

## Viscous Fingers in Superheated Geothermal Systems

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### Abstract

In this paper we investigate the physical controls upon the rate of vaporization of liquid as it is injected into a porous layer containing superheated vapour. We develop a simple model of the process and show that if liquid is injected at a relatively high rate, a small fraction of the liquid vaporizes and the porous layer becomes filled with hot liquid. In contrast, at low rates of injection a large fraction of the liquid may vaporize. We also describe a new and fundamental instability that can develop at a migrating liquid-vapour interface if the rate of injection is sufficiently small. This phenomenon is manifest in the form of liquid fingers growing from a liquid-vapour interface and is investigated through the use of analytical, experimental and numerical techniques.

### Introduction

Vapour-saturated geothermal reservoirs are recharged as liquid invades the pore spaces in the hot rock and vaporizes (Truesdell & White 1973). However, the rapid development of the vapour-dominated geothermal fields like The Geysers in California, and the significant fluid extraction in excess of the natural recharge has caused a depletion of the fluid levels within the reservoirs (Enezy 1989). The generating capacity of power plants installed within The Geysers region as of mid-1991 was approximately 2000 MW. As a result of the depletion of fluid reserves, the reservoir can only supply 1500 MW (Kerr 1991). This shortage of 500 MW has alarmed many commercial operators since this has affected the economics drastically. Depletion of fluid reserves has been observed in nearly all exploited geothermal reservoirs and this is to be expected locally around production

wells. However, if exploitation of a reservoir is to be economical, the overall reservoir pressure needs maintaining to a certain degree. Hence, an exploitation scheme which addresses this problem is necessary for all reservoirs.

Active water injection schemes have been designed to regenerate the vapour and hence maintain the vapour pressure at The Geysers (Enezy et al. 1991). However, the optimal injection rate depends upon the particular situation. Understanding the underlying physical controls upon the rate of vaporization of liquid and the mass fraction of liquid which vaporizes helps to determine these conditions.

As a simple model we consider the injection of liquid into a porous layer as a mechanism to provide artificial recharge of fluid. Most of the energy within a geothermal reservoir is stored in the rock, and by increasing the rate of recharge into the reservoir, the rate at which hot fluids can be extracted from it can be maintained or increased (Schroeder et al. 1982; Pruess & Enezy 1993).

In this paper we describe results of our analysis of the physical controls upon the rate of vaporization of liquid as it is injected into a geothermal reservoir. The paper is divided into two parts. In the next section we analyse the vaporization of liquid at a planar liquid-vapour interface and describe how both the rate of vaporization and fraction of liquid which vaporizes change as the rate of injection is varied. In the subsequent section we investigate the stability of such planar vaporizing interfaces and show that for sufficiently slow rates of injection the interface actually becomes unstable. A series of analytical,

experimental, and numerical investigations identify that this instability is manifest in the form of fingers at the liquid-vapour interface.

### Vaporization of a planar interface

As liquid migrates through hot porous rock and vaporizes, the rate of vaporization is governed by the amount of heat released by the rock and by the ability of the vapour to migrate ahead of the interface. In order to quantify the rate of vaporization and the mass fraction of liquid which can vaporize, equations for the conservation of mass and energy at the moving liquid-vapour interface are coupled with the Clausius-Clapeyron equation relating the interfacial pressure and temperature, and an equation describing the motion of the vapour ahead of the interface. Details of the physical processes and relevant equations are given by Woods & Fitzgerald (1993).

By solving the system of equations described above, we find that as the liquid flow rate increases, the interfacial pressure also increases (Figure 1). As a result, the interfacial temperature increases, reducing the heat released by the hot rock for vaporizing liquid. This in turn lowers the mass fraction of liquid which vaporizes (Figure 2); however, the total mass of vapour produced per unit time is greater owing to the greater flow rate (Figure 3).

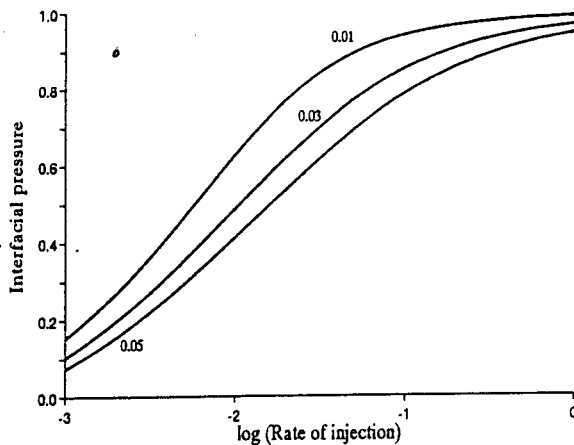


Figure 1. Interfacial pressure as a function of rate of injection from a vertical well. Curves are given for various reservoir porosities.

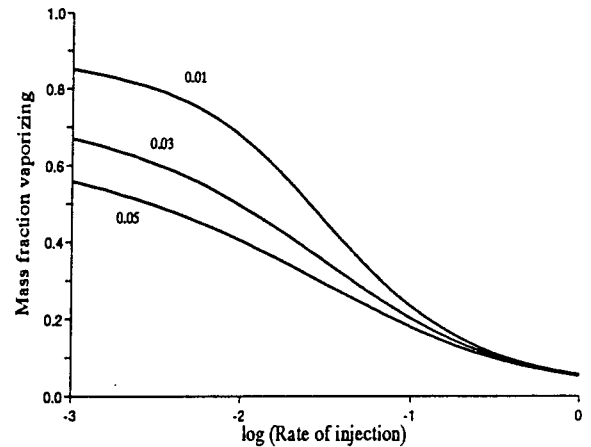


Figure 2. Mass fraction vaporizing as a function of rate of injection from a vertical well. Curves are given for various reservoir porosities.

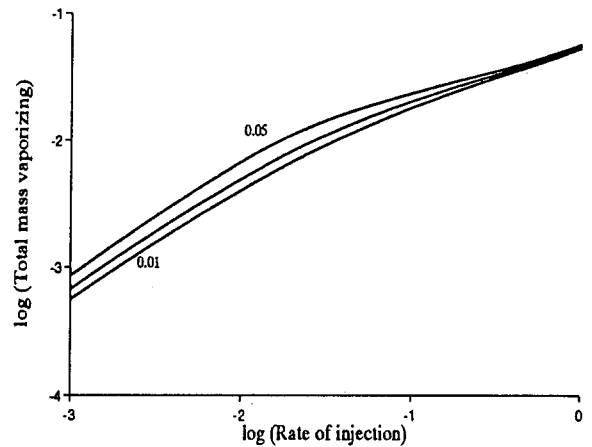


Figure 3. Rate of production of vapor as a function of rate of injection from a vertical well. Curves are given for various reservoir porosities.

These issues are discussed in greater detail by Woods & Fitzgerald (1993). They identify a fundamental paradox between the short term need for a high rate of vapour regeneration and the longer term objective of removing the thermal energy from the reservoir through extraction of vapour. In the next section we show that at low flow rates, for which the mass fraction of liquid which vaporizes is high, the situation is somewhat more complex. The assumption that the liquid vapour interface remains planar is not always satisfied. This affects

the above results concerning the fraction of the liquid which vaporizes, the rate of vaporization and the residual thermal energy stored in the reservoir.

### Interface Stability

The results we described above hinge upon the assumption that the liquid vapour interface remains planar. However, this need not be the case if the pressure gradient in the vapour ahead of the interface exceeds that in the liquid just behind the interface. This effect is somewhat analogous to the Saffman-Taylor instability which develops in a porous layer when a more viscous fluid is displaced by a less viscous fluid (Saffman & Taylor 1958). The instability arises when a finger of the fluid behind the interface advances ahead of the interface. Because the pressure gradient in this finger is smaller than that of the surrounding fluid, the finger advances.

Appealing to the analogy with the Saffman-Taylor instability, we can derive a simple condition which identifies the flow rates for which the interface may be unstable. The pressure gradient in the liquid just behind the interface is given by Darcy's Law (Bear 1972)

$$\vec{\nabla}P_l = -\frac{\mu_l}{k} u_l \quad (1)$$

where  $u_l$  is the Darcy velocity of the liquid,  $k$  the permeability of the rock and  $\mu_l$  the dynamic viscosity of the liquid. Similarly in the vapour, the pressure gradient is given by

$$\vec{\nabla}P_v = -\frac{\mu_v}{k} u_v \quad (2)$$

We can relate these pressure gradients by noting that, if the mass fraction which vaporizes is  $F$ , then the mass flux of vapour  $\rho_v u_v = F \rho_l u_l$  and so

$$\frac{\vec{\nabla}P_v}{\vec{\nabla}P_l} = F \frac{\mu_v \rho_l}{\rho_v \mu_l} = F \frac{\eta_v}{\eta_l} \quad (3)$$

If this ratio is greater than unity an instability may develop. Typically, the ratio  $\eta_v / \eta_l \approx 10$  for water and water vapour at pressures of 6 - 30 atmospheres and temperatures of 150 - 250°C. Therefore, if

the typical mass fraction vaporizing,  $F$ , exceeds about 10 - 20% the liquid-vapour interface may be unstable.

A more detailed calculation (Fitzgerald & Woods, 1994) including the effects of thermal diffusion and the differential rate of vapour production between the crests and base of the fingers shows that very short and very long wavelength perturbations may be stabilized. Therefore, there is typically a band of wavelengths which are linearly unstable if  $F$  is sufficiently large (Figure 4).

From Figure 2, we see that the instability may arise for dimensionless injection rates  $Q < 0.1$  since at such low rates of injection the mass fraction vaporizing is sufficiently large. Once the fingering develops the surface area of the liquid-vapour interface increases, enabling more vapour to migrate ahead of the liquid. As a result, the mass of liquid which vaporizes can increase and the injection process becomes more effective.

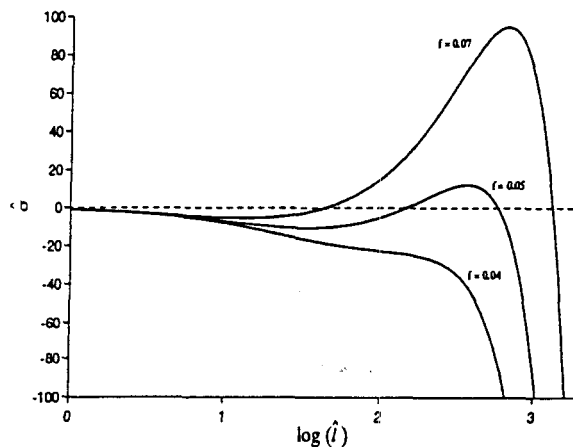


Figure 4. Exponential growth rate of a finger as a function of wavenumber. Curves are given for three values of the mass fraction vaporizing. In these calculations the thermal diffusivity  $k=3 \times 10^{-6}$ ,  $k/\alpha=5 \times 10^{-4}$  and  $v_l/v_v=0.042$ .

In order to demonstrate that the instability can indeed occur, we have conducted a series of experiments in which ether was injected into a cylindrically symmetric porous layer of hot sand bounded by two clear perspex plates (Fitzgerald & Woods 1994). As the ether spread radially from the source, a boiling front developed and ether vapour migrated ahead of the interface.

When the fraction vaporizing was sufficiently small,  $F < (\eta_l / \eta_v)$ , the advancing liquid-vapour front remained stable and circular. However, when a larger fraction of the ether vaporized, the liquid-vapour front developed fingers.

We plan to carry out a series of detailed numerical simulations of vaporizing liquid-vapour interfaces. Once we have reproduced the analytical stability criterion, we will then examine the nonlinear growth of the interfacial stability. This will enable us to determine the extent of the two-phase zone (the region in which liquid fingers are interspersed with vapour) and its impact upon the vaporization efficiency. We have conducted a series of preliminary calculations using the geothermal simulator TETRAD (Vinsome 1991).

Initial simulations of the case in which liquid is injected from a line source and in which the model domain is 2-dimensional indicate that for relatively high rates of injection the interface remains planar (Figure 5), while at lower flow rates the interface appears to become unstable.

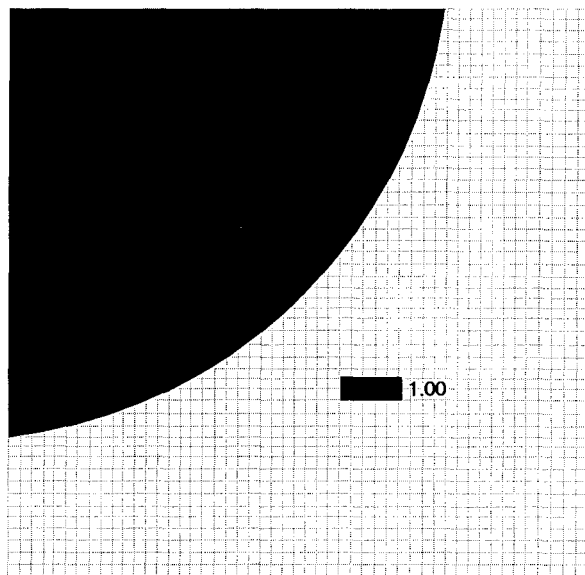


Figure 5. Liquid saturation profile at 0.43 Vp injected. Injection rate  $q=0.833$  kg/s. Fraction vaporizing = 0.04. Five-point finite differencing.

However, these simulations have identified a number of difficulties inherent in the numerical simulation of a moving, vaporizing liquid-vapour interface and must, therefore, be regarded with caution.

The numerical predictions are highly sensitive to the finite difference scheme employed, the orientation and refinement of the grid and the boundary conditions. For example, simulations of the unstable case using 5-point and 9-point finite differencing schemes (Figures 6 and 7 respectively) lead to very different predictions of the evolution of the model interface. It is of paramount importance to resolve these dramatic discrepancies in the numerical simulations before we may build upon our analytical results and investigate the nonlinear growth of the interface numerically.

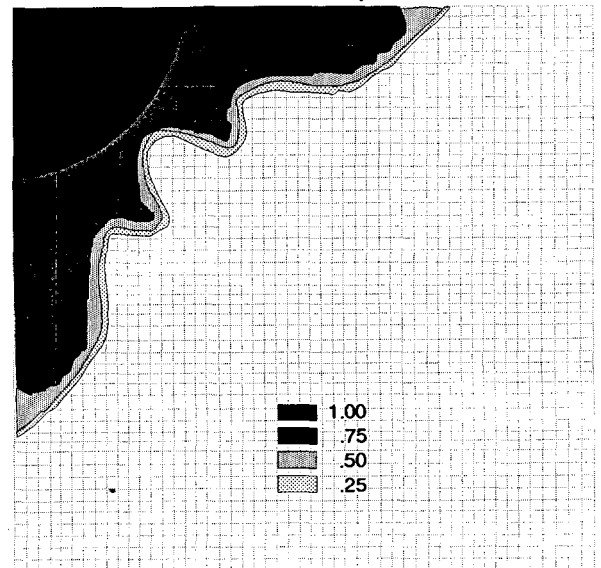


Figure 6. Liquid saturation at 0.43 Vp injected. Same conditions as in Figure 5, but  $q=0.0167$  kg/s. Average fraction vaporizing = 0.58. 5-point finite differencing.

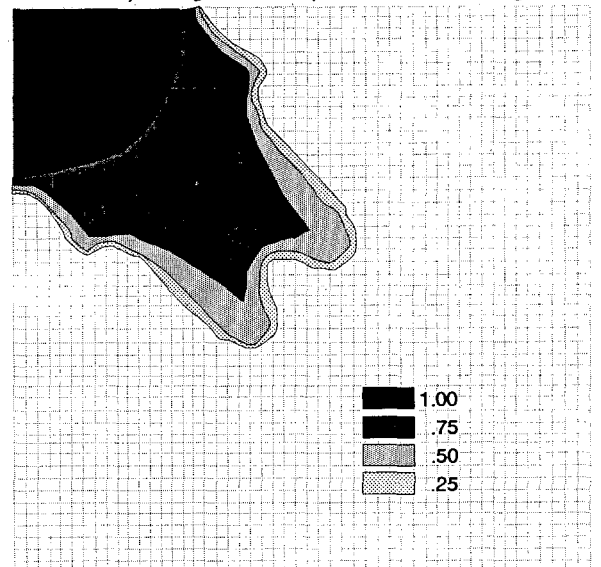


Figure 7. Liquid saturation profile at 0.43 Vp injected. Same conditions as in Figure 6, but 9-point finite differencing. Fraction vaporizing = 0.58.

## Conclusions

Vapour-saturated geothermal reservoirs may be recharged through the active injection and subsequent vaporization of liquid into the reservoir. As the rate of injection increases, the mass fraction of the liquid which vaporizes decreases although the overall rate of production of vapour increases. Furthermore, as the rate of injection increases, the temperature of the liquid-vapour interface increases. Therefore, owing to the thermal inertia of porous layers (Woods & Fitzgerald 1993) the liquid remaining in the reservoir is hotter, rendering the overall energy extraction through the vapour less efficient. We have described how, at low rates of injection, the liquid-vapour interface may become unstable and break up into fingers. This further increases the mass fraction of the liquid which can vaporize at low injection rates.

## Acknowledgments

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