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INTRINSIC RANDOM FUNCTIONS OF HIGH ORDER AND THEIR APPLICATION TO THE MODELING OF NON-STATIONARY GEOTHERMAL PARAMETERS

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ABSTRACT

Geothermal reservoirs are typical examples of natural systems having incomplete and discontinuous information. Data obtained show heterogeneity, frequently, at a high degree. Any geothermal reservoir manifests the following paradox: it is a natural phenomenon whose existence is uniquely determined in time and in space, but that can only be known or measured in an incomplete and fractional manner. Advanced simulators used to study and predict the behavior of such systems require petrophysical and thermodynamical continuous parameters, even at places where they have never been measured.

Traditional statistics used to calculate average values of those parameters, becomes evidently insufficient because the geothermal processes involved, are non-stationary. For example, temperature and pressure increase with depth, while porosity and permeability decrease. In this document we introduce practical applications of a numerical technique based on the theory of Intrinsic Random Functions of order k ($k \geq 1$), for the optimum spatial interpolation of geothermal parameters. This method allows the construction of geo-statistical generalized estimators composed by two functions: one portion can be totally random, while the second portion is deterministic, containing the spatial trend of the non-stationary parameter. This methodology has great potential usefulness in the risk analysis of any geothermal project.

INTRODUCTION

Geothermal processes originate and evolve partially in random form. This means that it is impossible to make exact predictions about the future behavior and final state of geothermal zones subject to exploitation. Any change in fluid extraction conditions, any variation in liquid reinjection rates and even the decisions concerning new wells location, could affect irreversibly the later evolution of the natural system. The use of mathematical models is a routinary activity to simulate several exploitation scenarios under different initial conditions and parameters. The most probable system's final state is then inferred, using or not probability laws. If probabilistic techniques are used, it is possible to propose different scenarios making predictions about geothermal field's capacity and reservoir's longevity, within tolerables ranges of error.

More commonly, more or less reasonably deterministic predictions are made about the probable evolution of the reservoir. These results are not presented very often in a probabilistic way, by showing the original parameter uncertainties and the way they propagate from the basic data to the final results, through the model equations. Perhaps this is due to the fact that probability analysis is still lacking of tradition in geothermal reservoir assessment. Probabilistic modeling **is** a process, not a final outcome. It has various stages with techniques and strategies related to each stage. International experience in geothermal reservoir engineering, spread in literature worldwide, shows that there are three classes of technical decisions to be considered, since the beginning of geothermal reservoir exploitation:

1)- What kind of basic information will be necessary to understand the reservoir ?
2)- what type of optimal mechanisms must be designed to pick up the basic **data,** including location and frequency of measurements ?
3)- What kind of information analysis will be performed and what type of models will be used or constructed to predict reservoir performance ?

These decisions are crucial for the optimum assesment and management of any geothermal field. **As** long as it is impossible to predict the exact evolution of these systems, it becomes necessary to take uncertainties and magnitude of error into account in the final decision concerning the power plant capacity. Risk analysis must be a routinary methodology for geothermal projects development. The purpose of this paper is to introduce a mathematical method that could be useful to treat uncertainties that **are** implicit in every geothermal parameter.

THE ORIGIN OF RANDOMNESS IN GEOTHERMAL RESERVOIRS

"The conception of chance enters into the very first steps of scientific activity, in virtue of the fact that no observation is absolutely correct. I think chance is a more fundamental conception than causality; for whether in a concrete case a cause-effect relation holds or not can only be judged by applying the laws of chance to the observations". (Max Born, 1949).

A random phenomenon is an empirical or natural event characterized by the property that its observation does not always lead to the same observed outcome, so that there is no ***deterministic regularity,*** but rather to different outcomes in such a way that there is ***statistical regularity*** (Parzen, 1960).

The natural heterogeneity of the terrestrial crust is well known and selfevident to every earth scientist. In naturally fractured or simply porous reservoirs, such heterogeneity affects every important parameter such as porosity, permeability, thermal conductivity, temperature, pressure, enthalpy, steam quality, concentration of non-condensible gases, etc. Heterogeneity manifests itself at different scales: pores, matrix blocks, fractures, formation, regional neighborhood, etc. If we assume that such variations appeared as consequences of unpredictable geological processes, then we have to accept that reservoir parameters got their values at least partially in random form during stochastic processes. We understand a stochastic process to be a family of space-time functions depending on natural random variables. But actually, the reservoir is uniquely determined in time and in space. The clue problem is that it can only be uncompletely known , because it is impossible to mesure continuouslly all its properties at every point of space and in all directions. The reservoir is therefore, at each time, partially unknown.

The foundation of the scientific understanding of a hydrothermal system is information. Continuous spatial distribution of parameters forms the basis to construct models and perform predictions on the reservoir. But reservoir **data** are always insufficient. Numerical modeling requires the use of available data to determine unknown values at places where there are no observations. The estimated evolution is thus biased by initial distribution of numerical values. Uncertainties generated by reservoir's conceptual model, propagate inside the model equations and brings out uncertain numerical outcomes within unknown error ranges. On the other hand, classic matchmg techques done by comparing numerical results against production parameters, are valid only at places where confidential measurements are done. Therefore, it is useful to make some kind of analysis on the basic information inaccuracies.

CLASSICAL AND ORDINARY GEOSTATISTICS

Classical methods to calculate spatially distributed parameters in natural systems, cover a wide variety of techques: simple statistics, linear regression, splines, polynomials, weighted least squares, trend surfaces, time series, factorial analysis, distance weighting, ordinary geostatistics, inverse theory, mobile averages, Fourier series, etc. All those techniques have advantages and disadvantages. In some problems all of them give good results, while in others most of them can fail, specially when the data look chaotic. There is no perfection in the spaces of incomplete information and a single technique providing perfect solutions to any problem does not exist. Particularly, classical statistics introduces the following disadvantages when applied to geothermal phenomena: its application requires to assume that parameter samples drawn from the reservoir are independent random variables. The outcome of an event should not influence other realizations of the same event (Sani, 1979). This means that the probability of any member of the population to appear in a sample, is independent of the appearance of other members in the sample (Kreyszig, 1973). Classical statistics assumes that there is no spatial relationship among the sampling values.

As most of great mathematical ideas, the theory of geostatistics emerged from very practical problems. In 1950 mining engineer Daniel G. Krige invented a techmque in order to evaluate veins of gold in South Africa. With his method he estimated mineral quality by means of empiric coefficients assigned to sampled values. This method considered the vein geometry, the spatial localization of samples and the relationship among them. This mining technique is known since then as ***"kriging".*** In the 60's, a young French mathematician, Georges Matheron, established the theoretical bases of Krige's method, developing a new rigorous technique of spatial interpolation under the general name of Theory of Regionalized Variables (Matheron, 1971). This theory is applicable to the approximate calculation of non-measured parameters, under the general condition that they possess some spatial structure, detectable from the samples. For example, a field's temperature T. In Matheron's theory a phenomenon of this class is said to be regionalized and every function representing it is called a regionalized variable (RV). This term does not have anything to do *a priori* with any probabilistic concept. It is simply the name of a special function that could be very irregular and exhibits two contradictory aspects:

1).- The RV has a spatial structure, which reflects the correlated characteristics of the regional phenomenon
2).- The RV is random because it shows unpredictable variations of the phenomenon when observed in different points.

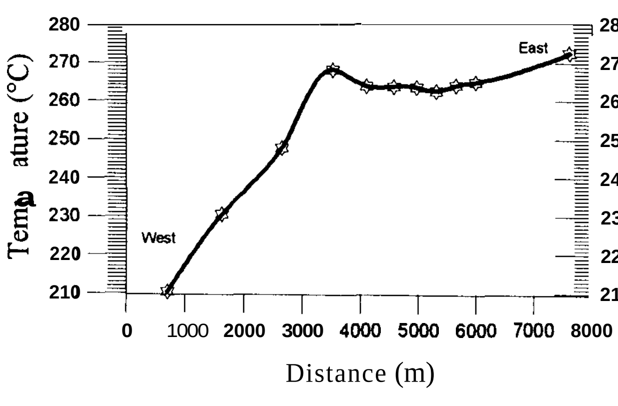
Ordinary geostatistics makes two assumptions: a) the differences between every pair of samples (T_i, T_j) are stationary; b) their covariance is only a function of the distance **h** between samples. The first hypothesis implies that the mathematical expectation or mean of the RV is constant = $E[T_i] = E[T_j]$. Because of the linearity of the expectation this is equivalent to $E[T_i - T_j] = 0$. The second hypothesis implies that: Cov (T_i, T_j) = $K(\mathbf{h})$ only. In order to describe the RV's spatial structure, an important auxiliary function is introduced: the variogram $\gamma(\mathbf{h})$. Under these conditions, the variable $T(\mathbf{r})$ is called ***intrinsic.*** This is the simplest form of stationary geostatistics (Journel & Huijbregts, 1978).

Since 1971 a wide range of methods based on the aforementioned ideas have been developed in distinct areas. From the estimate of ore reserves, they are presently applicable to any field of science and of industry, where is necessary to evaluate incomplete correlated data in space and/or in time. Most of those techniques have been summarized by Delhomme (1976). **A** detailed treatement on the subject, including a wide range of more advanced related topics can be found in the book written by Wackernagel (1995). Our objective from now is to introduce the necessary tools to solve the more general problem when the RV's mean is not a constant and the parameters are non-stationary.

NON-STATIONARY PROCESSES IN GEOTHERMAL RESERVOIRS

During the exploitation and study of geothermal reservoirs, phenomena are found that cannot be considered as stationary in the whole space. Temperature and pressure increase almost linearly with depth, while porosity and permeability decrease. Non-condensible gas concentration increases when steam quality is high. The expected amount of available energy is not the same in shallow fractured zones, than in deep sealed zones. Figure 1 shows a real example of temperature drift.

Fig. 1.- TEMPERATURE DRIFT W-E, 1750 m asl, Los Azufres.



Matheron himself (1973) developed the geostatistical tools for the mathematical representation of non-stationary processes. The fundamental concept is the Intrinsic Random Function of order k (IRFk). The application of this type of function to a non-stationary phenomenon, leads to a process having stationary differences starting from certain order k. For example, the random functions of stationary geostatistics correspond to k= 0.

General knging for non-stationary phenomena, was called universal knging by the first authors. It is a linear functional representing a parameter distribution whose values are only partially known. Some of its values can be deterministic, forming a trend or drift, while others are random residuals or fluctuations. Graphical representation of contours and 3D surfaces, is an obvious application of universal knging; but this is not the essential objective of the method. Applications of knging cover a wide variety of fields: mining, underground hydrology, propagation in underground and air of pollutants, climatic variables, oil and gas reservoirs, forests, gravimetry, agricultural soils, orientation of faults, seismic risk, etc. It seems that geothermal reservoir engineering lacks for these techques. **A** catalog of complete references can be found in Journel & Huijbregts (1978); and in Suárez (1**988).**

NON-STATIONARY KRIGING SHORT THEORY

The sample description space of a random geothermal phenomenon is the set of all possible outcomes of the phenomenon. Let S be a population **S** = $\{T_1, T_2, ..., T_N\}$ subset of that space. $T_i = T(\mathbf{r}_i)$, (i = 1,N) represents any reservoir geothermal parameter, located in the reservoir at the vectorial position $\mathbf{r}_i = (x_i, y_i, z_i, t)$ at time t, with respect to an arbitrary origin. Relationship intensity between every pair of samples $\{T_i, T_j\}$, $T_j = T(\mathbf{r}_i + \mathbf{h})$, is measurable by the covariance of their difference. The vector $\mathbf{h} = \mathbf{r}_j - \mathbf{r}_i$ represents the spatial distance between both samples. The typical problem is the following:

Given N values of the parameter $T(\mathbf{r}_i)$, temperature for example, it is required to calculate approximately the unknown value $T_0 = T(r_0)$ at point $\mathbf{r}_0$ (punctual knging), so that all available information be optimally used. More generally one needs to estimate average values $\int_\Omega T(\mathbf{r})\,dr$ at some small reservoir's portion Ω (block knging), in order to assign values at each element conforming a mesh for numerical simulation. Parameter T is a RV showing a deterministic trend in its spatial behavior (Fig. 1).

In ordinary geostatistics, the random function itself does not need to be stationary, but only its first increment. This implies that the mathematical expectation or mean value of T is a **constant,** or: $E[T(r+\mathbf{h}) - T(\mathbf{r})] = 0$. Any function I calculating primary differences, filters every constant c: $I(c) = c - c = 0$.

The clue idea in solving the estimation problem of non-stationary phenomena, is the definition of general increments filtering the trend of the RV, giving a constant result at some final order k. If the trend could be approximated by a polynomial, we can define generalized **differences** of order k+1 in such a way that they filter every polynomial of order **k.** Let x_i be a one dimensional variable and h be a constant. With $x_{i+1} = x_i + h$, $x_{i+2} = x_i + 2h$, and so on. The succesives increments of x_i are:

$$I^0(x_i) = x_i \ , \ I^1(x_i) = x_{i+1} - x_i = x_i + h - x_i = h \ ,$$
$$I^2(x_i) = I^1(I^1(x_i)) = x_{i+2} - 2x_{i+1} + x_i = 0$$

Thus, every increment $I^{k+1}(x_i) = 0$, for $k \geq 1$. Consider the increments of the linear polynomial: $P_1(x) = c_0 + c_1 x$:

$$I^1[P_1(x)] = c_0 + c_1(x+h) - (c_0 + c_1 x) = c_1 h$$
$$I^2[P_1(x)] = I^1[I^1[P_1(x)]] = I^1[c_1 h] = 0$$

Any increment of order greater than 1 filters every linear polynomial. We can go now to more general ideas. Let P, be a general polynomial of order **k,** represented as a linear combination of basis functions $b_j(r)$:

$$P_k(\vec{r}) = \sum_{j=1}^{m_k} c_j b_j(\vec{r}) \ , \qquad \ldots\ldots \quad (1)$$
$$where: \vec{r} = (x,y,z) \ , \ m_k = \frac{(k+1)(k+2)}{2}$$

The functions b_j must be linearly independent, the c_j's are unknown constants. In a polynomial basis, if k = 1, $\{b_j\} = \{1, x, y\}$; if k=2, $\{b_j\} = \{1, x, y, xy, x^2, y^2\}$. Any linear functional I^{k+1} is a generalized increment of order k+1 if it filters out general polynomials of order k: $I^{k+1}(P_k) = 0$:

$$I^{k+1}[P_k(\vec{r})] = \sum_{j=1}^{m_k} c_j I^{k+1}[b_j(\vec{r})] = 0 \ , \ \ldots\ldots \quad (2)$$

We assume that for every RV under study in the region Ω, it exists a generalized covariance $K(\mathbf{h})$, representing the spatial structure of the non-stationary parameter. Let $T(\mathbf{r})$ be a function satisfying the following conditions:

$$a)\ E[I^k(T)] = 0 \ , \ where\ \vec{r}, (\vec{r}+\vec{h}) \in \Omega \qquad \ldots\ldots \quad (3)$$
$$b)\ E[(I^k(T))^2] = \sum_i \sum_j \beta_i \beta_j K(\vec{h}) \geq 0 \qquad \ldots\ldots \quad (4a)$$
$$c)\ E[(I^k(T))^2] = \int_\Omega K(\vec{h}) \cdot I^k(d(\vec{r}+\vec{h})) \cdot I^k(d\vec{r}) \quad \ldots\ldots \quad (46)$$

Equation **(4a)** holds for the discrete case or point knging, while (4b) must be applied in the continuous case or block **knging.** Then T is an intrinsic random function of order k or IRFk (Matheron, 1973; Delfiner, 1975). T is random if and only if r is random. We assume that T can be expressed as: $T(\mathbf{r}) = D(\mathbf{r}) + R(\mathbf{r})$. Where $D(\mathbf{r})$ is a deterministic function representing the non-stationary portion or drift of T; it can be approximated by equation (1): $D(\mathbf{r}) \approx P_k(\mathbf{r})$. The residual $R(\mathbf{r})$ is a weakly stationary random function representing the fluctuation of T. We can build a linear interpolator L in order to estimate an unknown value $T_0 = T(\mathbf{r}_0)$, or an average value of T in Ω. L must have the following properties (*ibid.*):

i)- $L = I^k + I_d$ (I_d is the identity function)
ii)- $T_0 \approx L(T_0) = \sum \beta_i T_i$, i = 1, n ≤ N
iii)- L is unbiased: $E[L(T_0)] = E[T_0]$
iv)- The error variance σ^2 is a minimum.

This estimator is defined as a generalized increment. Definition $I^k = L - I_d$ and the second hypothesis ii) show that I^k is a linear combination of known samples T_i. Thus $I^k[T_0] = (L - I_d)[T_0] = L(T_0) - T_0 = L(D_0 + R_0) - (D_0 + R_0) = L(R_0) - R_0$, because of the fact that I^k filters D. This last property can be expressed as:

$$D_0 = L(D_0) = L\left(\sum_{j=1}^{m_k} c_j b_j^0\right) = \sum_{j=1}^{m_k} c_j L(b_j^0) \quad \ldots\ (5)$$

$$Thus: \ \sum_{j=1}^{m_k} c_j b_j^0 = \sum_{j=1}^{m_k} \sum_{i=1}^{n} \beta_i c_j b_j^i \qquad \ldots\ldots \quad (6)$$

Where β_i are unknown coefficients, $b_j^i = b_j(r_i)$, i=0, n ≤ N. Notice that N is the total number of data but the quantity of samples considered in the estimation of T_0 can be a lower number. $\sigma^2 = Var[L(T_0) - T_0] = E[L(T_0) - T_0]^2$ is the variance of the estimation error. By definition:

$$\sigma^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} (\beta_i \beta_j K_{ij} - 2\beta_i K_{i0}) + K_{00} \quad \ldots \quad (7)$$

Where $K_{ij} = K(r_i, r_j) = K(\mathbf{h})$ is simply the generalized covariance of the random portion of T or fluctuation R. Minimization of property (iii) is accomplished by using the classical method of Lagrange multipliers μ_l ($l = 1, m_k$) in the expression:

$$F(\mu_l, \beta_j) = \sigma^2 + \sum_{l=1}^{m_k} \mu_l \sum_{j=1}^{n} (\beta_j b_l^j - b_l^0) \quad \ldots \quad (8)$$

$$Assuming: \ \frac{\partial F}{\partial \mu_l} = \frac{\partial F}{\partial \beta_j} = 0$$

Setting to zero the partial derivatives of F and from equation **(6)** we obtain:

$$\sum_{i=1}^{n} \beta_i K_{ij} + \sum_{l=1}^{m_k} \mu_l b_l^j = K_{0j} \quad \ldots \quad (9)$$

$$\sum_{i=1}^{n} \beta_i b_j^{\,l} = b_j^0 \quad , j = 1, n \quad \ldots \quad (10)$$

Equations (9) and (10) are called traditionally the Universal Kriging system (Matheron, 1973). This system corresponds to a more general application of knging to estimate non-stationary parameters. The problem of the effective calculation of the generalized covariance K(**h**), was solved experimentally by Matheron (1973) and Delfiner (1975), when they discovered that a large variety of structures are satisfactorily represented by functions of the form:

$$K_D(\vec{h}) = \alpha_0 \delta(\vec{h}) + \sum_{i=1}^{k} (-1)^{i+1} \alpha_i |\vec{h}|^{2i+1} \quad \ldots \quad \mathbf{(11)}$$

Where k = 1, 2, 3, ... δ(**h**) is the Dirac function which takes into account the nugget effect, indicating that T could change abruptly at very small scale, which is the case for porosity or for permeability. Wackernagel (1995) and Delfiier (1975) suggest another possible model for a k-th order generalized covariance, using a Gamma function in the form:

$$K_W(\vec{h}) = \Gamma(-\frac{w}{2}) \, |\vec{h}|^{w} \quad \ldots\ldots \quad (12)$$

$$\Gamma(-\frac{w}{2}) = \int_0^{\infty} t^{-\frac{(w+2)}{2}} e^{-t} \, dt \,, \quad 0 < w < (2k+2)$$

There are no theoretical restrictions considering another class of functions such as exponentials, trigonometric functions, Tchebyschev polynomials, Bessel functions, and so on, to represent generalized covariances. For every particular data set other functions should be tested.

IRFk ESTIMATION OF THE TEMPERATURE DISTRIBUTION IN A GEOTHERMAL FIELD

The previous ideas were programmed in a FORTRAN-90 code called IRFk2.f90, in order to estimate the distribution of temperatures (and other parameters). The data correspond to measurements made between 1980 and 1995 at the Los Azufres, Mexico geothermal field. The portion of the reservoir where the techmque was tested, presents two clear trends: the highest observed temperatures are toward the center-east of the field, while the lower temperatures are found toward the west following a complex non-linear decrement. On the other hand, reinjection is performed since several years close to the field's center. This distribution is also affected by the southern boundary, where the reservoir is subjected to a more intense exploitation. Figure 1 shows this E-W trend.

This first version of the program process data in one and two dimensions. To test different generalized covariances, known values of the original data group were eliminated, and considered as unknown. To verify by hand that the supposed covariance characterizes the spatial structure of the observed data and therefore, that estimates correctly the value $T(\mathbf{r}_0)$, a very simple way consists to krige a known parameter and then observe if the estimated value differs little from the real value by means of a simple numerical criterion: $|L[T_0] - T_0| \leq \varepsilon$. For practical reasons in the case of temperature, it is enough to take $\varepsilon \leq 1$°C.

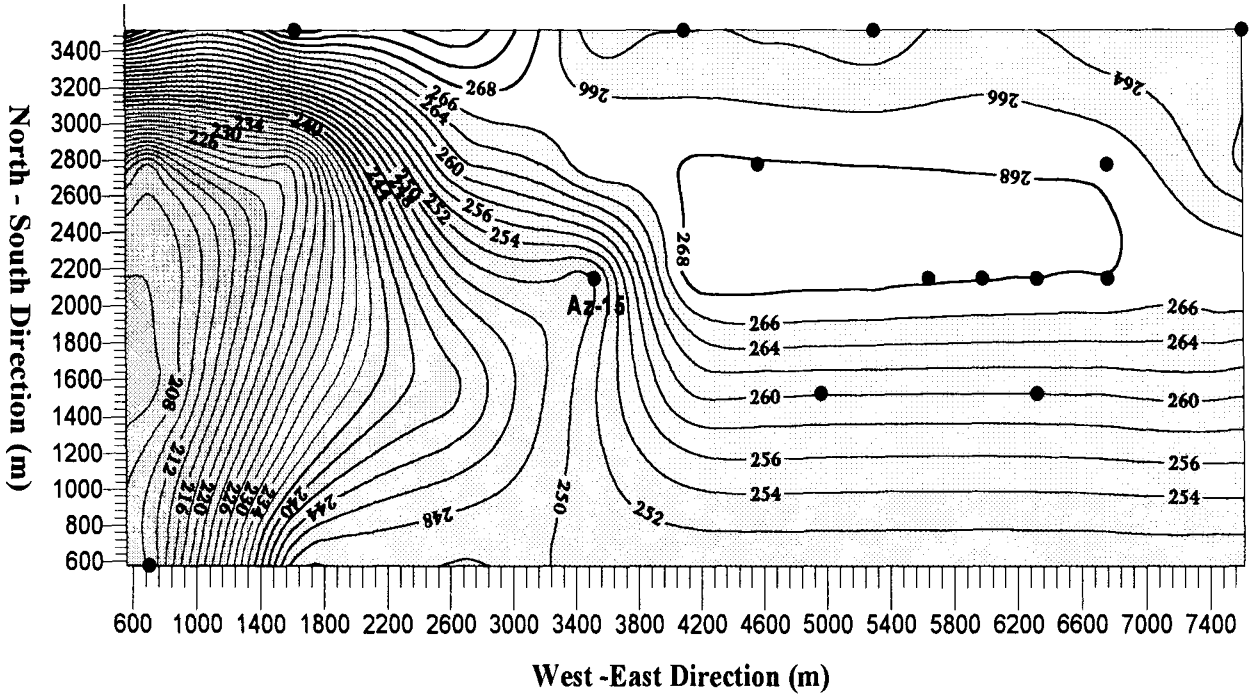
The best covariance model is that one presenting the least differences with regard to real data. But each model is not unique, because different models can approach reasonably well the same parameter. **As** final criterion, we calculated the variance of the estimation error from equation (7), for each one of the **tested** covariances K. The best final model is that one presenting the lowest variance at the majority of estimated points. Later on several temperature values were estimated at places never measured.

The following table summarizes some results obtained from the application of IRKF2.f90. Column T_0 contains observed values, $L(T_0)$ are the corresponding kriged results.

Table 1.- Some results of point kriging

X	Y	TO	L(TO)	σ^2	K(h)
702.70	578.40	210.33	210.33	0.1686E-12	K1
2652.30	2151.40	247.80	247.80	-0.2028E-11	K1
4097.30	3513.50	263.88	263.88	-0.5637E-05	K2
792.80	2151.40	206.61	206.61	-0.4185E-11	K3
4097.30	2151.40	268.87	268.87	0.1927E-11	K2
684.70	2778.40	209.42	209.42	0.9393E-06	K1
6765.80	2778.40	267.58	267.58	0.9025E-06	K3
792.80	1527.90	205.99	209.69	0.7136E+01	K3
1623.40	1527.90	230.43	229.83	0.5126E+01	K1
2652.30	1527.90	241.41	246.74	0.2101E+02	K2
1623.40	2151.40	224.60	229.52	0.9714E+01	K3
3518.90	2151.40	248.63	252.08	0.9161E+01	K3
1623.40	2778.40	221.53	223.46	0.1442E+02	K2
6765.80	2778.40	267.58	265.51	0.2168E+02	K2
5646.90	3513.50	264.02	264.19	0.1256E+02	K1
7614.40	2151.40	268.11	267.48	0.2856E+02	K3
4565.80	2778.40	268.13	264.61	0.1693E+02	K1
1623.40	3515.50	273.86	274.96	0.2078E+02	K3
7614.40	3513.50	262.55	268.09	0.2722E+02	K1
100.20	200.50	?	196.76	0.2522E+02	K2
200.20	1800.50	?	193.96	0.2053E+02	K2
5980.20	3513.50	?	264.24	0.1640E+02	K2
6980.20	4513.50	?	252.57	0.6946E+02	K1
8580.20	4513.50	?	261.56	0.4653E+02	K2

Fig. 2.- TEMPERATURE ("C) CONTOURS AFTER KRIGING 45 POINTS at 1750 masl



In the preceding table σ^2 is the variance of the estimation error and $K(\mathbf{h})$ represents the type of generalized covariance used, as reported in Table 2. A general contour obtained from this application is shown in figure 2. The main W-E drift is clearly observed. The technique reproduces exactly the known temperature values at this depth, represented by black points.

Table 2.- Some Tested Generalized Covariances

Orderk	Type of $K(\mathbf{h})$
(K1) 1	$164.59\,\delta(\mathbf{h}) - 0.13671\,\mathbf{h}$
(K1) 1	$102.86\,\delta(\mathbf{h}) - 0.17166\,\mathbf{h}$
(K2) 2	$\mathbf{0.23377}\ 10^{-6}\,\mathbf{h}^3$
(K3) 2	$-0.61111\ 10^{-13}\,\mathbf{h}^5$

CONCLUSIONS

The main aim of this work has been to present a practical application of a numerical technique based on the theory of Intrinsic Random Functions of order **k** ($k \geq 1$), for the optimum spatial interpolation of geothermal parameters. From the results of this *study* the following conclusions are established:

- Frequently in the modeling of geothermal reservoirs, the initial data processing is made with little rigor, by guessing or performing simple averages, and assigning values to portions of the mesh where there is no information. That lack of rigor in the treatement of basic information, could lead to results with errors whose magnitude is uncertain.

- Non-stationary Kriging is a mathematical method of interpolation, useful to estimate unknown geothermal data irregularly distributed in space, with an associated mesure of uncertainity. The estimate is based on measurements that could be randomly distributed or partially known in deterministic form.

- During the experience obtained in the modeling and simulation of mexican geothermal reservoirs, we have found that non-stationary kriging based on the theory of IRFk is a **very** powerful tool, specially appropriate to estimate diverse geothermal parameters with drift.

- Any geothermal project development involves high initial costs. Making wrong decisions on the basis of erroneous reservoir potential evaluation is costly. It is important to estimate ***system's*** capacity by optimally using all available information.

- Approximate knowledge of the range of confidence of the basic information, permits to estimate simulation results error margins . In this way, the implicit reservoir's uncertainties could be transferred to the risk analysis of the geothermal project.

NOMENCLATURE

$b_j(r)$ arbitrary basis functions of the functional space representing reservoir's mathematical properties.
c_j unknown constants in a polynomial expression.
d (r) differential form of vector r.
D(**r**) deterministic function representing the drift of T.
$E[T_i] = \Sigma_i T_i p_i$, $p_i > 0$ mathematical expectation of the RV, it is the mean of the probability law of T.
$h = \mathbf{r}_j - \mathbf{r}_i$ vectorial distance between samples (i,j)
$I_d(X) = X$ identity function
I^{k+1} generalized increment of order k+1
k order of the Intrinsic Random Function T.
$K(\mathbf{h}) = K_{ij} = Cov(R_i, R_j) = E[R_i R_j] - E[R_i]\, E[R_j]$, generalized covariance measuring the spatial correlation between random variables, R_i and R_j
$L = (I^k + \mathbf{I}_d)$ non-stationary optimal linear estimator.
$m_k = (k+1)(k+2)/2$ total number of basis functions b_j
n number of points to perform kriging at point T_0
N total number of samples or known values T_i.
P_k general polynomial of order k.
$\mathbf{r}_i = (x_i, y_i, z_i, t)$ Cartesian position vector at time t.
R(**r**) stationary random variable or fluctuation of T. R is a function defined on the outcome of a random geothermal phenomenon. It is weakly stationary because its mean is constant and its covariance does not depend on the location r.
RV Regionalized Variable in Matheron's theory.
$S = \{T_1, T_2, ..., T_N\}$ population of the sampled parameter T.
$T_i = T(\mathbf{r}_i)$ ith sample of the parameter T at postion $\mathbf{r}_i$
(x_i, y_i, z_i, t) Cartesian coordinates of any point in 3D space
$Var[T] = E[(T - E[T])^2]$ the variance is a measure of the dispersion of the probability law for T.

Greek **Symbols**

α_i experimental coeficients to determine K **(h).**
β_i unknown coefficients in the linear estimation of T.
Γ the Gamma function.
$2\gamma(\mathbf{h}) = E[(T(\mathbf{r} + h) - T(\mathbf{r}))^2]$ variogram of T.
$\delta(\mathbf{h})$ Dirac distribution or generalized function.
$\sigma^2 = Var[L(T_0) - T_0]$ variance of the estimation error.
μ_l lagrange multipliers.
Ω = Any portion of the geothermal reservoir.

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