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ON THE PRACTICAL ASPECTS OF DETERMINATION OF THE TRUE RESERVOIR TEMPERATURE UNDER SPHERICAL HEAT FLOW CONDITIONS

Ascencio, F. (1), Rivera, J.(2) and Samaniego, F.(2)
(1) **Escuela** de Ingenieria Mecánica. Universidad Michoacana.
(2) Universidad Nacional Autónoma de México.

Abstract

This paper presents additional unpublished aspects related to a recenty published method to calculate equilibrium formation temperatures, under spherical flow conditions in geothermal reservoirs. This method considers that the evolution of equilibrium bottom–hole temperatures following a cold fluid circulation time during drilling, follows a radialspherical pattern. F'rom the mathematical solution to this heat transfer boundary–value problem, it can be concluded that formation temperature recovery after a cold fluid circulation period, follows a straight line relationship on a T vs. $1/\sqrt{t}$ plot. The interception of this plot provides the formation equilibrium temperature. An advantage of this method compared to similar methods previously published, is that no explicit knowledge of fluid circulation lime previous to shut–in is required. On the other hand, it usually estimates equilibrium formation temperatures somehow higher than those obtained from the standard Horner–type of analysis. This is due to the fact that the proposed method takes into account the additional vertical heat transfer component at bottom–hole, while the Horner type of anlysis neglects any vertical heat transfer contribution, since it assumes a purely radial–cilindrical heat flow in the formation. Field data are used to illustrate the proposed technique.

INTRODUCTION

Proper knowledge of undisturbed producing formation temperature is a factor of importance in several earth sciences disciplines. Among them, in geothermal reservoir engineering, it is an important factor to properly define thermodynamic properties of reservoir fluids; or in the assessment of heat content in the reservoir. On the other hand, in petroleum engineering, it is an important parameter in designing deep–well cementing jobs, or in the design of hydraulic fracturing operations, as well as in the correct interpretation of well logs. It is common practice to calculate extrapolated static formation temperatures by means of a Horner–type of analysis [4, 6, 7, 9, 11,13], which is based upon a graph of recorded bottom–hole temperature, T, vs $\log(t_c + \Delta t)/\Delta t$, where t_c is the fluid circulation time, and Δt is the time elapsed since circulation stopped. This graph is suposed to exhibit a semilog straight line, whose interception with the temperature axis should render the undisturbed formation temperature.

It is convenient to point out, that the Horner–type of analysis of temperature build–up within a wellbore, is based upon an apparent similarity between this thermal process and that of pressure build–up within a wellbore, at the end of a fluid flow period. One of the assumptions in Horner's pressure build–up analysis is that of a fully–penetrating well in a homogeneous, isotropic formation, confined between two impermeable boundaries. For the thermal and pressure build–up problem to be equivalent, it would imply that the bottom of the well has to be an adiabatic boundary, which is not necessarely the case for most wells drilled in geothermal reservoirs, that are actually partially penetrating wells.

The purpose of this paper is to present several examples of the application of an alternative technique to calculate undisturbed formation temperatures, previously presented by Ascencio *et al.* [1994] [1], which considers that heat transfer from the formation at the bottom of the well takes place following a spherical–radial pattern.

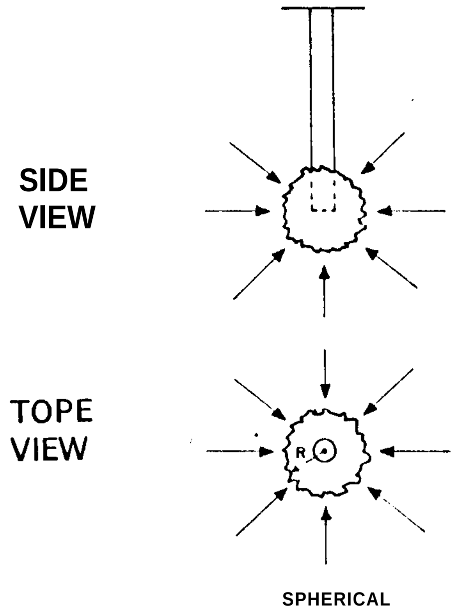


Figure 1: Conceptual model of heat flow lines following a spherical pattern at bottom–hole, during the warm–up period after fluid circulation stops.

SPHERICAL HEAT FLOW MODEL AT BOTTOM–HOLE

Recently, Ascencio et al. [1994] [1], presented an alternative method to calculate static formation temperatures from bottom-hole measurements taken during the temperature build–up period that follows fluid circulation during the drilling stage of a well. Main assumptions considered in this model are as follows.

1. Heat flow lines in the formation at the bottom of the well during the temperature build–up period, following cold fluid circulation, converge towards the bottom of the hole following a spherical–radial pattern, as shown in Fig. 1.

2. At bottom–hole and within a formation of initial static temperature, T_i, there exists a cooled region due to the effect of cold fluids circulation of temperature T_f, where $T_f < T_i$. This region can be approximated by a sphere of radius R, having a mean temperature T_f, while outside this thermal disturbance formation temperature remains equal to its initial value T_i. The center of this spherical region is taken right at the bottom–hole.

Further assumptions are: the medium is considered as homogeneous, isotropic and of infinite extent in the radial direction, with constant physical and thermal properties. It is also considered that when temperature measurements are taken, enough time has elapsed so that any heat transient effect due to rate changes during drilling, as well as heat transfer contributions from free and forced convection inside the wellbore, are negligible.

Taking into account the assumptions mentioned above, thermal recovery at bottom–hole during the thermal build–up period, following cold fluid circulation can be described by the following equations:

$$\frac{\partial^2 T_D}{\partial r_D^2} + \frac{2}{r_D}\frac{\partial T_D}{\partial r_D} = \frac{\partial T_D}{\partial t_D}, \quad 0 < r_D < \infty; \qquad (1)$$

$$T_D(t_D = 0, r_D) \begin{cases} 1 \text{ for } 0 < r_D < 1, \\ 0 \text{ for } 1 < r_D < \infty. \end{cases} \qquad (2)$$

where dimensionless variables are defined as:

$$T_D = \frac{T_i - T}{T_i - T_f} \qquad (3)$$

$$t_D = \frac{at}{R^2} \qquad (4)$$

$$r_D = \frac{r}{R}. \qquad (5)$$

It is convenient to mention that boundary conditions as defined by eq. (2), are similar to those previously applied by several authors (2, 15, 14, 16, 18, 19] to an equivalent problem, where the cooled region of temperature T_f around the wellbore was approximated as a cylinder of radius R, where heat flow lines follow an aproximate radial–cylindrical pattern.

Solution to the boundary–value problem given by eqs. (1) and (2) has already been published by several authors [5,12, 20]. This solution for $r_D = 0$ can be expressed as:

$$T_D = \text{erf}(\frac{1}{2\sqrt{t_D}}) \qquad (6)$$

where

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-u^2) du \qquad (7)$$

As previously shown by *Ascencio* et al. [1994], for sufficiently long times, eq. *(6)* can be approximated as:

$$T_D \approx \frac{1}{\sqrt{\pi t_D}} \quad (8)$$

w ich in terms o real variables can be expressed **as:**

$$T = T_i - m\frac{1}{\sqrt{t}} \quad (9)$$

where

$$m = \frac{R(T_i - T_f)}{\sqrt{\pi\alpha}} \quad (10)$$

From eq. (9), it follows that by p otting T *vs.* $1/\sqrt{t}$ a straight line is obtained whose interception with the ordinate **axis** ($t \to \infty$) will give the undisturbed formation temperature, T_i, and whose slope is inversely proportional to the square root of formation thermal diffusivity, α. The conditions that have to be met by the fitted field data to apply this aproximation, are described in the next section.

As previously stated by Ascencio **et** *al.* **[1994]**, strictly speaking, this approach should only be applied to calculate temperatures at the bottom of the well. At other depths, the typical cylindrical–radial type of geometry should be prefered.

CRITERIA TO APPLY THE MODEL PROPOSED

As it can be observed from Fig. **2,** for $t_D > 2$, ($1/\sqrt{t_D} < 0.7$) and/or $T_D < 0.4$, which corresponds to large times (late transient period), the behavior **of** T_D *vs.* $1/\sqrt{t_D}$ under spherical heat flow conditions, follows a straight line pattern. For field data that are fitted in this heat flow region, the following criteria can be established:

a) From eq. **(4)** the condition $t_D > 2$ can be *ex*pressed **as:**

$$t > \frac{2R^2}{\alpha} \quad (11)$$

On the other hand, from *eq.* **(10):**

$$R = \frac{m\sqrt{\pi\alpha}}{(T_i - T_f)} \quad (12)$$

Substituting eq. **(12)** in (11), the time criterion that field data have to meet, in order to apply the model proposed in this paper is:

$$t > \frac{2\pi m^2}{(T_i - T_f)^2} \quad (13)$$

b) Similarly, a limiting criterion regarding recorded temperatures can be obtained from the condition

$$T_D < 0.4 \quad (14)$$

From **eq.** (3), a straight line behavior should be followed by recorded temperatures such that

$$T > T_i - 0.4(T_i - T_f) \quad (15)$$

It should be pointed out that field data not adjusting to either limiting criteria given by eqs. (13) y (15), can still be analyzed in order to obtain the undisturbed formation temperature T_i. These data can be analyzed by either a type curve matching procedure, or by means of a non–linear regression analysis procedure. The former can be **ac**complished by using the type-curve given en Fig. **2,** meanwhile for the latter the objective function can be obtained from eq. **(6) as:**

$$T = T_f + (T_i - T_f)[1 - \mathrm{erf}(\frac{1}{2\sqrt{t/\tau}})] \quad (16)$$

where

$$\tau = R^2/\alpha \quad (17)$$

The parameters to be adjusted in eq. **(16)** are T_f and the time constant τ.

ILLUSTRATIVE EXAMPLES

PROBLEM A

Field data for this example were taken from a paper previously published by Brennand **[1984] [3], for** a geothermal well located at Leyte, Phyllip ines (Table **1).** Mean fluid flowing temperature, T_f, and circulation time, t_c, were **65 °*C*** and 15 *h*, respectively, while the equilibrium formation temperature **was** estimated **as 208** "C. The radial–cylindrical model for temperature recovery proposed by this author is:

$$T = T_f - \frac{m'}{\Delta t + ptc} \quad (18)$$

where ***At*** is the elapsed time since circulation stopped, and m' is the slope of the resulting

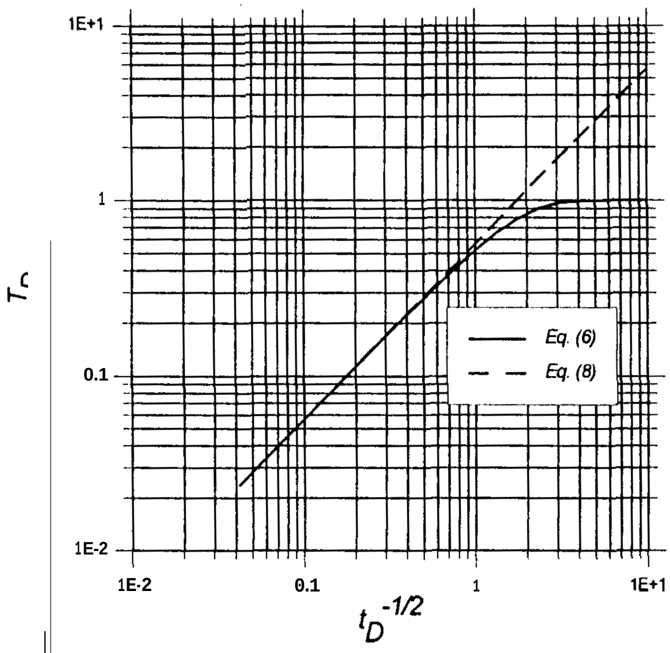


Figure 2: Variation of bottom–hole temperature, as given by eqs. (6) and (8), in terms of dimensionless temperature, T_D, and dimensionless time t_D.

straight line fitted through the field data when T is plotted versus $1/(\Delta t + pt_c)$, and is given as:

$$m' = \frac{\lambda(T_i - T_f)}{\alpha} \tag{19}$$

where λ and p are characteristic constants that have to be determined for a given field. For the field case considered they were estimated by the author as 6.8 y 0.785, respectively. Calculated undisturbed formation temperature with this model was reported as 208 $°C$.

Plotting reported field data on a T vs $1/\sqrt{t}$ graph (Fig. 3), according to the model given by eq. (9), an undisturbed formation temperature of 209 $°C$ and a straight line slope of $-258\ °C\ h^{-1/2}$ were determined. Checking the time restriction given by eq. (13), the straight line section predicted by Ascencio's et al. model should start after $t > 20$ h; however, the length of the field test was only of 15.58 h; therefore, recorded field data were still within the early time section of the type-curve of Fig. 2, probably close to its end.

Since the straigh line approach analysis described above proved to be non-valid, a non–linear regression analysis with the objetive function given by eq. (16) was performed by means of the Marquardt technique [1963] [17]. Results obtained from this analysis were T_i = 211 $°C$ and τ = 12.32 h. Good agreement with the reported for-

t (h)	T $(°C)$
2.58	93
3.58	88
4.58	99
5.58	108
6.58	112
7.58	117
8.58	120
9.58	126
10.58	133
11.58	133
14.58	
15.58	146

Table 1: Bottom–hole recorded temperatures for well of problem A (After *Brennand* [1984].)

mation equilibrium temperature of 208 °C was obtained with both *Brennand* [1984] (208 $°C$) and *Ascencio et al.* [1984] model (211 °C). A major advantaje of the latter model is that no explicit knowledge of circulation time is required to calculated T_i.

PROBLEM B

For this example, field data from well LC-1, Kyushu geothermal field in Japan are considered. These data were previously presented by *Hyodo and Takasugi* [1995] [10] (Table 2); total recording time was 24.5 h with a reported fluid circulation temperature, T_f, of 65.38 $°C$. Calculation of undisturbed formation temperature by these authors was performed by means of a model proposed by *Middleton* [1979] [19], which considers the wellbore as an elongated parallelepiped, assuming that after circulation stops, a region of total length 2R adjacent to the wellbore reaches instantaneously the fluid circulating temperature, T_f. The model proposed by Middleton is:

$$T = T_f + (T_i - T_f)[1 - \mathrm{erf}(\frac{1}{2\sqrt{t/\tau}})]^2 \tag{20}$$

Using this model and applying non–linear regression analysis to the recorded field data, the authors were able to calculate a predicted temperature of 170.5 $°C$ after 72.5 h of warm–up, which

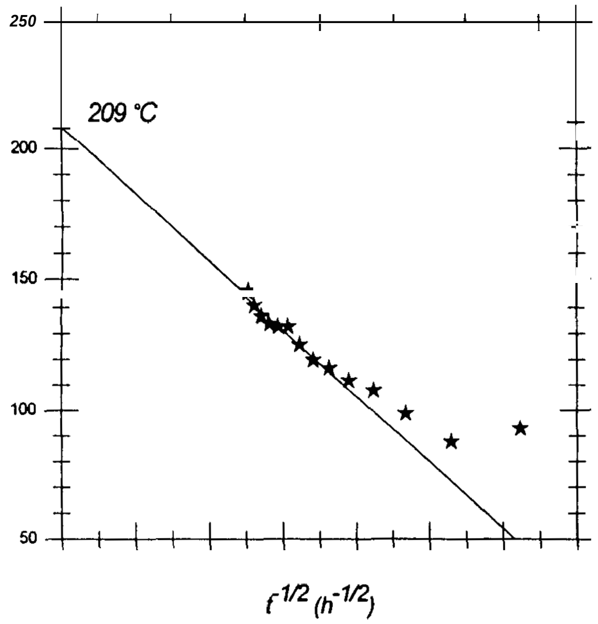


Figure 3: Analysis of field data €or problem A by means of Ascencio et al. model. Field data taken from Brennand [1984].

compared well with a measured temperature of 170.9 $°C$. Extrapolated undisturbed formation temperature was calculated as 215.9 "C.

It should be pointed out that Leblanc *et al.* [1981] [15] stated that Middleton's model given by eq. (20) is not correct, showing that the correct form of Middleton's model should be:

$$T = T_f + (T_i - T_f)\{1 - [\text{erf}(\frac{1}{2\sqrt{t/\tau}})]^2\} \qquad (21)$$

As these authors pointed out, eqs. (20) and (21) appear similar to each other; however, calculated temperature by means of eq. (21) converges asymptoticly faster than eq. (20) towards equilibrium temperature T_i. These authors also provide an alternative form of Middleton's model, given as:

$$T = T_f + (T_i - T_f)[\exp(-\frac{1}{4t/\tau})] \qquad (22)$$

According to Leblanc et al., both equations (21) and (22) provide similar results with convergence ratios also similar.

Fig. 4 shows a graph of Hyodo and Takasugi's field data plotted on a T vs $1/\sqrt{t}$ graph. Extrapolated equilibrium formation temperature considering the last five data points was 209 "C, it should be mentioned that the time criterion given by eq. (13) was fulfilled. Field data were also analyzed

t (h)	T ($°C$)
5.5	92.0
6.5	98.5
7.5	103.0
8.5	107.4
9.5	110.4
12.5	119.6
15.5	126.6
18.5	132.8
24.5	142.4
72.5	170.9

Table 2: Bottom–hole recorded temperatures for well LC-1 (after Hyodo and *Takasugi* [1995]).

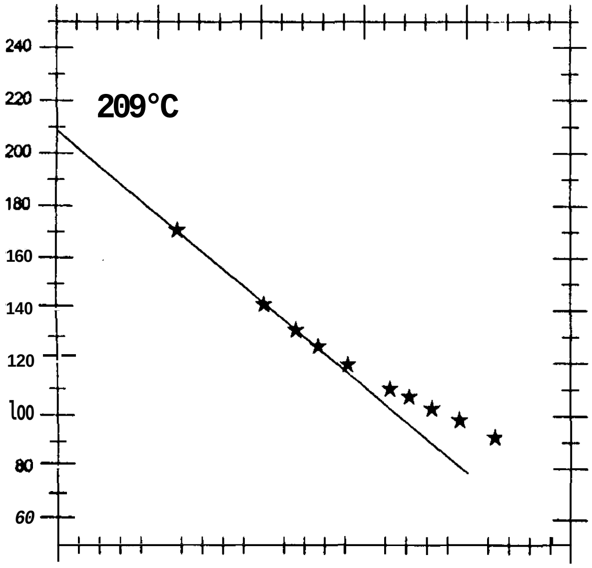
	Middleton	Leblanc	Ascencio	T vs $1/\sqrt{t}$
T_i, $°C$	179	178	212	209
τ, h	29	30	20	–

Table 3: Undisturbed formation temperatures calculated by means of several models. Well LC-l.

by means of non–linear regression analysis [17] and an undisturberd formation temperature of 212 $°C$ was obtained. Both results agree well with tbe value of 215.9 $°C$ estimated by Hyodo and *Takasugi* [1995]. Table 3 shows a summary of the results obtained by using the previously discussed three models to fit the field data.

PROBLEM C

Field data for this example correspond to well Az-44 from Los Azufres geothermal field in México, and were previously reported by García [1986] [8]. Temperatures were recorded during drilling at depths of 302 m, 584 m, 1600 m, 1970 m, 2440 m and 3029 m; recorded temperatures are given here in Table 4. Garcia calculated equilibrium formation temperatures by means of Horner, Middleton, Leblanc et al., and Brennand models; his results are reproduced here as Table 5. He considered a circulation time of 3 h for Horner's method, while for Middeleton's and Leblanc's et *al.* models, he took 0.1 m and 0.35 $\times^{-6}$ m^2/s for R y α, respectively. For Brennand's model Garcia used the same values for λ and p as those suggested by the author in his paper.



z (m)	t (h)	T $(°C)$
302	4	90.26
	6	104.51
	9	
584	3	65.03
	6	79.82
	9	
1600	3	129.45
	6	149.10
	10	158.92
1970	3	136.29
	6	156.97
	9	171.00
	12	184.91
2440	3	211.21
	7	224.97
	10	231.45
3029	4	228.37
	8	240.82
	12	254.23

z (m)	Horner $(°C)$	Middleton $(°C)$	Leblanc $(°C)$	Brennand $(°C)$
302	146	252	146	146
584	121	241	121	121
1600	191	259	176	177
1970	–	–	–	–
2440	245	299	242	242
3029	266	316	284	264

z (m)	T vs $1/\sqrt{t}$ $(°C)$	Non–linear regression $(°C)$
302	173	190
584	144	185
1600	195	198
1970	226	234
2440	254	255
3029	280	281

Table 6: Formation equilibrium temperatures for well A544 calculated by means of Ascencio's et al. model.

As it can be easily observed from Table 5 above, equilibrium formation temperatures calculated with Middleton's model, are far out of range compared with those temperatures calculated by means of all other models. This is due to the inconsistency problem of the model given by eq. (20), that was previously discussed.

The analysis of field data for well **Az-44** by means of Ascencio's et *al.* model is given in **Table** 6, where formation equilibrium temperatures were calculated by means of both the T *vs.* $1/\sqrt{t}$ plot (Fig. 5) and using the non–linear regression technique [17]. It is convenient to mention that the restriction given by the time criterion of eq. **(13)** was fulfilled by **all** data analyzed by means of the graphical technique.

It can be observed that undisturbed formation temperatures predicted by Ascencio's et al. model are somehow greater than those predicted by models previously used by Garcia, except for Middleton's model.

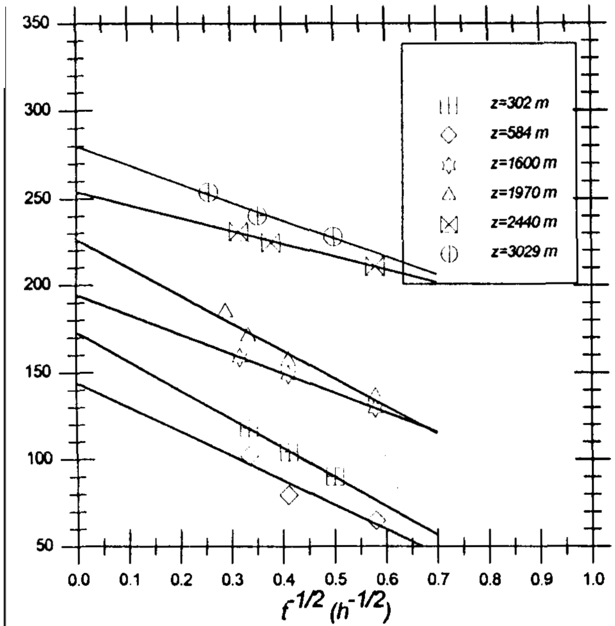


Figure 5: Analysis of field data from well Az-44 by means of Ascencio's *et al.* model.

CONCLUSIONS

This paper discusses new aspects related to a model previously presented by Ascencio et al. [1994] [1], to calculate undisturbed formation temperatures. This model assumes that temperature recovery at bottom–hole **follows** a radial–spherical heat flow pattern. Criteria for application of this model were presented and discussed and were illustrated through the application of the model to three field cases. F'rom the results obtained the following conclusions can be drawn:

1. Application of Ascencio ***et al.*** model should be restricted to those cases when temperatures are recorded at the very bottom–hole; when the recording instrument is not at the bottom of the well, previously reported models based upon radial-cylindrical heat **flow** patterns should be considered.

2. No explicit knowledge of previous fluid circulation time is required to apply the model discussed in this paper.

3. Criteria to be met for field data to be suitable for analysis by means of the model discussed are $t > 2\pi m^2/(T_i - T_f)^2$ and/or $T > T_i - 0.4(T_i - T_f)$.

4. When the time criterion is no met, field data should be analyzed by means of either a type–curve analysis using Fig. 2, or performing a **non–linear regression** analysis.

5. Ascencio's et al. model predicts undisturbed formation temperatures slightly higher, than those obtained with any of the models based on a Horner's type of analysis. This difference could be due to the contribution of heat recovery from the formation transfered in the near vertical direction at the bottom of the hole, which is neglected in models that assume radial–cylindrical heat flow patterns.

NOMENCLATURE

- m slope of a straight line on a T versus $1/\sqrt{t}$ (eq. (10))
- m' slope on a straight line of Brennand's model (eq. (19))
- p constant (equals to 0.785)
- r radial distance
- R radius of thermal disturbance due to cold fluid circulation
- r_D dimensionless radius (eq. (5))
- t time
- t_c circulation time
- t_D dimensionless time (eq. (4))
- T temperature
- T_D dimensionless temperature (eq. (3))
- T_f fluid circulation temperature
- T_i undisturbed formation temperature
- α formation thermal diffusivity
- At elapsed time after circ)lation stopped
- λ constant (equals to 6.8)
- τ time constant (eq. (17))

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