the caprock as propagating fractured boundaries, are referred to as thermo-mechanical fracture shock waves. It is found that the speed of thermo-mechanical fracture shock waves propagation and their amplitude are governed by a coefficient Γ, whose value depends on the role played by fluid-rock thermal diffusivity and fluid diffusivity variations. As a result, the final effect of thermo-mechanical fracture shock waves is the migration of the top of the aquifer towards a shallower depth with associated increases in the superficial thermo-mechanical outputs. Moreover, if thermo-mechanical fracture shock waves are maintained from below by the arrival of other waves, the fractured boundary can further migrate upwards and eventually trigger a catastrophic event such as a phreatic eruption.

INTRODUCTION

Thermo-mechanical instabilities such as ground surface displacements in hydrothermal domains have been interpreted in terms of a buried thermo-mechanical source. Generation has been currently considered by either a shallow magma body characterised by episodic mafic and silicic magma mixing, as it is supposed to have happened for example in Yellowstone, Long Valley and Campi Flegrei silicic calderas, or due to the growth of a batholith (Dvorak and Dzurisin, 1997 and bibliography therein).

Recently, an alternative interpretation to this current hypothesis has been proposed, for which thermo-mechanical instabilities may even be triggered by a buried thermo-mechanical source related to subsurface water-rock coupling dynamics (Bonafede, 1991 and 1997; Merlani et al., 1996 and 1997; Natale and Salusti, 1996). As a consequence of such a new tendency and in order to interpret the last ground surface uplift at Campi Flegrei (1982-84, Italy), Bonafede (1991) initiated the application of modern thermo-poro-elasticity theory to water-rock coupling dynamics at the thermo-mechanical buried source. This was considered to be generated at the bottom of an aquifer and formally interpreted in terms of two non-linear heat-like equations as already stated by Rice and Cleary (1976) and McTigue (1986). From this, he perturbatively modelled the observed ground surface uplift in terms of migration through the Phlegraean aquifer of hot and pressurised supercritical water able to thermo-elastically deform the overburden rock. In the light of Bonafede's study, Natale and Salusti (1996) analytically found that water-rock coupling dynamics at the buried source could generate thermo-mechanical shock waves, migrating throughout either deformable pervious or deformable semi-pervious horizons.

Here we examine the case in which modern thermo-poro-elasticity theory can even hold to describe rock fracturing through low permeability horizons. Such horizons are thought of as overlying hydrothermal aquifers and acting as caprocks. With these considerations rock fracturing is supposed to start at the aquifer-caprock boundary. By analytically developing the two non-linear one-dimensional heat-like equations upon which modern thermo-poro-elasticity theory is based, namely the stress-diffusion equation and the fluid-rock energy equation, it is assumed that at the onset of rock failure fluid-rock thermal diffusivity and fluid diffusivity changes in the heat-like equations. The stress-diffusion equation is here adopted as stated by McTigue (1986). The fluid-rock energy equation is formulated as recently proposed by Natale (1998). The model is based upon data from the last ground surface uplift at Campi Flegrei in Italy (1982-84).

However, it is applicable to any hydrothermal domains as long as conditions for rock failure exist at depth. Finally, Westerly granite is taken into account as the rock type representing the caprock.

THE SYSTEM AND THE HEAT-LIKE EQUATIONS

Let **us** consider a homogeneous, isotropic and laterally boundless water-saturated porous horizon acting as cap rock on an underlying aquifer in the underground of a hydrothermal domain (fig. 1). Such a horizon has a thickness $z = b$, with the

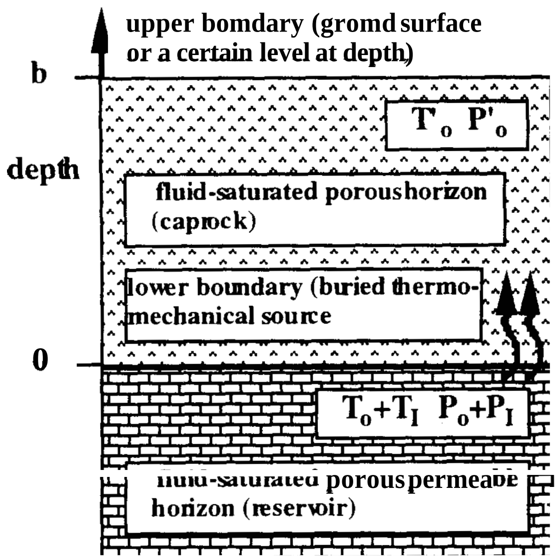

*Fig. 1. Conceptual geological section of a stratified system for a hydrothermal domain. Westerly granite **is** assumed as a rock type representative of the horizon overlying the aquifer. Rock fracturing **is** supposed to start on the boundary aquifer-caprock as a buried thermo-mechanical source supplies a temperature $T_0 + T_I$ and an associated fluid pore pressure $P_0 + P_I$ which approaches the breakdown pressure of **the** overburden **rock** at depth*

lower boundary at $z = 0$ corresponding to the frontier with the aquifer and the upper one at $z = b$ coinciding with either the ground surface or a certain level at depth. The upper boundary $z = b$ has a temperature T_0^I and a pressure P_0^I respectively supplied by the thermal conductive regime and the hydrostatic pressure gradient existing through the fluid-saturated caprock. Let us then suppose that a steady state regime exists in the aquifer such that at $z = 0$ fluid-rock temperature and fluid pore pressure are maintained at T_0 and P_0, respectively. Furthermore, the system is subjected to zero tectonic stress, so that it **is** in an isotropic **state** of stress, and failure occurs when the fluid pore pressure has reached a value P_F such that the effective vertical stress **is** negative and exceeds the tensile strength of the overburden rock. Let us now envisage that at the boundary $z = 0$ such a value P_F is generated by a buried thermo-mechanical source arising in terms of fluid-rock temperature T_I and fluid pore pressure P_I perturbations. **As** before, fluid-rock local thermal equilibrium is considered and rock fracturing is assumed to be one-dimensional along the z-direction only. In the present one-dimensional model the choice of the z-direction is due to the fact that for natural fracturing within the earth's crust the vertical gradient stress is proportional to rock density and is larger than the vertical fluid pore pressure gradients. This means that the effective stress is decreasing in the upwards direction promoting upwards propagation of fractures.

As already discussed by Natale and Salusti (1996), if τ is the time scale of the process and λ its space scale, it follows that $t \Rightarrow t/\tau$ and $z \Rightarrow$ **2/13,** thus the two non-linear heat-like equations describing the thermal $T \Rightarrow T/T_I$ and mechanical $P \Rightarrow P/P_I$ process can be adimensionalised and written as:

(1) $$\frac{\partial P}{\partial t} - H\frac{\partial^2 P}{\partial z^2} - A\frac{\partial T}{\partial t} = 0$$

(2) $$\frac{\partial T}{\partial t} - K\frac{\partial^2 T}{\partial z^2} - B\frac{\partial P}{\partial z}\frac{\partial T}{\partial z} - X\left(\frac{\partial P}{\partial z}\right)^2 +$$

In equations (1) and (2) above the parameters are given by (Tables I and 11):

(3) $$H = h\frac{\tau}{\lambda^2} = \left(\frac{K_I}{\mu}\frac{2GB_S^2(1+v_u)^2(1-V)}{9(1-v_u)(v_u-v)}\right]\frac{\tau}{\lambda^2}$$

i.e. the adimensional fluid diffusivity,

(4) $$A = \alpha\frac{T_I}{P_I} = (G\alpha_m\frac{4(1+V)}{3(1-V)} + GB_S\phi(\alpha_f - \alpha_m)\frac{2(1+v)(1+v_u)}{3(v_u - V)})$$

$$\frac{B_S(1-v_u)(1-v)}{3(1-v)(1+v_u) - 6(v_u - V)}\frac{T_I}{P_I}$$

i.e. the adimensional source term due to the differential fluid-rock thermal expansivities,

(5) $$K = k\frac{\tau}{\lambda^2} = \frac{K_T}{\phi\rho_f c_f + (1-\phi)\rho_m c_m}\frac{\tau}{\lambda^2}$$

i.e. the adimensional average thermal diffusivity due to diffusion,

$$B = \beta \frac{P_I \tau}{\lambda^2} = \frac{K_f}{\mu} \frac{P_I \tau}{\lambda^2} \quad (6)$$

i.e. the adimensional average thermal diffusivity due to convection.

$$X = \chi \frac{P_I^2}{T_I} \frac{\tau}{\lambda^2} = \frac{\beta}{\rho_f c_f} \frac{P_I^2}{T_I} \frac{\tau}{\lambda^2} \quad (7)$$

i.e. the adimensional average dissipative diffusivity due to fluid-rock friction, and finally

$$\Xi = \xi \frac{P_I \tau}{\lambda^2} = \frac{\rho_f R K_f}{\phi\mu\left[\phi\rho_f c_f + (1-\phi)\rho_m c_m\right]} \frac{P_I \tau}{\lambda^2} \quad (8)$$

i.e. the adimensional expansion work term related to the fluid thermal expansivity.

Material property	Westerly granite	Units SI
ϕ	10^{-2}	
K_f	$4\ 10^{-19}$	m^2
K_T	3	$J/m.s\cdot{}^0C$
G	$1.5\ 10^{10}$	Pa
B_S	0 85	
ν	0 25	
ν_u	0 34	
α_m	$2.4\cdot10^{-5}$	${}^0C^{-1}$
α_f	10^{-3}	${}^0C^{-1}$
μ	$8\ 10^{-5}$	$Pa\cdot s$
ρ_m	$2.7\ 10^3$	Kg/m^3
ρ_f	$3.6\ 10^2$	Kg/m^3
c_m	10^3	$J/Kg\cdot{}^0C$
c_f	$2.1\ 10^3$	$J/Kg\cdot{}^0C$
R	$4.6\cdot10^2$	$J/Kg\cdot{}^0C$

ϕ: porosity, K_f: permeability, K_T: average thermal conductivity, G: shear modulus, B_S: Skempton parameter, ν: drained Poisson's ratio, ν_u: undrained Poisson's ratio, α_m: rock thermal expansivity, α_f: fluid thermal expansivity, μ: viscosity, ρ_m: rock density, ρ_f: fluid density, c_m: rock heat capacity, c_f: fluid heat capacity, R: gas constant

Table I. Definition and values of the physical quantities for water-saturated Westerly granite at fluid-rock temperature $T_0 = 400°C$ and fluid pore pressure $P_0 = 3\cdot10^7\,Pa$

As boundary conditions we assume:

$$T = T_0 + T_I,\quad P = P_0 + P_I,\quad \text{at } z = 0$$
$$T = T_0^I,\quad P = P_0^I,\quad \text{at } z = b \quad (9)$$

where T_I and P_I are respectively temperature and pressure perturbations at the buried source established at $z=0$ and $t=0$. Consequently the initial conditions correspond to:

$$T = T_0^I,\quad P = P_0^I,\quad z \geq 0$$
$$T = T_0 + T_I,\quad P = P_0 + P_I,\quad z \leq 0 \quad (10)$$

Material property	Westerly granite	Units SI
α	$1.7\cdot10^6$	$Pa/{}^0C$
$\beta \cong \beta^I$	$5\cdot10^{-13}$	$m^2/Pa\cdot s$
h	$2.7\cdot10^{-4}$	m^2/s
k	10^{-6}	m^2/s
χ	$2\cdot10^{-21}$	$m^2/Pa\cdot{}^0C\cdot s$
ξ	$3\cdot10^{-14}$	$m^2/Pa\cdot s$
A	4	
$B \cong B^I$	0.2	
H	6	
K	$2\cdot10^{-2}$	
X	$3\cdot10^{-4}$	
Ξ	10^{-2}	
λ	10^3	m
τ	$2\cdot10^{10}$	s

$\tau = \lambda^2/\alpha(\beta^I + \xi)T_I$: time scale, λ : space scale. $H = h\tau/\lambda^2$: dimensionless fluid diffusivity, $K = k\tau/\lambda^2$: dimensionless average thermal diffusivity due to diffusion, $A = \alpha T_I/P_I$: dimensionless source term due to the differential fluid-rock thermal expansivity, $B = \beta P_I\tau/\lambda^2$: dimensionless average thermal diffusivity due to convection, $B^I = B + AX$, $X = \chi P_I^2\tau/T_I\lambda^2$: dimensionless average dissipative diffusivity due to fluid-rock friction, $\Xi = \xi P_I\tau/\lambda^2$: dimensionless fluid expansion work term

Table II. Definition and values of the parameters in equations (1) and (2) for water-saturated Westerly granite at fluid-rock temperature $T_0 = 400^0C$ and fluid pore pressure $P_0 = 3\cdot10^7\,Pa$ with perturbations $T_I = 50^0C$ and $P_I = 2\cdot10^7\,Pa$

THERRIO-MECHANICAL SHOCK WAVES AS A SOLUTION OF BURGERS' EOUATION FOR ROCK DEFORMATION

As already reported by Natale and Salusti (1996) for rock deformation, when temperature and pressure perturbations arise within a fluid-saturated porous medium, in the case of small fluid diffusivity h the stress-diffusion equation (1) reduces to:

$$(11)\quad \frac{\partial P}{dt} \cong A\frac{\partial T}{\partial t}$$

and the energy equation *(2)*can be rewritten as:

$$(12)\quad \frac{\partial T}{at} - K\frac{\partial^2 T}{\partial z^2} - AB'\left(\frac{\partial T}{\partial z}\right)^2 - A\Xi T\frac{\partial^2 T}{\partial z^2} = 0$$

with $B' = B + AX$. The derivative of equation *(12)* in terms of z is:

$$(13)\quad \frac{\partial}{\partial t}\frac{\partial T}{\partial z} - K\frac{\partial}{\partial z}\frac{\partial^2 T}{\partial z^2} - C\frac{\partial T}{\partial z}\frac{\partial^2 T}{\partial z^2} = 0$$

with $C = A(2B' + \Xi)$. The above expression is Burgers'equation for:

$$(14)\quad c(z,t) = -C\frac{\partial T}{\partial z} \cong -\frac{C}{A}\frac{\partial P}{\partial z}$$

with the general solution given by (Whitham, 1974):

$$(15)\quad \frac{\partial T(z,t)}{\partial z} = -\frac{c(z,t)}{C} = -\sqrt{\frac{K}{t}}\cdot\frac{1}{C}\,\frac{\left(e^{\Re}-1\right)e^{-z^2/4Kt}}{\sqrt{\pi} + \left(e^{\Re}-1\right)\int_{z/\sqrt{4Kt}}^{\infty} e^{-\varsigma^2}\,d\varsigma}$$

ruled by the number $\Re = CT_I/2K$, i.e. by the ratio of fluid convective effects to fluid-rock diffusive ones. If $\Re$ is small the diffusive regime prevails over the convective one and equation (15) above reduces to the diffusive case (Natale et al., 1998):

$$(16)\quad \frac{\partial T}{\partial z} \propto \frac{1}{\sqrt{t}}\exp\left(-\frac{z^2}{4Kt}\right)$$

Instead, if $\Re$ is large it yields convective effects prevailing over diffusive ones (Table III) and the solution of Burgers' equation **(13)** has the following form defining the propagation of a fluid thenno-mechanical solitary shock wave through the porous medium (Whitham, 1974):

$$(17)\quad c(z,t) \cong \begin{cases} z/t & 0 \le z \le \sqrt{2CT_I t} \\ 0 & \text{otherwise} \end{cases}$$

A Darcy's velocity front $v_D = \lambda/\tau$ is formed at $z_I = \sqrt{2CT_I t}$, the velocity value across the wave front jumps from zero to $c_f = \sqrt{2CT_I/t}$ and the wave front moves at speed $v_I = \sqrt{CT_I/2t}$.

Material properties	Westerly granite	Units SI
v_D	$5\cdot 10^{-8}$	$m\cdot s^{-1}$
$\Re$	$2\cdot 10^{3}$	
C	1.6	

$v_D = \lambda/\tau$: Darcy's velocity, $\Re = CT_I/2K$, C: coefficient governing the amplitude and speed of the wave

*Table III. Dejinition and **values** of the parameters for **rock** deformation*

THERMO-MECHANICAL FRACTURE SHOCK WAVES AND THEORETICAL DEFINITION OF THE COEFFICIENT Γ

Let **us** now consider the case in which istantaneous fluid-rock temperature and fluid pore pressure changes generate a buried thermo-mechanical source at the boundary aquifer-caprock, such that rock deformation leads to rock failure and in particular the caprock starts failing (fig. 1). With these considerations we investigate the case in which fluid pore pressure $P_0 + P_I$ approaches the breakdown pressure P_F of the overburden rock. Let **us** further assume that at the onset of rock failure the parameters in equations (1) and *(2)*vary and that such variations might be explained in terms of a series expansion in $T - T_0$ and $P - P_0$. In the case of small fluid diffusivity h, equation (1) becomes $\partial_t P = A\partial_t T$ and the variability of the parameters yields (Natale et al., 1998):

$$(18)\quad S = S_0\left(1 + S_I\frac{T - T_0}{T_I} + \ldots\right)$$

where S denotes one of the parameters, S_0 and S_I respectively its initial values and its variation at the onset of rock failure. **At** first order in H and for $T \approx T_0$ the stress-diffusion equation (1) becomes:

$$(19)\quad \frac{\partial P}{\partial t} - A_0H_0\frac{\partial^2 T}{\partial z^2} + \frac{A_0H_0H_I}{T_I}\left(\frac{\partial T}{\partial z}\right)^2 + -A_0\frac{\partial T}{\partial t} = \partial\left(H^2\right)$$

which can be solved as (Natale et al., 1998):

$$(20)\quad P = P_0 + A_0(T - T_0) + A_0 H_0 \int_0^t \frac{\partial^2 T}{\partial z^2} dt' + \frac{A_0 H_0 H_I}{T_I}\int_0^t \left(\frac{\partial T}{\partial z}\right)^2 dt' + \partial(H^2)$$

Now by inserting solution (20) and relation (18) into the fluid-rock energy equation (2), disregarding terms in H^2 and again assuming $T \approx T_0$, we arrive at:

$$(21)\quad \frac{\partial T}{dt} - K_0 \frac{\partial^2 T}{\partial z^2} - \frac{K_0 K_I}{T_I}\left(\frac{\partial T}{\partial z}\right)^2 \qquad \partial T$$
$$+ D_0 H_0 \frac{\partial T}{\partial z}\frac{\partial}{\partial z}\int_0^t \left[\frac{\partial^2 T}{\partial z^2} + \frac{H_I}{T_I}\left(\frac{\partial T}{\partial z}\right)^2\right] dt'$$

with $D_0 = A_0\left(B_0^I + \Xi_0\right)$. This equation is rather complex since it consists of an integro-differential nonlinear partial differential equation. However, we now show that if one is interested in non-linear signals, i.e. for large $\Re$, the temperature is again a solution of:

$$(22)\quad \frac{\partial T}{\partial t} - K_0 \frac{\partial^2 T}{\partial z^2} - \Gamma\left(\frac{\partial T}{\partial z}\right)^2 = 0$$

with the coefficient Γ to be determined. Caiiing $\tilde{T}$ the solution of (22) above, we have to discuss the integrals:

$$(23)\quad D_0 H_0 \frac{\partial \tilde{T}}{\partial z}\frac{\partial}{\partial z}\int_0^t \left[\frac{\partial^2 \tilde{T}}{\partial z^2} + \frac{H_I}{T_I}\left(\frac{\partial \tilde{T}}{\partial z}\right)^2\right] dt'$$

For strong non-linear signals the first integral in (23) above gives no contribution as shown in the Appendix B. To estimate the second integral, from equation *(22)* we have:

$$(24)\quad \frac{\partial}{\partial z}\int_0^t \left(\frac{\partial \tilde{T}}{\partial z}\right)^2 dt' = \frac{1}{\Gamma}\frac{\partial}{\partial z}\int_0^t \frac{\partial \tilde{T}}{\partial t} dt' - \frac{K_0}{\Gamma}\frac{\partial}{\partial z}\int_0^t \frac{\partial^2 \tilde{T}}{\partial z^2} dt' = \frac{1}{\Gamma}\frac{\partial \tilde{T}}{\partial z}$$

with Γ given by:

$$(25)\quad \Gamma = D_0 + \frac{K_0 K_I}{T_I} + \frac{D_0 H_0 H_I}{T_I \Gamma} \cong D_0 + \frac{K_0 K_I}{T_I} + \frac{H_0 H_I}{T_I}$$

where $D_0 = A_0\left(B_0^I + \Xi_0\right)$, H_0 and K_0 are the initial values of the parameters before the arrival of the perturbation and H, and K_I the variations of fluid and thermal diffusivities at the onset of rock failure. By deriving equation (22) in terms of z a renormalised Burgers' equation:

$$(26)\quad \frac{\partial}{\partial t}\frac{\partial T}{\partial z} - K_0 \frac{\partial}{\partial z}\frac{\partial^2 T}{\partial z^2} = 2\Gamma \frac{\partial T}{\partial z}\frac{\partial^2 T}{\partial z^2}$$

with the following solution:

$$(27)\quad c(z,t) = -2\Gamma\frac{\partial T}{\partial z} \approx \begin{cases} z/t, & 0 \le z \le \sqrt{4\Gamma T_I t} \\ 0, & \text{otherwise} \end{cases}$$

Γ is the overall effect of the above quantities governing the rapidity of thermo-mechanical fracture waves propagation. This new Burgers' equation is such that $\tilde{\Re} = \Gamma T_I / K_0 > 1$. A Darcy's velocity front is formed at $z_f = \sqrt{4\Gamma T_I t}$, the Darcy's velocity value across the front, moving at speed $v_{front} = \sqrt{\Gamma T_I / t}$, jumps from zero to $c_f = \sqrt{4\Gamma T_I / t}$. We underline that the above solution is valid for $T \approx T_0$, i.e. near the wave head where the greatest temperature gradient occurs and where fracturing processes can hold if the fluid pore pressure carried up by the wave approaches the breakdown pressure of the overburden rock (fig. *2).*

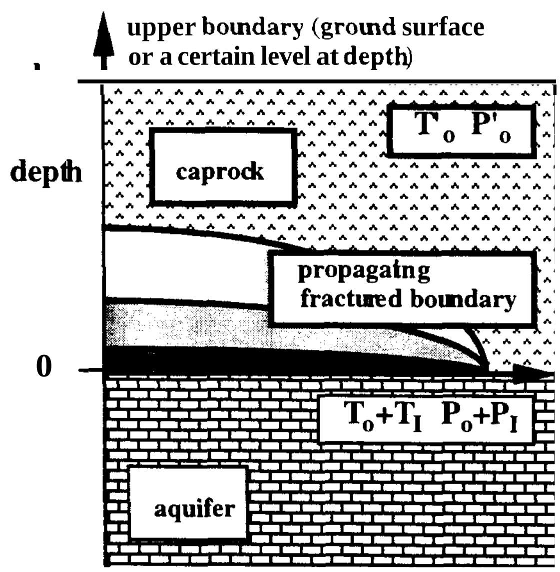

Fig. **2.** *Qualitative diagram of* ***the*** *integral of solution (27).*

CONCLUSIONS

It has been shown that the one-dimensional non-linear heat-like equations (1) and (2), recently adopted to interpret ground surface displacements caused by hot and pressurised water migration in the underground of hydrothermal domains (Bonafede, 1991 and 1997; Merlani et al., 1996, 1997; Natale and Salusti, 1996), can even hold to describe fracturing processes of a caprock overlying an aquifer (Natale et al., 1998), when fluid pore pressure $P_0 + P$, reaches the breakdown pressure P_F at the top of the aquifer under an isotropic state of stress.

Under the assumption of constant elastic parameters G. B_S, v and v_u of the considered rock type, constant thermal expansivities, rock density, fluid and rock heat capacitites and finally constant fluid viscosity, fluid-rock thermal diffusivity K and fluid diffusivity H variations in equations (1) and *(2)* have been studied at the onset of rock failure. By assuming K and H variations in terms of a series expansion in $T-T_0$ and $P-P_0$, a re-normalised Burgers' equation has been defined (expression (26)), with the term Γ called thermo-mechanical coefficient controlling the amplitude and speed of thermo-mechanical fracture shock waves (expression (25)).

In conclusion, it has been proposed that rock fracturing can be explained in terms of thermo-mechanical shock waves associated with a fractured boundary migration given by a solution of Burgers' equation for strong non-linear signals. These non-linear signals have been thought of as being generated by temperature and pressure perturbations at the buried source at the bottom of the caprock. The fractured boundary migration is governed by the thermo-mechanical coefficient Γ which causes a rapid temporal weakening of the amplitude of the induced fractured wave fronts. An attempt to estimate Γ at rock failure from expression (25) may be given by evaluating porosity change ϕ_I, which determines fluid-rock thermal diffusivity variation K,, and permeability change K,, , which determines fluid diffusivity variation H,. These estimations are intrisically empirical and they depend on both laboratory experiments on the specific rock type and the specific natural system. **It** must be noted that the series expansion adopted to study K and H variations restrains the validity of the model at the beginning of the fracture wave arrival, i.e. at the wave head where $T-T_0 < T_I$ and $P-P_0 < P$, and the greatest temperature and pressure gradients occur. The final effect of these fractured wave fronts is the migration of the top of the aquifer towards a shallower depth. If these fractured wave fronts are maintained by the arrival of a series of shock waves from ,below, a catastrophic event such as a phreatic explosion may happen.

APPENDIX A

The stress-diffusion is adopted as stated by McTigue (1986). The fluid-rock energy equation is formulated as recently proposed by Natale (1997). They are derived as a combination of the continuity equation of the fluid, Darcy's flow, the ideal gas state equation, the first law of continuum thermodynamics in the case of negligible termo-elastic coupling terms representing sources due to adiabatic deformation (McTigue, 1986), the equilibrium equation of elasticity and the constitutive relation between stress, strain, fluid-rock temperature and fluid' pore pressure with constant elastic parameters.

Continuity eauation

(A1) $$\frac{\partial \rho_f}{\partial t} + \frac{\partial(\rho_f v_{f_z})}{\partial z} = 0$$

where ρ_f and v_{f_z} are the density and the vertical component of fluid velocity.

Darcy's law

(A2) $$v_D = \phi v_{f_z} = \frac{K_f \ \partial P}{}$$

where inertia and body forces, and rock velocity are neglected. In **(A2)** above K, is the permeability of the rock, ϕ its porosity, μ the viscosity of the fluid and P the added fluid pressure onto the equilibrium hydraulic pressure.

. irst law of continuum thermodvnamics

The complete expression of the energy balance for the fluid-rock mixture, in which a local thermal equilibrium between fluid and rock is assumed, rock velocity and thermo-elastic coupling terms representing sources of adiabatic deformation are neglected, can be written as (Bejan, 1984; McTigue, 1986; Natale and Salusti, 1996; Merlani et al., 1997):

(A3) $$\left[\phi\rho_f c_{p_f} + (1-\phi)\rho_m c_m\right]\left(\frac{DT}{Dt}\right)_{(z)} = K_T \frac{\partial^2 T}{\partial z^2} + \alpha_f T\left(\frac{DP}{Dt}\right)_{(z)} - \phi v_{f_z}\frac{\partial P}{\partial z}$$

Ideal gas state eauation:

The thermodynamic behaviour of supercritical water is simplified by the ideal gas state equation. The ideal gas state equation is used, inasmuch as the properties of the gaseous state ***per se*** are considered:

(A4) $$P = \rho_f RT$$

where R is the gas constant and T the temperature.

Equilibrium equation of elasticity

The momentum balance for the fluid-rock mixture or the equilibrium equation of elasticity is simply given by:

$$\text{(A5)} \quad \frac{\partial \sigma_{ij}}{\partial x_j} = 0$$

where the total stress σ_{ij} is defined by the following constitutive relation between stress, strain, pressure and temperature (Rice and Cleary, 1976; McTigue, 1986):

$$\text{(A6)} \quad \sigma_{ij} = 2G\left(e_{ij} + \frac{v}{1-2v} e_{kk}\delta_{ij}\right) + - \frac{3(v_u - v)}{B_S(1+v_u)(1-2v)} - \frac{2G(1+v_u)}{3(1-2v)} \alpha_m T \delta_{ij}$$

with e_{ij} the strain, G the shear modulus, B_S Skempton parameter, α_m the rock thermal expansivity, v (v_u) drained (undrained) Poisson's ratio and with inertia and body forces neglected.

APPENDIX B

As discussed in Natale and Salusti (1996) and Natale et al. (1998), to compute the first integral in (23) it must be noted that it refers to two zones. In the first zone, characterised by points (t, z) before the arrival of the shock wave $\left(t_f, z_f = \sqrt{2C_0 T_I t_f}\right)$, one has $c(z,t) = 0$. In the second zone after the arrival of the wave the solution is related to Burgers' equation, namely $c(z,t) \cong z/t$. It follows that for $t > t$, we can write $\partial^2 T/\partial z^2 = -1/C_0 t$. The term containing the integral gives:

$$\text{(B1)} \quad \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\frac{\partial}{\partial z}\int_{t_f}^{t}\frac{\partial^2 T}{\partial z^2}dt'\right) = -\frac{1}{C_0}\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\frac{\partial}{\partial z}\int_{t_f}^{t}\frac{1}{t'}dt'\right|$$

In computing this integral we must remember that on the boundary between the two zones separated by the shock, $z_f = \sqrt{2C_0 T_I t_f}$ and equation (B1) gives:

$$\frac{\partial}{\partial z}\left[-\frac{1}{C_0}\frac{1}{t}\ln\left(\frac{t}{t_f}\right)\right] = -\frac{1}{C_0}\frac{\partial}{\partial z}$$

$$\text{(B2)} \quad \left[\frac{z}{t}\frac{\partial}{\partial z}\ln\left(\frac{t}{z^2}2C_0\right)\right] = \frac{1}{C_0}\frac{\partial}{\partial z}\left[\frac{z}{t}\frac{\partial}{\partial z}\ln\left(z^2\right)\right] =$$

$$= \frac{1}{C_0}\frac{\partial}{\partial z}\left(\frac{z}{t}\frac{2}{z}\right) = 0$$

which finally yields the same functional solution as the original Burgers' equation.

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REFERENCES

Bejan , **A.** (1984), "Convection heat transfer," J. Wiley & Sons, New York.

Bonafede, M. (1991), "Hot fluid migration: an efficient source of ground deformation of the 1982-1985 crisis at Campi Flegrei-Italy," J. Volcanol. Geotherm. Res., **48,** 187-198.

Bonafede, M. (1997), "Hot fluid migration in compressible saturated porous media," Geophys. J. Int., **128,** 383-398.

Dvorak, J.J. and Dzurisin, D. (1997), "Volcano geodesy: the seqrch for magma reservoirs and the formation of eruptive vents," Reviews of Geophys., **35/3**, 343-384.

McTigue, D. F. (1986), "Thermoelastic responce of fluid-saturated rock," J. Geophys. Res., **91/89,** 9533-9542.

Merlani, A.L., Natale, G. and Salusti, E. (1996), "Pressure and temperature dynamics in fluid-saturated porous-permeable rocks," Physica D, **90. 154-**169.

Merlani, A.L., Natale, G. and Salusti, E. (1997). "On the theory of pressure and temperature non-linear waves in compressible fluid-saturated porous rocks," Geophys. Astrophys. Fluid Dyn., **85,** 97-128.

Natale, G. (1998), "The effect of fluid thermal expansivity on thermo-mechanical shock waves propagating in the underground of volcanic domains," In press in Pure and Appl. Geophys..

Natale, G. and Salusti, E. (1996), "Transient solutions for temperature and pressure waves in fluid-saturated porous rocks.," Geophys. J. Intern., **124/3**, 649-656.

Natale, G., Salusti, E. and Troisi, **A.** (1998), "Rock deformation and fracturing processes due to nonlinear shock waves propagating in hyperthermal fluid-pressurised domains," in press in J. Geophys. Res..

Rice, J.R. and Cleary, M.P. (1976), " Some basic stress diffusion solutions for fluid-saturated elastic porous media with compressible constituents," Rev. Geophys. Space Phys., **14,** 227-241.

Whitham, G.B. (1974), "Linear and non-linear waves," Wiley Interscience, New **York,** NY.