

FREE-CONVECTIVE FLOW OF FLUID IN A THIN POROUS CONTOUR AND GEOTHERMAL ANOMALIES.

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ABSTRACT

The problem of free convection in a thin porous contour, placed in uniform impermeable massif is considered. The approximate analytical solution of conjugate problem is obtained. The critical Rayleigh number is determined, by exceeding of which the steady fluid circulation in an annulus is established. The computations of abnormal heat flow near surface are carried out, stipulated by thermoconvection in a contour.

INTRODUCTION

Urgency of hydrogeological problems formulation and solution with reference to underground fluids, stipulated by exploration and output of economic minerals, increases greatly in connection with development of new directions of mine technology including use of deep underground heat (geothermal power engineering) and development of deep stratal waters for extraction of valuable chemical combinations. General regularities of distribution and migration of thermal water are closely connected with geologic structure of either area of the Earth's Crust. Together with other factors the natural fractured zones play an important part in formation and development of natural process of heat and mass transfer. Such zones are sources of anomalies of rocks reservoir properties, temperature gradients of geologic sections, hydrochemical parameters of stratal waters and so on. Combination of natural fractures with artificial channels - wells, drilling in fluid saturated complexes - permits to find optimal solutions, increasing the effectivity of mine technology by development of underground fluid fields.

In this connection the simulation of head and mass transfer processes is of great interest, that take place not only in separate stratum and fractured zones, but also in different hydroconductive contours, formed by crossing of these zones with each other and with fluid saturated formations.

Numerical and experimental study of free-convective flows in an annular layer of porous material showed [1] existence of two-cellular structures in the ring. In [4,5] the

thermoconvective processes in thin closed porous contours in simplified positing were investigated - in suggestion, that the tangential velocity and temperature are uniform over the contour cross section, and that the heat exchange with the walls obeys Newton's law. The conditions of the free-convective flow onset were determined and it was shown, that steady flow in the contours is established. In the work [6] natural convection in a thin porous ring was considered. On the basis of the two-dimensional equations an integro-differential equation is obtained in the zeroth approximation in terms of a small parameter - namely the relative thickness of a ring. It was shown, that under specific conditions the steady flow of liquid in the ring is established with small velocity variations in radial direction. In the work [3] the analysis of the conditions of onset of thermoconvective movements was fulfilled in limited zones of sedimentary mantle. It was shown that existence of convection intensifies the thermal anomalies above inclusions of liquid fluids in comparison with anomalies, generated by conductive transport.

FORMULATION OF THE PROBLEM

Let us suppose, that a porous ring of inner radius R and thickness h ($h \ll R$) is placed in impermeable massif at depth H from the surface (fig.1).

Let us put in signs:

$$D = \{(x, z) : |x| < \infty, 0 < z < \infty\}$$

$$B = \{(x, z) : 1 \leq [x^2 + (z-H)^2]^{1/2} \leq 1+h\} =$$

$$= \{(r, \varphi) : 1 \leq r \leq 1+h, 0 \leq \varphi \leq 2\pi\},$$

where r and φ - the polar coordinates, connected to ring's centre. Let's through $\Omega = D \setminus B$ the additions B to D , i.e. the totality of all points $\xi(x, z)$ from D , which is not contained in B .

The steady free-convective fluid flow in the ring in the Darcy-Oberbeck-Boussinesq approximation in the dimensionless variables is described with the set of equations [2]:

$$\nabla u = 0, \mathbf{u} = -\nabla P + Ra(\delta - T)\mathbf{e}_z, (r, \varphi) \in B,$$

$$\mathbf{u} \nabla T = \Delta T, \quad (1)$$

$$\mathbf{un} = 0, \quad (r, \varphi) \in \partial B,$$

$$\{\mathbf{u}, P, T\}(r, \varphi) = \{\mathbf{u}, P, T\}(r, \varphi + 2\pi),$$

where $\mathbf{u} = (u_r, u_\varphi)$ is vector of filter velocity; P - the pressure in the liquid phase; T - the temperature in porous medium; ∂B - the boundary of the area B ; \mathbf{n} - the vector of the normal to ∂B ; \mathbf{e}_z - the single vector, directed along the axis z .

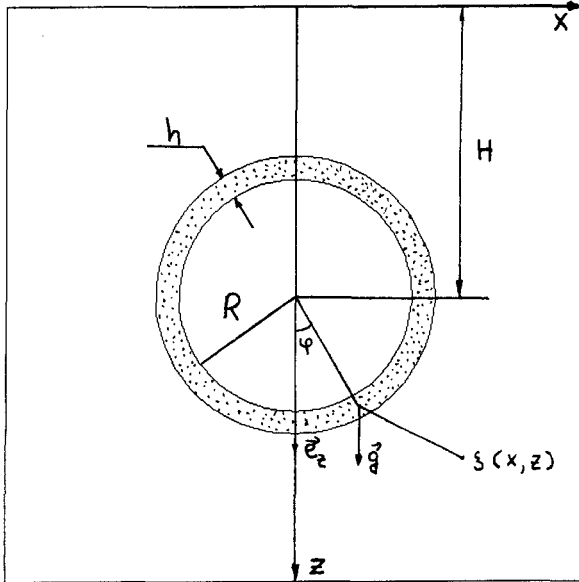


Fig.1. Fluid saturated porous ring, placed in impermeable massif.

In the region, occupied with impermeable massif, the temperature field $T_m(x, z)$ is determined by solutions of the problem

$$\Delta T_m = 0, \quad (x, z) \in \Omega,$$

$$T_m|_{z=0} = 0, \quad \partial T_m / \partial z|_{z \rightarrow \infty} \rightarrow 1,$$

$$T_m = T, \quad \partial T_m / \partial n = \partial T / \partial n, \quad (x, z) \in \partial B, \quad (2)$$

It is suggested here, that the heat conductivities of porous medium and impermeable massif are the same.

In the problem (1) - (2) R is the characteristic scale of length; $k \rho_0 / \mu$ - of time; ΓR - of temperature; $\lambda / c_f R$ - of velocity; $\mu \lambda / c_f k$ - of pressure. Here is geothermal gradient; k , λ and c_μ - the permeability, thermal conductivity and volume heat capacity for the porous medium; c_f, μ, ρ_0, β are respectively, the volume heat capacity, dynamic viscosity, density (at temperature T_0) and the temperature expansion coefficient of the liquid; $g = 9.8 \text{ m/c}^2$. The problem in question contains dimensionless parameters

$$Ra = \frac{k \rho_0 g \beta \Gamma c_f R^2}{\lambda \mu}, \quad \delta = \frac{1 + \beta T_0}{\beta \Gamma R}, \quad (3)$$

where Ra - the filter Rayleigh number; δ - is the constant for fixed medium.

Mechanical equilibrium of fluid ($\mathbf{u} = 0$) is corresponded with linear distribution of temperature in the system and quadratic distribution of pressure in liquid phase

$$T^{(0)} = z = H + r \cos \varphi, \quad (r, \varphi) \in D, \quad (4)$$

$$P^{(0)} = -Ra[0.5(z-H) - (\delta - H)](z-H) + p_0 = \\ = -Ra[0.5r \cos \varphi - (\delta - H)]r \cos \varphi + p_0, \quad (r, \varphi) \in B,$$

where p_0 is some constant.

Let us reformulate the problem (1) - (2) in the more convenient form. Let us present fields of temperature and pressure in the form

$$T_m = T^{(0)} + T_m^{(1)}, \quad T = T^{(0)} + T^{(1)},$$

$$P = P^{(0)} + P^{(1)}. \quad (5)$$

Substituting (5) in (1) - (2) and omitting for short the upper index (1), we obtain

$$\Delta T_m = 0, \quad T_m|_{z=0} = 0, \quad \partial T_m / \partial z|_{z \rightarrow \infty} = 1$$

$$\nabla \mathbf{u} = 0, \quad \mathbf{u} = -\nabla P - Ra T \mathbf{e}_z, \quad \mathbf{u} \mathbf{e}_z + \mathbf{u} \nabla T = \Delta T, \quad (6)$$

$$\{\mathbf{u}, P, T\}(r, \varphi) = \{\mathbf{u}, P, T\}(r, \varphi + 2\pi),$$

$$\mathbf{un} = 0, \quad T_m = T, \quad \partial T_m / \partial n = \partial T / \partial n, \quad (r, \varphi) \in \partial B,$$

In the work [6] it was shown, that free convection of the liquid in a thin porous ring ($h \ll 1$) is described with equations

$$u_r \partial T / \partial r + u_\varphi \partial T / \partial \varphi - u_\varphi \sin \varphi = \partial^2 T / \partial r^2,$$

$$u_\varphi = \partial \Psi / \partial r, \quad u_r = -\partial \Psi / \partial \varphi,$$

$$\Psi = \sin \varphi \int_0^r (T - \langle T \rangle) dr + u(r-1),$$

$$\langle T \rangle = \frac{1}{h} \int_0^1 T dr, \quad u = \langle u_\varphi \rangle =$$

$$\frac{1}{2\pi} \int_0^{2\pi} \langle T \rangle \sin \varphi d\varphi,$$

where Ψ is the stream function; $\langle T \rangle$ and $\langle u_\varphi \rangle$ are correspondingly, mean over the width of the ring temperature and tangential velocity.

Using the continuity equation $\partial u_r / \partial r + \partial u_\varphi / \partial \varphi = 0$ for thin ring (at the same approximations [6]), we write the first equation (7) in the form:

$$\partial(u_r T) / \partial r + \partial(u_\varphi T) / \partial \varphi - u_\varphi \sin \varphi = \partial^2 T / \partial r^2 \quad (8)$$

Averaging this equation over the width of the ring and taking into account, that $u_r|_{\partial B}=0$, we will obtain

$$d(\langle u_\varphi T \rangle)/d\varphi - u \sin\varphi = -1/h [\partial T / \partial r], \quad (9)$$

where

$$[\partial T / \partial r] = \partial T / \partial r|_{r=1} - \partial T / \partial r|_{r=1+h}, \quad (10)$$

as it was noted above the tangential fluid flow is established in a thin porous ring [6]. Therefore let us suppose, that

$$\langle u_\varphi T \rangle \approx \langle u_\varphi \rangle \cdot \langle T \rangle = u \langle T \rangle \quad (11)$$

Collecting (6), (9), (11) and taking into account, that the porous ring it may regard (mathematically) as a contour of zeroth thickness, we have finally

$$\Delta T_m = 0,$$

$$u d\langle T \rangle / d\varphi - u \sin\varphi = -1/h [\partial T / \partial r] = -1/h [\partial T_m / \partial r], \quad (12)$$

$$u = Ra / 2\pi \int_0^{2\pi} \langle T \rangle \sin\varphi d\varphi,$$

$$T_m|_\gamma = \langle T \rangle, \quad \langle T(\varphi) \rangle = \langle T(\varphi + 2\pi) \rangle,$$

$$T_m|_{z=0} = 0, \quad T_m|_{z \rightarrow \infty} \rightarrow 0, \quad (13)$$

here

$$[\partial T_m / \partial r] = \frac{\partial T_m^+}{\partial r} \Big|_\gamma - \frac{\partial T_m^-}{\partial r} \Big|_\gamma, \quad (14)$$

is the jump of limit quantities of normal derivative of the temperature when approaching the contour, respectively, from the inside and from the outside.

SOLUTION OF THE PROBLEM AND DISCUSSION

First let us find the solution for unlimited massif, i.e. the solution, corresponding to the boundary condition instead (13). This solution has the form

$$T_m(r, \varphi) = \begin{cases} A \cos\varphi + B \sin\varphi, & r < 1, \\ 1/r, & r > 1 \end{cases} \quad (15)$$

where

$$A = 4/Rah - 1, \quad B = \pm 2/Rah \sqrt{Rah - 4}$$

moreover the distribution of the temperature over the contour and filter velocity of fluid filtration are given by formulae

$$\langle T(r, \varphi) \rangle = A \cos\varphi + B \sin\varphi, \quad (16)$$

$$u = \pm 1/h \sqrt{Rah - 4} \quad (17)$$

In the formula (17) the different directions of liquid movement in contour correspond to different signs in front of radical.

Now we can find the solution, that satisfies the boundary conditions (13). It is represented in the form

$$T_m(x, z) = A \left[\frac{(z-H)}{(x^2 + (z-H)^2)} + \frac{(x+z)}{(x^2 + (z+H)^2)} \right] + B \left[\frac{2x}{(x^2 + (z-H)^2)} - \frac{2x}{(x^2 + (z+H)^2)} \right] \quad (18)$$

Using the new temperature distribution along contour, one can make the temperature field in massif more precise. This procedure one may repeat until be obtained the required precision of computations. But we confine ourself to approximation (18). One of unknown quantities, that is of interest for applications is the heat flow on the surface, generated by hydrothermal convection in a contour. From (18) follows, that it is expressed with a formula

$$q_0 = -\partial T_m / \partial z|_{z=0} = -[2A(x^2 - H^2) + 4BHx] / r(x^2 + H^2)^2 \quad (19)$$

Finally we write the expression for jump of derivative $\partial T_m / \partial r$. Using (15) and (16) we obtain

$$[\partial T_m / \partial r] = K \langle T \rangle, \quad K = 2. \quad (20)$$

Such are presentation of the derivative jump may be used successfully for determination of temperature field in massif when existing the convection in porous contours of free geometrical form [4].

From the expression (17) follows, that the steady-state circulation is possible only at $Rah > 4$, since the filter velocity must be material quantity. Hence the critical Rayleigh number, when exceeding of which the convective fluid movement in a contour arises is equal to $Ra = 4/h$. The expression (18) shows that the temperature in the upper part of the contour (when existing the convection) is much over than the temperature in a system. Therefore the output of thermal water from such regions is more perspective in comparison with its output from "common" stratal systems. Besides, even if the free convection in the contour is absent, drilling-in in the latter may contribute to origin of fluid flow, spreading all over the system "contour - wells" (with carrying out the heat from lower aquifers) and to increasing of temperature of water in productive wells.

Therefore when developing the fields of thermal water of type in question the technical and economic indices will improve in time, whereas when developing the "common" stratal systems they be worse in time.

In Fig. 2,3 we plotted are the graphs of dependences of heat flow density on daylight area against the horizontal coordinate x at different Rayleigh numbers.

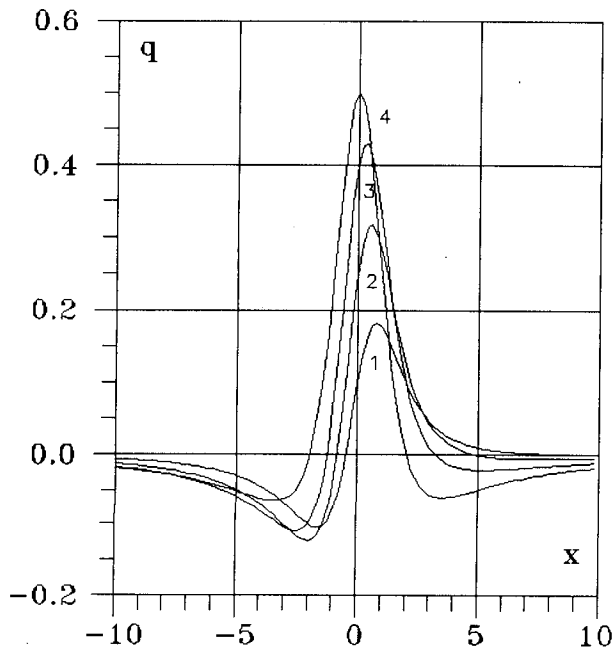


Fig. 2. Nature of variation of abnormal heat flow density on the daylight area: $H=2$, 1 - $Rah = 5$, 2 - $Rah = 8$, 3 - $Rah = 20$, 4 - $Rah = 10000$.

From these data on can see, that the curve of dependence of abnormal heat flow on x -coordinate is asymmetric relatively axis z and has three extrema - one maximum and two minima. With increase of the Reyleigh number (Ra) the degree of asymmetry reducts and

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amplitudes of extrema rises to some limit maquantities, which are determined by depth of contour position. Maximum always is placed above of rising fluid flow and minimum - always out of contour boundaries. Similar results were obtained and for contour in the form of square [4]. From analysis of the data, obtained for different contours, follows, that geothermal anomalies to a

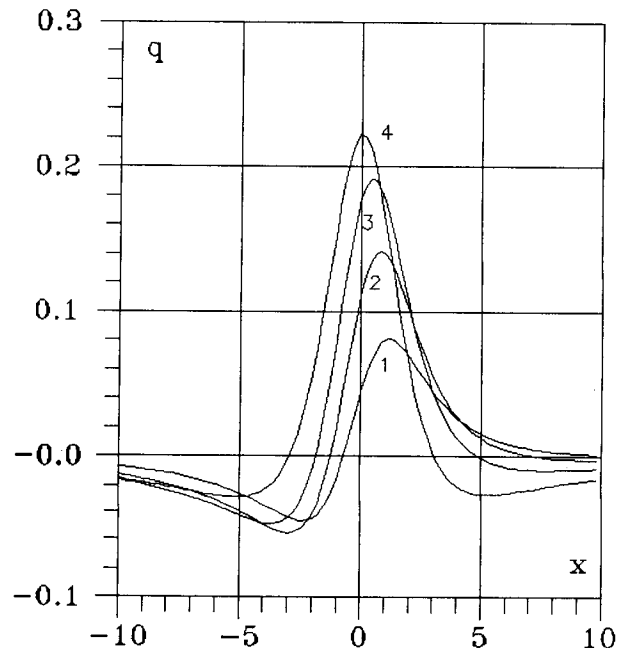


Fig. 3. Nature of variation of abnormal heat flow density on the daylight area: $H=3$, 1 - $Rah = 5$, 2 - $Rah = 8$, 3 - $Rah = 20$, 4 - $Rah = 10000$.

sufficient extent depend on dimension of contours and weakly depend on their geometrical form. These results may be used both for interpretation of measured quantities of heat flow and localization in geophysical medium of geothermal objects of type in question and for solution of inverse problems.

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