PROCEEDINGS, Twenty-First Workshop on Geothermal Reservoir Engineering
Stanford University, Stanford, California, January 22-24, 1996
SGP-TR-151

SIMPLE NUMERICAL SIMULATION FOR LIQUID DOMINATED GEOTHERMAL RESERVOIR

Djoko Wintolo[1], Sutrisno[1], Sudjatmiko[1], and S. Sudarman[2]
[1] Geothermal Research Center, Faculty of Engineering, Gadjah Mada University, Yogyakarta, Indonesia
[2] Directorate of Exploration and Production, PERTAMINA, Indonesia

ABSTRACT

A numerical model for geothermal reservoir has been developed. The model used is based on an idealized, two-dimensional case, where the porous medium is isotropic, nonhomogeneous, filled with saturated liquid. The fluids are assumed to have constant and temperature dependent viscosity. A Boussinesq approximation and Darcy's law are used. The model will utilize a simple hypothetical geothermal system, i.e. graben within horsts structure, with three layers of different permeabilities. Vorticity plays an importance roles in the natural convection process, and its generation and development do not depend only on the buoyancy, but also on the magnitude and direction relation between the flow velocity and the local gradient of permeability to viscosity ratio. This model is currently used together with a physical, scaled-down reservoir model to help conceptual modeling.

INTRODUCTION

During a stage conceptual modeling for a geothermal field only very limited data is available. Since three-dimensional modeling is not a practical choice in this case, two-dimensional modeling will be more favorable (O'Sullivan, 1985; Pestov, 1993).

It is very well known also that the higher the pressure, water requires higher temperature to be in a vapor phase. For instance, at 15 bars it remains as liquid for temperatures up to 200^{0}C. At depth of 1500 meters, where the hydrostatic pressure is about 150 bars, the fluid is in liquid phase for temperatures up to 340^{0}C. Therefore reservoir modeling with liquid phase medium gives a reasonable approximation in most modeling especially in the stage of conceptual modeling development.

An inhomogenity affects in reservoir modeling receive less attention simply because of its simple definition. Due to these reasons a simple, two dimensional, liquid-phase reservoir model is developed and analyzed to gain the basic physical understanding to the fluid flow dynamics and heat transfer mechanism in geothermal systems.

In this study the liquid dominated geothermal reservoir is simulated as an unsteady convective flow development toward a natural state condition in non-homogeneous, isotropic porous medium filled with saturated liquid. When the asymmetrical component of the Darcy's law is investigated, it is found that the vorticity generation, leading to fluid flow pattern development, is not only caused by local, horizontal temperature gradient but also by vector product between the gradient of the permeability to fluid viscosity ratio and the fluid velocity. Therefore in numerical modeling, this vector product, along with the horizontal temperature gradient, are responsible for the resulting temperature distribution.

A concept of power density distribution is introduced to show the regions in the geothermal reservoir with high power capacity per unit area. A local power density is found when a local relative enthalpy is multiplied by the fluid density and the local fluid velocity. Together with temperature distribution, the power density pattern can be used to help in conceptual modeling in justifying the method of exploitation.

FORMULATION OF THE MODEL

In this numerical model, the liquid dominated geothermal reservoir is simulated as an unsteady convective system in a non-homogeneous, isotropic porous medium filled with saturated liquid. The fluid is assumed to be incompressible, following the

Boussinesq approximation (Cheng, 1978; Bejan, 1984). The governing equation for the two-dimensional convective flow can be written as

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0\ ,\quad \rho'=\rho'_0\,[1-\beta\ (T-T_0)] \qquad (1a,b)$$

$$u'=-\frac{K'\partial p'}{\mu'\partial x'}\ ,\quad v'=-\frac{K'}{\mu'}\left(\frac{\partial p'}{\partial y'}-\rho' g'\right) \qquad (2a,b)$$

$$\sigma'\frac{\partial T'}{\partial t'}+u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}=\frac{1}{\rho'_0 C'_V}\left[\frac{\partial}{\partial x'}\left(k'\frac{\partial T'}{\partial x'}\right)+\frac{\partial}{\partial y'}\left(k'\frac{\partial T'}{\partial y'}\right)\right] \qquad (3)$$

The law of mass conservation is expressed in equation (1a) for incompressible fluid, while (2a,b) show that fluid motion is governed by Darcy's law, that is dependent on pressure gradient, and the ratio between permeability and fluid viscosity. For the case of vertical motion, vertical velocity is affected by buoyancy which is strongly related to the Boussinesq approximation (1b).

Energy balance is expressed in (3) showing the relation between the rate of change in temperature $\sigma'\,\partial T'/\partial t'$, the convective heat transfer $u'\,\partial T'/\partial x' + v'\,\partial T'/\partial y'$, and heat diffusion. It is to be noted that that rock permeability K', thermal conductivity k', specific heat are assumed to be space-dependent and that the liquid viscosity varies with temperature, $\mu' = \mu'(T')$.

From computational point of view, a more suitable expression for mass conservation can be formulated by defining stream function (ψ'), as

$$u' = \partial\psi'/\partial y' \text{ and } v' = -\,\partial\psi'/\partial x' \qquad (4).$$

Physically the stream function can be used to show the flow pattern, such that the difference between two stream functions denotes the confined fluid flux (Bejan, 1984). Correspondingly, the strength of the local vortices (ζ') can be expressed in terms of stream function as

$$\partial^2\psi'/\partial x'^2 + \partial^2\psi'/\partial y'^2 = -\,\zeta' \qquad (5).$$

This expression clearly shows that the local vortex strength dictates the fluid flow pattern. The expression for vorticity (ζ') generation and development can be found from the expression of Darcy's law (2a,b),

$$\bar{\zeta}'=\frac{K'}{\mu'}\left[\nabla'\left(\frac{K'}{\mu'}\right)\times\bar{V}'\right]+\frac{\rho'_0\,g'K'\beta}{\mu'}\frac{\partial T'}{\partial x'}\hat{j} \qquad (6).$$

This equation shows that the local vortex, which governs the fluid flow pattern, is caused not only by local, horizontal temperature gradient but also by vector product between the gradient of the permeability to fluid viscosity ratio and the fluid velocity. Therefore in numerical modeling, this vector product, along with the horizontal temperature gradient, are responsible for the resulting temperature distribution.

For the case when (K'/μ') is not homogeneous and the horizontal temperature gradient is absent, the equation (6) shows that the fluid flow will be affected by the strongest vorticity when $\nabla'(K'/\mu')$ is perpendicular to $\bar{V}'$. This vortex disappears when the two vectors are parallel.

NUMERICAL FORMULATION

In this preliminary work, the governing equation is expressed in non dimensional form

$$\Omega=-\frac{\mu}{K}\left[\frac{\partial\Psi}{\partial X}\frac{\partial}{\partial X}\left(\frac{K}{\mu}\right)+\frac{\partial\Psi}{\partial Y}\frac{\partial}{\partial Y}\left(\frac{K}{\mu}\right)\right]-R_{aq}\frac{\partial\theta}{\partial X}$$

$$\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}=-\Omega \qquad (7,8)$$

$$\frac{\partial\theta}{\partial\tau}+u\frac{\partial\theta}{\partial X}+v\frac{\partial\theta}{\partial Y}=\alpha\left[\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right]+\left[\frac{\partial\alpha}{\partial X}\frac{\partial\theta}{\partial X}+\frac{\partial\alpha}{\partial Y}\frac{\partial\theta}{\partial Y}\right] \qquad (9)$$

where

$(X,Y) = (x',y')/H'$, $(U,V) = (u',v')/(\alpha'_0/H')$,
$\tau = t'/(\sigma' H'^2/\alpha'_0)$, $\Psi = \Psi'/\alpha'$,
$\Omega = \zeta'/(\alpha'_0/H'^2)$, $\theta = (T' - T'_0)/(q'_0 H'/k'_{m0})$
$\sigma = \sigma'/\sigma'_0$, $\alpha = \alpha'/\alpha'_0$, $\mu = \mu'/\mu'_0$

and the modified Rayleigh number,

$$R_{aq} = (k'_0 g'\beta H'^2_0\, g'_0)/(\nu'_0\,\alpha'_0\, k'_{m0})$$

Equation (8) is discretized based on explicit method, namely line successive over relaxation technique (LSOR) to yield the fluid flow pattern. The resulting velocity distribution is then used in (9) which is discretized based on upwind differencing technique to solve the temperature distribution. Vorticity distribution in the system is calculated using (7), and proceed to the next step.

THE CONCEPT OF POWER DENSITY

In analyzing the results of conceptual models involving mass and energy conservations (1), (2), and (3), one can find simultaneously temperature distributions as well as fluid flow pattern. Generally only the temperature distribution is used to localize zones of high enthalpy.

It is to be noted that when only the temperature distribution is used, one can only identified zones of high enthalpy (J/kg). Nevertheles, in conceptual modeling one actually tries to identify zones with high power production (Watts) over a range of geothermal area. Therefore one should multiply the relative enthalpy (J/kg) with fluid density (kg/m^3) and fluid flow rate per unit area ((m^3/s)/m^2) in order to get the power density (W/m^2). Even though some regions have high relative enthalpy, if the local fluid flow rate is low, the resulting power production will be low.

Along with the temperature distribution, the power density distribution can be used to determine the regions of high enthalpy with high power production.

PROBLEM GEOMETRY

This numerical model is applied on a hyphothetical geothermal system, similar to Kamojang field. The geometry of the area is derived from conceptual models developed based on geophysical, geological observation with limited available data. The boundaries between layers are curve-fitted and expressed in functional forms. Due to the limited availability of the data, some parameters for rock matrices in each region are defined and assumed by trial and error.

It is to be noted that one should pay attention to the (K'/μ') discontinuity on the boundary between region. As explained in equation (6), that this discontinuity might cause numerical problem due to large gradients of (K'/μ').

NUMERICAL RESULTS

A simple geothermal system is considered, in the form of a graben within horsts structure with three layers having different permeabilities, as illustrated in Figure 1.

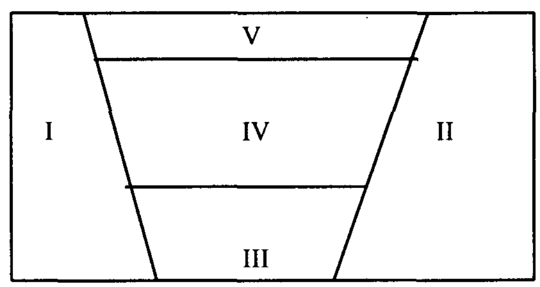

Fig. 1. The geometry of the problem .

Parameters involved in the five regions are tabulated in Table 1. The temperature dependence of viscosity is defined as $\mu = T^a$, where a = 0 is used to illustrate for the case of constant viscosity.

In this numerical experimentation, the preliminary effort is concentrated to match the model with a hypothetical geothermal system, similar to Kamodjang field, especially to the wellbore temperature data.

Table 1. Thermophysical parameters for rocks matrices for the five corresponding layers.

Region	Porosity ϕ	Density ρ (kg/m^3)	Specific heat C_r (J/kg C)	Conductivity k_r (W/m C)	Permeability K (mD)
Region-I	3,5 %	2,65	1050	3,01	1,75
Region -II	3,8 %	2,64	1020	3,01	1,75
Region-III	3,4 %	2,63	1040	3,04	9,45
Region-IV	3,7 %	2,54	1050	2,69	7,35
Region V	4,5 %	2,55	1030	2,10	8,95

Numerical calculations are carried out by assuming that its initial temperature distribution is linear to the depth from the surface in order to immitate an initial stable conduction heat transfer situation. The temperature conditions at the boundaries are then modified by some trial and error processes to match the known gradient temperature tendency, especially near the boundaries.

The input parameters based on geological and geophysical data interpretation generally give some isothermals which deviate considerably from the wellbore temperature data. Some initial efforts to give better matching is carried out by altering the permeability distributions based on (7). The current results are presented in Fig. 2 and 3 in the forms of temperature distribution and the corresponding flow pattern.

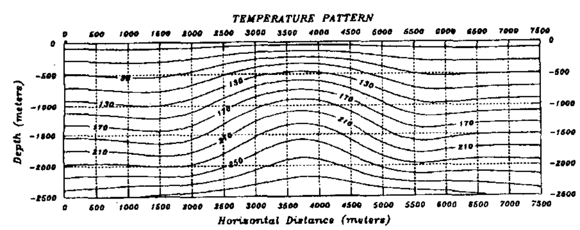

Fig.2. Temperature distribution for the hypothetical model.

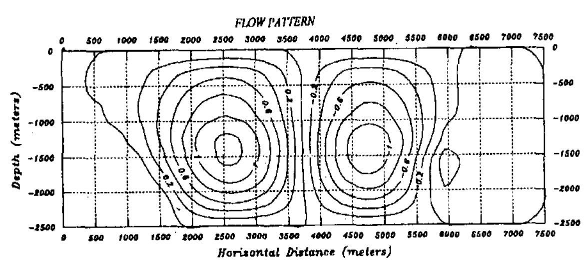

Fig. 3. Flow pattern for the hypothetical model..

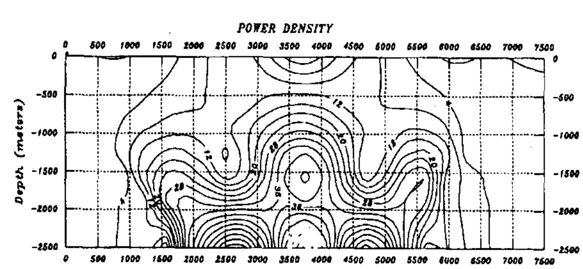

Fig. 4. Power density distribution for the hypothetical model..

Fig. 4 shows the resulting power density distribution. It is to be noted that zones of high power density correspond to regions of high temperature with high fluid velocity. Some regions could have low power density, even though they are located in the high enthalpy zones, due to their low fluid velocity. Fig. 4 also shows that the region of low enthalpy could also have high power density due to its high fluid velocity.

Some zones of high enthalpy having rather low power density suggest that exploration should be distributed in such away that matches to the local fluid fluxes in that regions.

Course mesh sizes could be used to speed up computation and minimize the tendency to become numerically unstable. Even though the results from the two dimensional, liquid sturated model is very crude, however the solution would be very useful for numerical models using more advanced methods. It is to be stressed also, since a numerical modeling is an inverse problem, the solution is not unique. Further investigation using vorticity development observation is currently in progress.

SCALED-DOWN RESERVOIR MODELS

An attempt to conduct laboratory-size, experimental reservoir modeling has been initialized. The preliminary model is constructed based on the basic physical understanding of the mathematical model for reservoirs using nondimensionalization analysis applicable for fluid dominated systems.

In this mathematical study, the hydrostatic pressure and the reference temperature are extracted, while the characteristic time, velocity, and pressure are isolated in order to develop analogous temperature and flow patterns in both model and the field situations.

A typical example, when the size is reduced by 10^{-3} and the permeability is multiplied by 10^3, the resulting velocity will be magnified by 10^3. Consequently, the characteristic time required for the fluid to flow from the bottom to the surface in the field situation will be simulated in the scaled-model by a characteristic time which is reduced by a factor 10^{-6}. It means that one-day experiment in the model simulation is equivalent to almost 3,000 years in the real field situations.

Basic characteristic investigations for the scaled-down models are currently in progress.

CONCLUSIONS

Preliminary efforts in numerical and experimental modelings for some hypothetical reservoir system has been conducted. It is concluded that,

1. Based on the basic physical understanding of the vorticity development in reservoir geothermal modelings, well temperature matching is possible to help in conceptual modeling step.

2. Power density distribution can be used along with temperature distribution to determine zones of high enthalpy which give high power output to help exploitation justifications.

3. Experimental modeling for geothermal reservoir in laboratories is theoretically possible, and accessible to be extended to three-dimensional modeling.

ACKNOWLEDGMENT

The authors would like to express their sincere gratitudes to Dr. Ir. Samsul Kamal, M.Sc., Dr. Ir. Indarto, DEA., Dr. Ir. Supranto, M.Sc., Ir. Budianto Toha, M.Sc., Dr. Ir. Soekrisno, MSME, and Dr. Ir. Heru Hendrayana for the fruitful discussions and helpful suggestions to this work. This work has been supported by research grant from PERTAMINA, to whom the authors are truly thankful.

REFERENCES

Bejan, A.; 1984: *Convection Heat Transfer*, John Willey&Sons, New York, 360p.

Bjornsson,S. and Stefansson,V., 1987: Heat and Mass Transfer in Geothermal Reservoir, in: J.Bear and M.J. Corapcioglu (Eds.): *Advance in Transport Phenomena in Porous Media*, NATO ASI Series no:128, Martinus Nijhoff Publishers, p.145-183.

Cheng,P., 1978: Heat Transfer in Geothermal System, *Advance Heat Transfer*, vol.14., pp.1-105.

Pestov,I., 1993: Mathematical Modeling of Two-Phase Geothermal Reservoir, *Proc. 15th New Zealand Geothermal Workshop*., p.285-289.

Pestov,I., 1994: Mathematical Modeling of Vapor Dominated Geothermal Systems., *Proc. 16th New Zealand Geothermal Workshop*, p.29-32.

O'Sullivan,M.J., 1985: Geothermal Reservoir Simulation, *Energy Research*, vol.9., p.319-332.