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ON WATER-STEAM PHASE TRANSITION FRONTS IN GEOTHERMAL RESERVOIRS

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ABSTRACT

The problems of steam (steam-water) production from hydrothermal reservoir and cold water injection into vapour-saturated reservoir are considered. The mathematical model includes equations of mass and energy conservation laws, the generalized Darcy law and thermodynamic relations. The formulation of each problem contains moving boundary at which the water saturation function suffers a discontinuity. The universal relations at discontinuity in combination with the thermodynamic equilibrium condition constitute the complete set of boundary conditions at the moving boundary. The analytical (linear approach) solutions of the one-dimensional problems for half space domain are presented. It is shown (production problem) that the introduction of an evaporation surface separating the steam and water leads to superheating of the water in a zone ahead of the front. This contradiction is removed by introducing of an extended phase transition zone between the single - phase zones. Furthemore, the boiling front separating the water zone and the two-phase zone is degenerated. In this case condition of continuity of water saturation function is satisfied. Finally, this function at the mixture - steam front varies discontinuously from the unknown value to zero. The analytical solution of condensation process is derived for cold water injection into vapour - saturated reservoir.

INTRODUCTION

The aim of the present study is to investigate the possible configurations of H_2O phase transitions fronts in geothermal reservoirs. Theoretical studies of the fronts have been carried out by several authors [Garg, Pritchett 1988, 1990], [Pruess, Calore, Celati, Wu 1987], [Udell 1985], [Woods, Fitzgerald 1993] among others. The present paper is the natural continuation of these investigations. Pruess (1995) enumerated a number of physical processes which should be taken into account: strong coupling between fluid flow and heat transfer; phase change processes (boiling and condensation) etc. There have been attempts to succeed in this way. Below we will consider the system of equations correlating the phenomena of heat and mass transfer.

An essential part of the mathematical description is the correct specification of boundary conditions. The formulation of the problem with phase transitions contains the unknown moving interface. This moving boundary is the surface where one or several parameters of the model have a discontinuity or discontinuous derivatives. The conservation laws may be written in term of "jump" at phase transition front. We assume that two phases of H_2O are locally at the thermodynamic equilibrium. These relations constitute the complete system of boundary conditions at the fronts.

It is vital to emphasize that in all zones it is possible to apply linear approach and derive the analytical solutions. On the whole the problems are nonlinear. Preliminary results demonstrate the strong nonmonotonicity of the water saturation function in two-phase region for nonlinear problems.

GENERAL MATHEMATICAL MODEL

We will assume that the hydrothermal reservoir is an incompressible porous medium at rest, saturated either with water or steam or a mixture of the two and the gravitational forces are negligible. In order to describe the processes of heat and mass transfer with equilibrium phase transitions we will use the following system of mass and energy conservation, the generalized Darcy law [O'Sullivan, 1985], the equations of state for water and steam, the equation of steam-water equilibrium curve [Vukalovitch, 1955], and the thermodynamic relations:

$$m\frac{d}{d t}S\rho_w + div\rho_w\vec{v}_w = M$$

$$m\frac{\partial}{d t}(1-S)\rho_v + div\rho_v\vec{v}_v = -A4$$

$$\frac{\partial}{\partial t}(\rho\, e), + div(\rho_w h_w\vec{v}_w + \rho_v h_v\vec{v}_v) =$$

$$= div(\lambda_m grad\, T)$$

$$\vec{v}_j = -\frac{k}{\mu_j}f_j(S) grad\, P \qquad j = w,v$$

$$\rho_w = \rho_{w0}(1+\alpha(P-P_0)-\beta(T-T_0))$$

$$P = \rho_v RT,\ \lg\frac{P}{P_a} = A + \frac{B}{T},\ e_j = h_j - \frac{P}{\rho_j}$$

$$dh_w = C_w dT + \frac{1-\beta T}{\rho_w}dP$$

$$dh_v = C_P dT, \quad de_s = C_S dT$$

$$\lambda_m = mS\lambda_w + m(1-S)\lambda_v + (1-m)\lambda_S$$

$$(\rho e)_m = mS\rho_w e_w + m(1-S)\rho_v e_v +$$
$$+(1-m)\rho_S e_S$$

Here. T is the temperature, P is the pressure, S is the water saturation, v is the filter velocity, M is the phase transition intensity, k permeability, μ is the viscosity. α is the water compressibility Coefficient, β is the water thermal expansion coefficient, ρ is the density, e is the intrinsic energy density, h is the enthalpy density, λ is the thermal conductivity, m is the porosity, C is the heat capacity,

$A = 5{,}44, \quad B = -2005{,}1K, \quad P_a = 10^5\,Pa$;

the subscripts denote: w - water, v - vapor, S - porous medium skeleton, m - effective value.

After identical transformations, the system of the basic equations reduce to the system of three equations in T, P and S [Tsypkin, 1994]. We will consider the solutions of the problems in linear approximation when the saturation, pressure and temprature variations in each zones are small. We represent this functions in the form

$$S = \dot{S} + S', \quad P = \dot{P} + P', \quad T = \dot{T} + T'$$

Here, $\dot{f}$ is the undisturbed value, f' is the perturbation of function. Then, in linear approximation, we have

$$\left(1-\frac{\dot P}{\dot\rho_w R\dot T}\right)\frac{\partial S'}{\partial t} + \left(\alpha\dot S + \frac{1-\dot S}{\dot\rho_w R\dot T}\right)\frac{\partial P'}{\partial t} -$$

$$-\left(\beta\dot S + \dot P\frac{1-\dot S}{\dot\rho_w R\dot T^2}\right)\frac{\partial T'}{\partial t} =$$

$$= \frac{k\mu_v}{m}\left(\frac{\mu_w f_w(\dot S)}{\mu_v} + \frac{f_v(\dot S)}{\dot\rho_w R}\frac{\dot P}{\dot T}\right)\Delta P',$$

$$\left((1-\dot S) - \frac{q\dot P}{R\dot T}\right)\frac{\partial T'}{d t} \quad \frac{q\dot P}{RT}\frac{\partial S'}{d t} +$$

$$+\left(\dot S(1-\beta\dot T) - 1 + q\frac{1-\dot S}{R\dot T}\right)\frac{\partial P'}{\partial t} =$$

$$-\frac{kq}{mR}\frac{f_v}{\mu_v}\frac{\dot P}{T}\Delta P' \frac{\lambda_m}{m}\Delta T' \qquad (1)$$

$$\frac{P'}{P} = F\frac{T'}{\dot T}, \qquad F = -2{,}3\frac{B}{\dot T}$$

$$(\rho C)_m = m\dot S\dot\rho_w C_w + m(1-\dot S)\dot\rho_v C_v +$$
$$+(1-m)\rho_S C_S$$

Here, q is the heat of transition. The equations of motion for single-phase zones saturated with steam or water can be obtained from the system (1) by formally setting $S = 0$ or 1, respectively. In these cases the equation of thermodynamic equilibrium of the water and steam is not considered.

In the system (1) the perturbation of the water saturation function is present under the time derivative sign only. Eliminating these derivatives and the pressure function, we obtain for two-phase flow an effective heat equation with coefficients depending on the initial values and parameters of the physical system:

$$(\rho C)_{ef}\frac{\partial T'}{\partial t} = \lambda_{ef}\Delta T', \quad (\rho C)_{ef} = R_1 + R_2$$

$$\lambda_{ef} = R_3\frac{q\rho_v}{\varepsilon} + R_4, \quad \varepsilon = 1 - \frac{\rho_v}{\dot\rho_w}, \qquad (2)$$

$$R_1 = \dot S(\beta\dot T - \alpha\dot F\dot P) + (1-\dot S)(1-\dot F)\frac{\dot\rho_v}{\dot\rho_w}$$

$$R_2 = \dot T\frac{(\rho C)_m}{m} + (1-\dot S)q\dot\rho_v +$$

$$+\dot P\dot F\left(\dot S(1-\beta\dot T) + \frac{q(1-\dot S)}{R\dot T} - 1\right),$$

$$R_3 = \frac{k\dot P\dot T}{m}\left(\frac{\dot S}{\mu_w} + \frac{1-\dot S}{\mu_v}\frac{\dot\rho_v}{\dot\rho_w}\right),$$

$$R_4 = \frac{kq}{m\mu_v}(1-\dot{S})\dot{P}\dot{F}\rho_v + \lambda_m \frac{T}{m}$$

n the single-phase zones the basic system of equations has the form

$$\frac{\partial P'}{\partial t} + \delta_j \frac{\partial T'}{\partial t} = \kappa_j \Delta P'$$

$$\frac{\partial T'}{\partial t} + \omega_j \frac{\partial P'}{\partial t} = a_j \Delta T' \qquad j = w, v \tag{3}$$

$$\delta_v = -\frac{P}{T'}, \quad \kappa_v = \frac{k\dot{P}}{m\mu_v}, \quad \omega_v = -\frac{m}{(\rho C)_v},$$

$$a_v = \frac{m\lambda_v + (1-m)\lambda_S}{m\dot{\rho}_v C_v + (1-m)\rho_S C_S}$$

$$\delta_w = -\frac{\beta}{\alpha}, \quad \kappa_w = \frac{k}{\alpha m\mu_w}, \quad \omega_w = -\frac{m\beta\dot{T}}{(\rho C)_w},$$

$$a_w = \frac{m\lambda_w + (1-m)\lambda_S}{m\dot{\rho}_w C_w + (1-m)\rho_S C_S}$$

Here, for the sake of simplicity, we set

$f_w(S) = S, \quad f_v(S) = 1 - S.$

The correctness (according to Petrovski) of the system (3) has been established [Barmin, Tsypkin 1996].

In the impermeable rock zone the usual heat quation is valid.

The formulations of the problems admit the existence of phase transition fronts. The conditions on these interfaces can be obtained from the mass conservation low for H_2O and the energy conservation law on discontinuities of the water saturation function

$$\left[\rho\,(V_n - u_n)\right]_-^+ = 0$$

$$\left[\rho h(V_n - u_n) + \lambda\,(grad\ T)_n\right]_-^+ = 0 \tag{4}$$

Here, V is the discontinuity velocity, u is the water or steam velocity; the subscripts denote: n - the normal, plus and minus signs - the quantities to the right and left of the front, respectively. The system of boundary conditions (4) must be supplemented with the thermodynamic relation between the pressure and the transition temperature:

$$\lg\frac{P_*}{P} = A + \frac{B}{T_*} \tag{5}$$

$$T_+ = T_- = T_*, \quad P_+ = P_- = P_*$$

An asterisk denotes the values of the quantities at the front.

THE PRODUCTION PROBLEM

Let us consider the simple model problem of heat-transfer medium extraction from a contact boundary between a hydrothermal reservoir and the surrounding rocks. This situation arise, for example, when the heat-transfer medium flows out into a fracture between permeable and impermeable rocks. The pressure fall in the process of extraction leads to vaporization and, consequently, to a decrease in the temperature of the reservoir which, as estimates show, may be considerable. In this case heat inflow from the surrounding rocks develops resulting in intensification of the evaporation process and the formation of the steam - filled zones (or mixture zone) of considerable dimensions.

Let impermeable rocks occupy the half-space $x < 0$ while the initial substance (steam-water mixture or water) at a temperature T_0 and a pressure P_0 occupies the half-space $x > 0$. The initial pressure must satisfy the thermodynamic condition for the existence of water $\lg P_0 > A + B/T_0$ or mixture $\lg P_0 = A + B/T_0$

We assume that on the stationary wall $x = 0$ corresponding to the extracting well (system wells) the pressure drops to a quite small value $P^0 < P_0$. Then the boiling front $x = X(t)$ $(\dot{X}(t) = V_n)$ propagates to the right from the surface $x = 0$.

The initial and boundary conditions have the form:

$$t = 0: \quad X(0) = 0, \quad x < 0: \quad T = T_0$$

$$x > 0: \quad T = T_0, \quad P = P_0, \quad S = S_0$$

$$x = 0: \quad P = P^0 \ (P^0 < P_0),$$

$$(\lambda\ grad\ T)_{n+} = (\lambda\ grad\ T)_n$$

If T_0, P_0, S_0, P^0 are constant quantities, then the problems have self-similar solutions of the form

$$T = T(\zeta), \quad P = P(\zeta), \quad S = S(\zeta),$$

$$X_j(t) = 2\gamma_j\sqrt{a_w t}, \quad (j = 1,2), \quad \zeta = \frac{x}{2\sqrt{a_w t}}$$

The solutions in all regions can be expressed in terms of linear combinations of probability integrals [Tsypkin,1990]. By substituting these solutions into the boundary conditions at the moving fronts we obtain the systems of transcendental equations for T_*, P_*, S_+, γ etc., which are solved numerically.

Mixture-steam front

($S_- = 0$, S_+ is the unknown function)

The existence of this front corresponds to the problem of steam production from **two-phase** geothermal reservoir.

We will assume that the steam zone is located to the left of the phase transition surface, and the water-steam zone to the right. From (4) there follow the relations on the moving boundary whch have the form

$$mS_+\left(1-\frac{P_*}{\rho_w RT_*}\right)V_n = \frac{kP_*}{\mu_v\rho_w RT_*}(grad\,P)_{n-} -$$
$$-k\left(\frac{f_w(S_+)}{\mu_w}-\frac{f_v(S_+)P_*}{\mu_v\rho_w RT_*}\right)(grad\ P)_{n+},$$

(6)

$$\frac{mS_+qP_*}{RT_*}V_n = (\lambda\ grad\,T)_{n+} - (\lambda\,grad\,T)_{n-} +$$
$$+\frac{kqP_*}{\mu_v RT_*}\left(f_v(S_+)(grad\,P)_{n+} - (grad\,P)_{n-}\right)$$

Relations (5), (6) form the complete system of conditions at the boiling front which separates steam and water-steam zones. The value of the water saturation function in the mixture zone varies from $S = S_0$ to the unknown value at the front on the right $S = S_+$. The computation results for $T_0 = 500K$, $S_0 = 0{,}5$, $\mathbf{m} = 0{,}2$, $\mathbf{k} = 10^{-16}M^2$ are shown in Fig. 1 and Fig. 2.

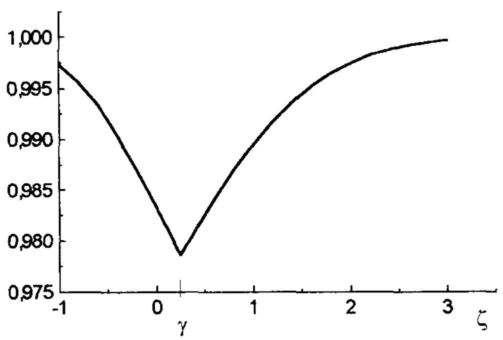

Fig. 1. The dimensionless temperature T/T_0 as a function of dimensionless length ζ

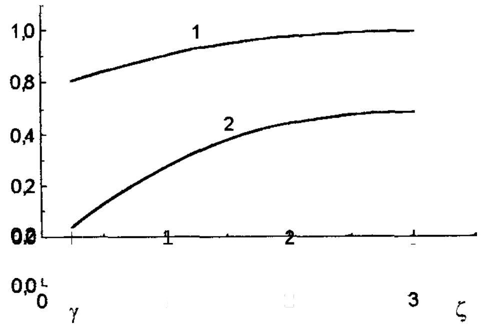

Fig. 2. The reservoir dimensionless pressure P/P_0 (curve 1) and the water saturation S (curve 2) as a functions of dimensionless length.

water-mixture front

(S_- is the unknown function, $S_+ = 1$)

In this section, following [Garg, Pritchett, 1988], we consider an initially water-saturated reservoir which evolves into a two-phase system as a result of fluid production.

The conditions on the moving boundary separating the mixture zone and the water zone have the form

$$m(1-S_-)\left(1-\frac{P_*}{\rho_w RT_*}\right)V_n = -\frac{k}{\mu_w}(grad\,P)_{n+} -$$
$$-k\left(\frac{f_w(S_-)}{\mu_w}+\frac{f_v(S_-)P_*}{\mu_v\,\rho_w\,RT_*}\right)(grad\,P)_{n-},$$

$$m(1-S_-)q\frac{P_*}{\rho_w RT_*}V_n = (\lambda\,grad\,T)_{n+} -$$
$$-(\lambda\ grad\,T)_{n-} - kq\frac{f_v(S_-)P_*}{\mu_v RT_*}(grad\,P)_{n-}$$

At **this** moving interface the water saturation varies discontinuously from $S_+ = 1$ to the unknown values- in the mixture zone. For the initial and boundary values

$$T_0 = 500\,K,\ S(0) = S^0 = 0{,}5,\ \mathbf{k} = 10^{-16}M^2,$$
$$P_0 = 4\cdot 10^6\,Pa,\quad P^0 = 2{,}2\cdot 10^6\,Pa$$

we have results presented in Fig. 3.

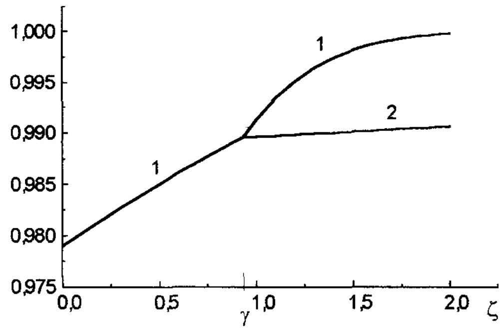

Fig. 3. Superheating of water in the zone ahead of the front γ. The temperature T / T_0 (curve 1) and phase transition temperature of water T_f / T_0 (curve 2).

The calculated values formally satisfy the mathematical problem, but curve of phase transitions temperature evaluated from equation *(5)* in the water zone lies below the temperature curve, i.e. the water is "superheated". We must suppose the continuity of the water saturation function $S_- = S_+ = 1$, which makes it possible to remove the thermodynamic contradiction in the liquid phase zone. In this case the relations at the jump degenerate and have the form

$$(grad\,T)_{n+} = (grad\,T)_{n-}$$
$$(grad\,P)_{n+} = (grad\,P)_{n-} \qquad (7)$$

(see [Garg, Pritchett , 1988])
Figures 4.5 show consistent results of calculations $(P^0 = 2{,}5 \cdot 10^6\ Pa)$

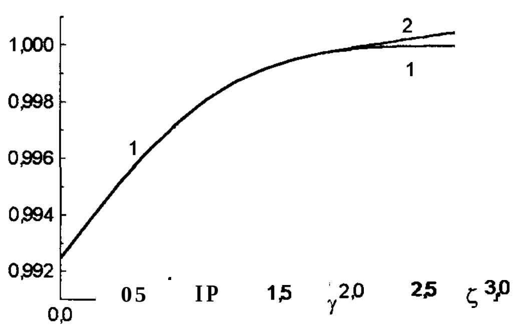

Fig. 4. The distributions of the temperature (curve 1.) and phase transition temperature (curve 2).

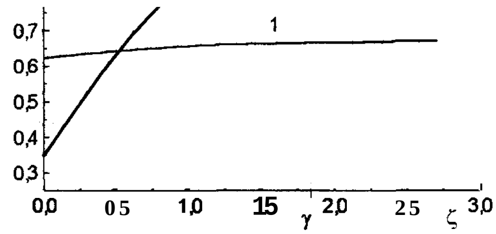

Fig. 5. The reservoir dimensionless pressure P / P_0 (curve 1) and water saturation function S (curve 2) as a function of dimensionless length.

<u>water-steam front</u>

$(S_+ = 1,\ \ S_- = 0)$

Here, we will consider the problem of the steam production from a water- saturated hydrothermal reservoir [Tsypkin, 1994]. In the water zone $(X(t) < x < \infty)$ the system of equations (3) is valid for $j = w$. In the zone $0 < x < X(t)$ the motion of the steam can be described by the system (3) for $j = v$. The relations at the moving water-steam interface have the form

$$m\left(1 - \frac{P_*}{\rho_w R T_*}\right) V_n = \frac{k P_*}{\mu_v \rho_w R T_*}(grad\,P)_{n-} - $$
$$-\frac{k}{\mu_w}(grad\,P)_{n+} \qquad (8)$$
$$m q \rho_w V_n = (\lambda\, grad\,T)_{n+} - (\lambda\, grad\,T)_{n-} -$$
$$-q \rho_w \frac{k}{\mu_w}(grad\,P)_{n+}$$

Relations (8), *(5)* constitute the complete system of boundary conditions at the boiling front. In Fig. 6 we have plotted the calculation results for

$T_0 = 500\,K,\quad P_0 = 2{,}7 \cdot 10^6\ Pa,\quad k = 10^{-17}\ M^2$

$m = 0{,}2 \quad P^0 = 10^6$ Pa.

The numerical experiments show that for all values, of the parameters in the zone ahead of the front the transition temperature $T_f = T_f(P)$ (curve 2) determined from *(5)* is less than the water temperature in that zone

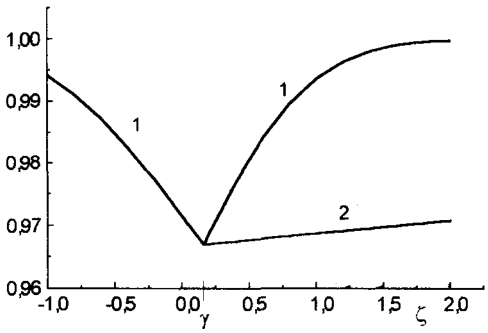

Fig. 6. Superheating of water in the zone ahead of the front γ . The temperature (curve 1) and phase transition temperature of water (curve 2).

Physically, this means liquid phase superheating ahead of the evaporation front. The mathematical model is based on the assumption of phase transition equilibrium. Consequently, this formulation of the problem contains a contradiction and cannot be used for describing the behavior of the hydrothermal system. We will construct a consistent theoretical description which removes this contradiction. For this purpose we introduce an extended phase transitions zone between the zones saturated with water and steam. In this case there are three zones with different phase composition separated by two moving boundaries: a steam zone $0 < x < X_1(t)$, a steam-water mixture zone $X_1(t) < x < X_2(t)$,and a water zone $x > X_2(t)$. In the intermediate zone the water and the steam are in a state of local thermodynamic equilibrium and the motion of the mixture can be described by the system (1). The boundary conditions on the rear moving interface $x = X_1(t)$ separating the steam zone and mixture zone have the form (6). For a leading front $x = X_2(t)$ boundary conditions can be described by (7). The calculation results are presented in Fig. 7,8.

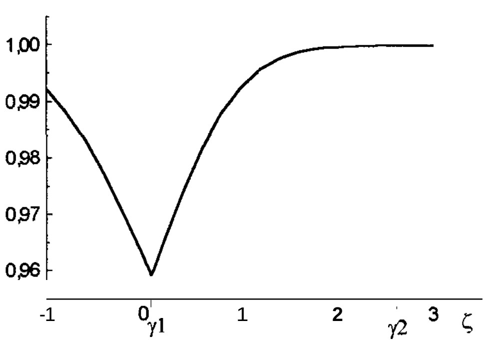

Fig. 7. The distribution of the dimensionless temperature.

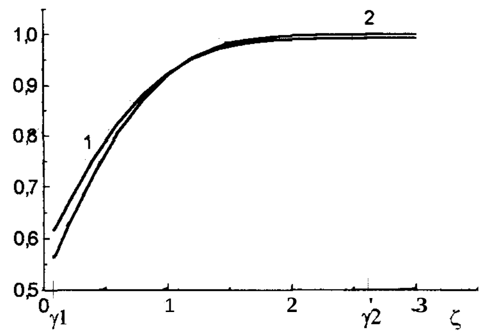

Fig. 8. The distributions of pressure (curve 1) and water saturation function (curve 2).

In accordance with the solution scheme proposed, water-steam transitions take place both in the extend region and in the narrow zone - the rear front separating the zones ocupied by steam and steam-water mixture. The value of the water saturation function in the mixture zone varies from $S = 1$ to the value at the rear front on the right $S = S_+$. As the leading boundary regards, this interface may be called the incipient boiling front because here the water saturation is equal to unity and there are no phase transitions. It should be noted that the velocity of the leading boundary is more than an order greater than the velocity of the total evaporation boundary and the distance of the latter from the stationary wall is insignificant. However, the question of the location of the interface between the steam and mixture zones is of fundamental importance because on this front the salts dissolved in the water are precipitated which may result in the choking of the reservoir and stoppage of the motion of the heat-transfer medium.

WATER INJECTION INTO VAPOUR-SATURATED RESERVOIR

This section is concerned with moving boundary problem that arises when cold water is injected into reservoir containing superheated vapour. Let the phase transition front $x = X(t)$ propagate to the right from the surface $x = 0$. We will show that for zones of reservoir with a permeability around $10^{-15} - 10^{-16} M^2$ the fact that the pressure decreases at a front and causes a flow of vapour toward the phase transition front can be analyzed on the basis of a simple model. If the permeability coefficient is less than $10^{-16} M^2$ this formulation of the problem leads to a solution which is incorrect. In the zone ahead of the front we have supercooled vapour. Furthemore, the linear approach is inapplicable to the reservoir which have a permeability above $10^{-15}\,M^2$. In this case the evaporation at the phase transition front must be taken into account.

vauour-water condensation front

$(S_+ = 0, \quad S_- = 1)$

We will assume that the vapour zone is located to the right of the interface, and water zone to the left. The condition at the moving front have the form [Barmin, Tsypkin 1996]

$$m\left(1-\frac{P_*}{\rho_w RT_*}\right)V_n = \frac{kP_*}{\mu_v \rho_w RT_*}(grad\,P)_{n+} - \frac{k}{\mu_w}(grad P),$$

$$mq\rho_w V_n = (\lambda\, grad\, T)_{n-} - (\lambda\, grad\, T)_{n+} - q\rho_w \frac{k}{\mu_w}(grad\,P)_{n-}$$

$$\lg\frac{P_*}{P_a} = A + \frac{B}{T_*}$$

This relations constitute the complete system of boundary conditions of the injection problem. The computation results for

$T_0 = 400\,K, \quad T^0 = 300\,K, \quad k = 0{,}5\cdot 10^{-15}\,M^2$

$m = 0{,}2, \quad P_0 = 2\cdot 10^5\,Pa, \quad P^0 = 3\cdot 10^5\,Pa$

are shown in Fig.9.

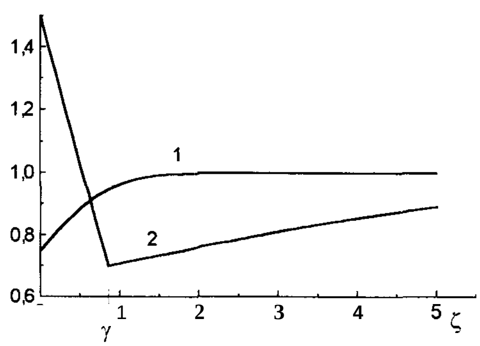

***Fig.** 9. The injection of water into vapour-saturated geothermal reservoir. The distributions of the temperature (curve 1) and pressure (curve 2).*

There are typical distributions of the temperature and the pressure in the two regions. The numerical simulations show that a change in the permeability has a strong influence on the value of the pressure at the interface. In this case the lowest pressure value is reached at the phase transition front that necessarily leads to condensation of vapour. Water is flowing away from the injection point while vapour is flowing towards it.

We have presented calculation results corresponding to small values of permeability coefficient. For high permeabilities the pressure and temperature distributions have various characteristic scales of length. In this cases the Figures lose obviousness.

ACKNOWLEDGEMENTS

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