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# THE EFFECTS OF HEAT CONDUCTION ON THE VAPORIZATION OF LIQUID INVADING SUPERHEATED PERMEABLE ROCK

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## ABSTRACT

We examine the role of conductive and convective heat transfer in the vaporization of liquid as it slowly invades a superheated permeable rock. For very slow migration, virtually all of the liquid vaporizes. As the liquid supply rate increases beyond the rate of heat transfer by thermal conduction, a decreasing fraction of the liquid can vaporize. Indeed, for sufficiently high flow rates, the fraction vaporizing depends solely on the superheat of the rock, and any heat transfer from the superheated region is negligible. These results complement earlier studies of vaporization under very high injection rates, in which case the dynamic vapour pressure reduces the mass fraction vaporizing to very small values.

## INTRODUCTION

Liquid recharge is an increasingly important mechanism of maintaining vapour pressures in commercially exploited superheated geothermal systems (Enedy *et al.* 1993; Barker *et al.*, 1995). Numerous experimental and theoretical studies have examined vapour production at migrating liquid-vapour interfaces in order to optimize liquid injection strategies (Fitzgerald and Woods 1995; Pruess *et al.* 1987; Woods and Fitzgerald 1993; Pruess 1995). However, clear understanding of the effects of heat conduction and convection is crucial for building up a picture of the fundamental controls on vaporization. Here we present a simple but powerful class of similarity solutions which describe the slow migration and vaporization of an axisymmetric front of liquid spreading from a radial source (Figure 1).

## MODEL

We consider a migrating liquid front, supplied with liquid of Darcy speed $u$, through a porous rock of porosity $\phi$ and permeability $k$ (figure 1). We assume the temperature of the liquid on injection is $T_0$, the rock temperature is $T_2$ and the vaporization temperature has value $T_s$.

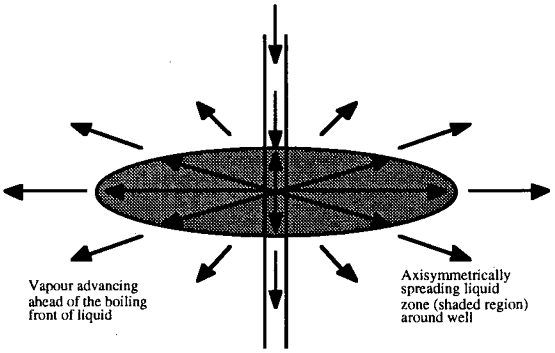

Figure 1. Cross-section of the model of liquid advancing radially outwards from a central well into the neighbouring permeable superheated strata. The advancing liquid front encloses a growing cylindrical region around the well. We analyze the radial motion of the liquid, assuming that the motion is independent of depth.

### Conservation Of Heat

In the liquid, the equation for the migration of the isotherms has the form

$$\frac{\partial T}{\partial t} + \lambda_w(\nabla u)T = \nabla(\kappa_w \nabla T) \tag{1}$$

where $\kappa_w$ is the effective thermal diffusivity of the porous matrix composite and

$$\lambda_w = \frac{\rho_l C_{pl}}{\rho_l C_{pl}\phi + \rho_s C_{ps}(1-\phi)} \tag{2}$$

In the vapour region, the isotherms migrate according to the relation

$$\frac{\partial T}{\partial t} + f\lambda_v(\nabla u_v)T = \nabla(\kappa_v \nabla T) \tag{3}$$

Here the isotherm advection speed, $f\lambda_v u$, is given from the conservation of mass across the interface

$$\rho_v u_v = f\rho_l u \tag{4}$$

where $u_v$ is the vapour speed ahead of the interface, $f$ is the mass fraction which vaporizes,

$$\lambda_v = \frac{\rho_l C_{pv}}{\rho_v C_{pv}\phi + \rho_s C_{ps}(1-\phi)} \tag{5}$$

and $\kappa_v$ is the average thermal diffusivity. In a typical low porosity system, the average thermal diffusivities will be comparable, and so we set $\kappa_w=\kappa_v=\kappa=$. Owing to the difference between the specific enthalpy of liquid and vapour, $C_{pl}>C_{pv}$, and so $\lambda_w>\lambda_v$. Equations (1) and (5) illustrate that the advection speed of isotherms through the porous layer, $\lambda_w u$, is much smaller than the interstitial speed of the liquid, $u\phi$, and this has an important influence on the heat transfer through the system (cf. Woods and Fitzgerald 1993).

## Interfacial Conditions

We assume that the temperature at the vaporizing interface equals the saturation temperature, as given by the Clausius-Clapeyron relation,

$$T=T_s(P_i) \qquad (6)$$

where $P_i$ is the interfacial pressure. The conservation of thermal energy across the vaporizing interface has the form

$$\left[\rho C_p \kappa \frac{\partial T}{\partial x}\right]_-^+ = ufL\rho_l \qquad (7)$$

where $f$ is the mass fraction of liquid which vaporizes and $L$ is the latent heat of vaporization per unit mass. The notation $[g]$ denotes the difference in the value of the function $g$ across the interface. For convenience we define the Stefan number of vaporization $\mathrm{St}=L/C_{pv}(T_2-T_s)$. This represents a measure of the heat required to vaporize the liquid compared to the superheat available in the reservoir for the vaporization.

## Flow Dynamics

We consider the situation in which a constant flux of cold liquid migrates from an injection well (figure 1). The vapour pressure may be found by analyzing the motion of the vapour as it advances from the interface (cf. Pruess *et al.* 1987; Woods and Fitzgerald 1993). However, for very slow vapour flow, the dynamic vapour pressures are small (Fitzgerald and Woods 1995). Therefore, throughout the vapour region, the pressure is approximately equal to that of the far field, and the temperature of the vaporizing interface equals the saturation temperature associated with the far field pressure.

## AXISYMMETRIC FLOW

We examine the situation in which a constant flux of liquid of magnitude $q=Q/2\pi$ per unit length spreads radially from a central source. The radial Darcy velocity in the liquid is

$$u=\frac{Q}{r} \qquad (8)$$

The thermal advection-diffusion equation in the liquid (1) takes the form

$$\frac{\partial T}{\partial t}+\frac{\lambda_w Q}{r}\frac{\partial T}{\partial r}=\frac{\kappa}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \qquad (9)$$

## Supercooled, Liquid Saturated Reservoir

Before analyzing the case of a vaporizing front, we consider the temperature profile in liquid invading a liquid-saturated, supercooled rock of different temperature. In this case, the temperature of the front of new liquid may vary from the input temperature $T_o$ to the far-field temperature $T_2$. For a constant source of liquid, $Q$, the system admits similarity solutions $T(\eta)$ where $\eta=r/2(\kappa t)^{1/2}$ and the interface is located at

$$\eta=\omega=\left[\frac{Q}{2\phi\kappa}\right]^{1/2} \qquad (10)$$

In similarity coordinates, the diffusion equation becomes

$$-\left(\frac{(1-\lambda_w Q/\kappa)}{\eta}+2\eta\right)\frac{dT}{d\eta}=\frac{d^2T}{d\eta^2} \qquad (11)$$

with solution

$$T(\eta)=T_0+\int_0^{\eta}\eta^{(\lambda_w\beta-1)}\exp\left(-\eta^2\right)d\eta \qquad (12)$$

This solution identifies that there is no net heat flux supplied to the origin for $Q>0$ since $\eta dT/d\eta\rightarrow 0$ as $\eta\rightarrow 0$. This is a result of the advection of fluid away from the source, and contrasts with the temperature field produced by a line source in a solid body, in which there is finite net heat flux transferred to the source (Carslaw and Jaegar 1986). In the present case, the absence of a net heat flux at the origin demands that the temperature at the origin equals the source temperature. The precise value of the interfacial temperature is given by $T_i=T(\omega)$and has the form

$$T_i=\frac{a+b}{c} \qquad (13)$$

where

$$a=T_2\int_0^{\omega}\eta^{(\lambda_w\beta-1)}\exp\left(-\eta^2\right)d\eta \qquad (13a)$$

$$b=T_0\int_{\omega}^{\infty}\eta^{(\lambda_w\beta-1)}\exp\left(-\eta^2\right)d\eta \qquad (13b)$$

$$c=\int_0^{\infty}\eta^{(\lambda_w\beta-1)}\exp\left(-\eta^2\right)d\eta \qquad (13c)$$

and the variable $\beta$ is a measure of the magnitude of the liquid flux in comparison with the thermal diffusivity

$$\beta=Q/\kappa \qquad (14)$$

In figure 2, we present some typical solutions of the temperature profile in the liquid, identifying the location of the interface between the original and new liquid, for three different injection rates. There are three different thermal regimes depending upon

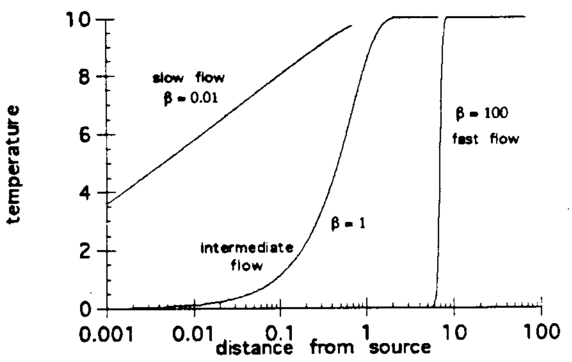

Figure 2 Comparison of thermal profiles in newly injected liquid which displaces colder liquid already in place in the reservoir.

whether the rate of thermal diffusion exceeds (i) the interstitial speed of the liquid, $\phi\kappa > Q$ and (ii) the advection speed of the isotherms, $\kappa > Q\lambda_w$.

### *Fast Injection*

For $\kappa << Q\lambda_w$ the interface migrates much more rapidly than heat may be conducted from the far-field. Therefore, the liquid is heated to the far-field temperature by cooling the rock near the source and the liquid attains the far-field temperature long before reaching the leading edge of the zone of newly injected liquid. Furthermore, since the isotherms are advected more rapidly than the rate of diffusion of heat from the far-field, the cold liquid injected at the source generates a cold region near the source. As a result, the temperature adjusts to that of the far-field across an internal boundary layer, ahead of the source.

### *Intermediate Injection*

If $\phi\kappa < Q < \kappa/\lambda_w$ then again the rate of advance of the leading edge of the liquid zone is greater than the rate of thermal conduction. As a result, the liquid again attains the far-field temperature long before reaching the leading edge of the liquid zone. However, the isotherms are advected more slowly than the rate of thermal diffusion, so that heat can be conducted towards the source, and no isothermal cold region develops at the source. Instead the temperature gradually increases from the source. As in the case of very fast injection, the thermal energy used to raise the temperature of the injected liquid originates from the cooling of the rock near the source.

### *Slow Injection*

In the case $\phi\kappa >> Q$, the liquid moves much more slowly than the rate of diffusion of isotherms. As a result, the rock far ahead of the interface may be cooled by thermal conduction heating up the newly injected liquid. Now the liquid only occupies a small inner part of the thermal boundary layer. Therefore, the temperature of the liquid interface decreases and the liquid is heated by conduction of heat from the far-field. However, in the limit of very small injection rates, the temperature at the leading edge of the liquid zone in fact returns to values close to that of the far-field.

These solutions provide valuable insight into the temperature profile which develops as liquid invades a superheated reservoir and vaporizes. In that case, the interface temperature is constant, but the mass fraction vaporizing changes according to the amount of heat conducted into the interface from the far field.

## **Liquid Injection Into A Superheated Reservoir**

If the reservoir is superheated, then a fraction $f$ of the liquid may vaporize as the liquid invades the reservoir. The advection-diffusion equation in the vapour ahead of the interface has the form

$$\frac{\partial T}{\partial t} + \frac{f\lambda_v Q}{r}\frac{\partial T}{\partial r} = \frac{\kappa}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{15}$$

while the liquid is governed by equation (9). At the interface between the liquid and vapour, the conservation of heat takes the form

$$\left[\frac{\partial T}{\partial r}\right]_-^+ = \frac{fQL}{\kappa C_p r} \tag{16}$$

Again, for a constant source of liquid, $Q$, the system admits similarity solutions $T(\eta)$ given by

$$T(\eta) = T_s + A\int_\omega^\eta \eta^{(\lambda_v fQ/\kappa - 1)} \exp\left(-\eta^2\right) d\eta \tag{17}$$

for $\eta > \omega$ and

$$T(\eta) = T_s + B\int_\omega^\eta \eta^{(\lambda_w Q/\kappa - 1)} \exp\left(-\eta^2\right) d\eta \tag{18}$$

for $\eta < \omega$ where $\eta = r/2(\kappa t)^{1/2}$ and the interface is located at

$$\eta = \omega = \left[Q(1-f)/2\phi\kappa\right]^{1/2} \tag{19}$$

The constants $A$ and $B$ are then found by noting that the conservation of heat at the interface has the form

$$B\omega^{[Q\lambda_w/\kappa]} \exp\left(-\omega^2\right) - A\omega^{[fQ\lambda_v/\kappa]} \exp\left(-\omega^2\right) = -\frac{LfQ}{\kappa C_p} \tag{20}$$

while the temperature at the origin equals the source temperature

$$T_o = T_s + B\int_\omega^0 \eta^{[\lambda_w Q/\kappa - 1]} \exp\left(-\eta^2\right) d\eta \tag{21}$$

and as $\eta \to \infty$, $T \to T_2$, so that

$$T_2 - T_s = A\int_\omega^\infty \eta^{[f\lambda_v Q/\kappa - 1]} \exp\left(-\eta^2\right) d\eta \tag{22}$$

In figure 3, we illustrate how the mass fraction which vaporizes varies with the injection rate, the reservoir superheat, the liquid undercooling and the latent heat of vaporization. The main effects on this figure may be understood by reference to the detailed thermal profiles which occur at the different flow rates (figure 4).

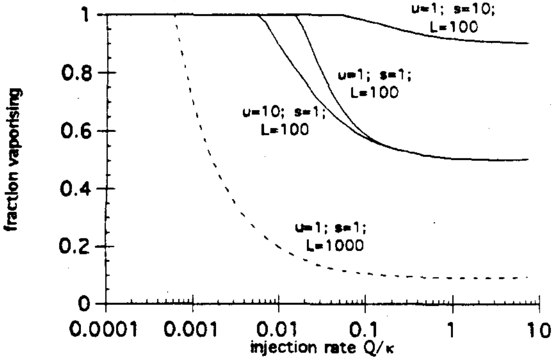

Figure 3 Fraction vaporizing as a function of injection rate. Parameters are dimensionless. u=undercooling of liquid; s=superheat of reservoir; l=latent heat of liquid

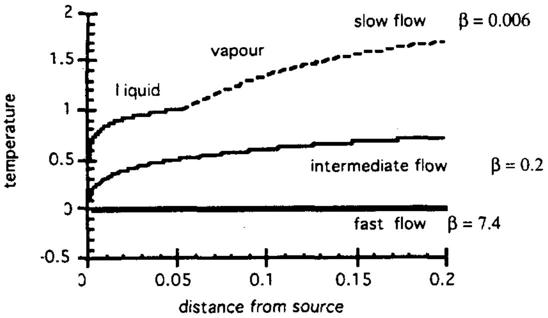

Figure 4(i) Temperature profile near the source. The fast flow retains the injection temperature ahead of the source. The slow flow is heated up and vaporizes much closer to the source.

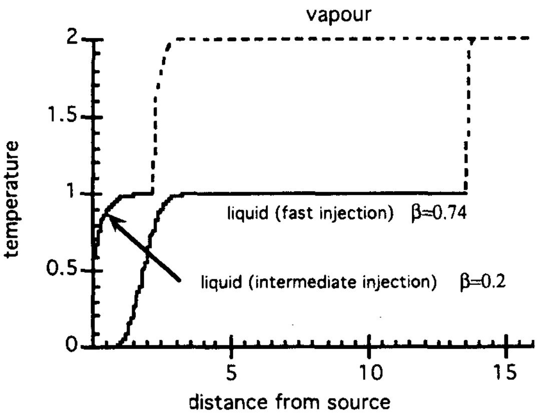

Figure 4(ii) The thermal profile further from the source. For fast flow, there is a sharp thermal boundary layer where the liquid is heated to the boiling point, and also where the liquid boils. All units are dimensionless.

### *Fast flow:* $\lambda Q > \kappa$

For high injection rates, the liquid Darcy velocity exceeds the thermal diffusion speed and so near the source, the liquid remains of temperature close to $T_0$ (figure 4). Only when the liquid has invaded a distance $\eta \approx O((\lambda_w \beta - 1)^{1/2})$ does the liquid heat up across a narrow thermal boundary layer to the temperature of the interface. The fluid then attains the interfacial temperature long before reaching the interface, and the mass fraction vaporizing is given by the asymptotic relation

$$f = \left[ 1 + \frac{\phi L}{C_p (T_2 - T_s)} \right]^{-1} \tag{23}$$

This corresponds to the slow injection regime studied by Woods and Fitzgerald (1993), and as seen in figure 3, the mass fraction vaporizing is independent of the initial temperature of the liquid. As the reservoir superheat increases, the mass fraction vaporizing also increases, since there is more energy released by the rock as it is cooled by the invading liquid. In contrast, as the latent heat increases, the mass fraction vaporizing decreases.

### *Intermediate Flow:* $\omega >> 1$; $\lambda_w Q < \kappa$

In this case, the liquid travels more slowly than the Darcy velocity of the interface, and therefore the liquid temperature increases towards that at the interface temperature immediately after entering the rock (figure 4). However, since the leading edge of the liquid zone migrates more rapidly than the rate of thermal diffusion, the liquid is heated up to the interfacial temperature long before reaching the interface. As a result, the mass fraction vaporizing still has the form (22).

### *Slow Flow:* $Q << \phi \kappa$

In the case of slow flow, the leading edge of the liquid migrates through the rock more slowly than the rate of thermal diffusion. In this case the liquid temperature varies from the source to the advancing interface, and there is a significant heat flux conducted from the region ahead of the interface into the liquid zone (figure 4). This cooling of the liquid zone increases the rate of vaporization (figure 3) so that in the limit $\beta \to 0$ corresponding to very small injection rates, the mass fraction vaporizing, $f \to 1$. Since there is no net heat flux supplied to the origin $\eta = 0$, the liquid always enters the rock with temperature $T_0$, and there is always a small region of liquid spreading from the origin. However, the conduction of heat from the superheated reservoir far ahead of the liquid provides additional heat to drive vaporization.

## IMPLICATIONS

These fundamental solutions illustrate the control that heat conduction has on the fraction of liquid which vaporizes, $f$, when the liquid slowly spreads from a central source into a superheated reservoir. When the radial flow rate $Q/2\pi$ per unit depth of the injection well is larger than the thermal diffusivity $\kappa \approx 10^{-7} m^2/s$, very little heat is conducted back towards the source, the liquid is heated to the vaporization temperature long before reaching the vaporizing front, and the mass fraction which vaporizes depends on the far-field reservoir superheat. When $Q/2\pi$ is smaller than $\kappa\phi$, heat is able to diffuse from the far-field superheated rock into the liquid region. In this case, the mass fraction of liquid which vaporizes increases towards unity, thereby producing a very effective means of recharging the reservoir without flooding the reservoir with liquid.

For a reservoir with average porosity 0.01-0.1 and thermal diffusivity $10^{-7}$ $m^2/s$, the injection rate into a 1km well needs to be smaller than $10^{-4}$-$10^{-5}$ $m^3/s$ for heat conduction to play an important part in the vaporization of the liquid. This corresponds to an injection rate of approximately 1-10 gal/min at the well. For higher injection rates, the maximum mass fraction which vaporizes depends on the reservoir superheat.

These results complement earlier work (Woods and Fitzgerald 1993; Pruess *et al.* 1987) which identified that for injection rates in excess of the effective vapour diffusivity, the dynamic pressure of the vapour raises the interface temperature and suppresses vaporization. The typical effective vapour diffusivity or vapour transport coefficient is of order $10^{-5}$-$10^{-2}$ $m^2/s$ per unit depth (Fitzgerald and Woods 1995). This corresponds to injection rates along a 1km well of 0.01-10 kg/s, depending on the effective permeability of the reservoir. Above this injection rate, the mass fraction vaporizing falls to very small values.

Combining the present results with those of Woods and Fitzgerald (1993), we deduce that the optimal injection rate is of order 0.01-10 kg/s, depending on the reservoir properties. Although for smaller injection rates, virtually all of the injectate vaporizes, for injection rates in the above range a significant fraction of the liquid will vaporize and a much larger vapour flux is produced. For higher liquid injection rates, the injection becomes less efficient and the reservoir floods with water much more rapidly. It is interesting to note that the upper values of the intermediate injection range correspond to the injection rates used at the wells in the NCPA steam field at the Geysers (Enedy *et al.* 1993). We note, however, that although our model exposes the fundamental controls on vaporization in a simple axisymmetric geometry, the process may be more complex, with geometrical or gravitational effects controlling the spreading of the liquid (e.g. Pruess 1995; Woods and Fitzgerald 1995).

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