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# PRELIMINARY STUDY OF DISCHARGE CHARACTERISTICS OF SLIM HOLES COMPARED TO PRODUCTION WELLS IN LIQUID-DOMINATED GEOTHERMAL RESERVOIRS

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## ABSTRACT

This paper presents a theoretical study of the discharge characteristics of slim holes compared to production wells. Assuming that (1) the boreholes feed from an all-liquid zone, and (2) the feedzone pressure and temperature are independent of borehole diameter, calculations have been carried out for a variety of borehole diameters. The wellhead pressure/flowrate relationships for the various borehole diameters do not collapse to a single curve, even when flow rates are adjusted to account for differences in cross-sectional area. The area-scaled discharge rate declines with a decrease in borehole diameter. Both frictional pressure gradient and heat loss effects are more significant for the smaller-diameter slim holes than for the larger-diameter wells. The difference in heat loss effects is probably responsible, at least in some cases, for the difficulty encountered in inducing deep slim holes (depths >> 300 m) to discharge. Scaling up the discharge capacity of slim holes to those of production wells by the cross-section area ratio provides a conservative estimate of production-size hole discharge.

## 1. INTRODUCTION

During the exploration of a geothermal prospect, so-called "slim holes" offer an attractive alternative to full-size production well drilling for preliminary characterization of the reservoir, owing both to lower cost and reduced drilling time requirements. The most important problem in reservoir exploration is to establish the underground temperature distribution and the presence of a thermal resource, and large-diameter wells are not required for downhole temperature measurements. Slim holes can also serve to help characterize geological structure, and many geothermal well logging tools can be run in relatively small-diameter holes.

The fluid discharge capacity of a typical slim hole will generally be significantly less than that of a corresponding production-size hole, however. This means, of course, that if a productive region is discovered using slim-hole exploration, it will be necessary to subsequently drill production-size wells to exploit the resource. Also, due to their relatively low fluid discharge capacity, slim holes are not usually useful as signal sources for pressure interference testing (they may, however, be used as shut-in observation wells in interference tests in which the flowing wells are of conventional diameter).

If a slim hole is drilled into a geothermal prospect and can be induced to discharge, the rate of steam production from the slim hole is presumably much less than would have occurred had the well been of conventional diameter. Accordingly, one question of interest is how one might estimate the steam delivery capacity of future production-size wells based upon results from testing slim holes. The discharge characteristics of a geothermal well involve the relationships among stable wellhead flowing pressure, total mass discharge rate, and wellhead fluid enthalpy (i.e., steam/water ratio). If these characteristics are known for one or more slim holes in a prospective geothermal field, is it possible to estimate the corresponding characteristics of future production wells in any meaningful way?

This paper describes a preliminary examination of this question from a purely theoretical point of view. The WELBOR computer program (Pritchett, 1985) was used to perform a series of calculations of the steady flow of fluid up geothermal boreholes of various diameters at various discharge rates. Starting with prescribed bottomhole conditions (pressure, enthalpy), WELBOR integrates the equations expressing conservation of mass, momentum and energy (together with fluid constitutive properties obtained from the steam tables) upwards towards the wellhead using numerical techniques. This results in computed profiles of conditions (pressure, temperature, steam volume fraction, etc.) as functions of depth within the flowing well, and also in a forecast of wellhead conditions (pressure, temperature, enthalpy, etc.). Pipe friction (for both single- and two-phase flow regions) is treated using the formulation of Dukler, Wicks and Cleveland (1964). Liquid holdup (that is, the relative motion between the liquid and vapor phases) is treated using Hughmark's (1962) correlation. Heat transfer through the casing between the fluid within the wellbore and the formation outside is treated using a method derivable from results presented by Minkowycz and Cheng (1976).

For all calculations performed in this preliminary study, it was assumed that:

(1) the reservoir fluid may be treated as pure $H_2O$ (negligible dissolved solids and/or noncondensible gases),

(2) the thermal conductivity of the rock outside the borehole is equal to 4 Watts/meter/degree Celsius,

(3) the borehole is vertical, of uniform interior diameter ($D$), and its interior surface is hydraulically smooth,

(4) the borehole has a single feedzone, at a depth of 1500 meters,

(5) the undisturbed reservoir pressure at 1500 meters depth is 80 bars absolute, and

(6) the distribution of reservoir temperature with depth may be obtained by linear interpolation among the following values:

| | |
|---|---|
| zero depth | ...10°C |
| 200 m | ...100°C |
| 500 m | ...180°C |
| 1000 m | ...230°C |
| 1500 m | ...250°C |

The borehole produces fluid from 1500 meters depth in the formation at a temperature of 250°C. Note that the bubble point pressure for water at 250°C is only 39.8 bars (compared to 80 bars reservoir pressure) so the reservoir fluid is all single-phase liquid near the borehole's feedpoint. For most of the calculations reported herein, the borehole's inside diameter $D$ was taken to be either 10 centimeters (typical of a "slim hole") or 25 centimeters (a typical "production well"). Several calculations were also performed for other values of $D$. Figure 1 illustrates a typical computed downhole profile.

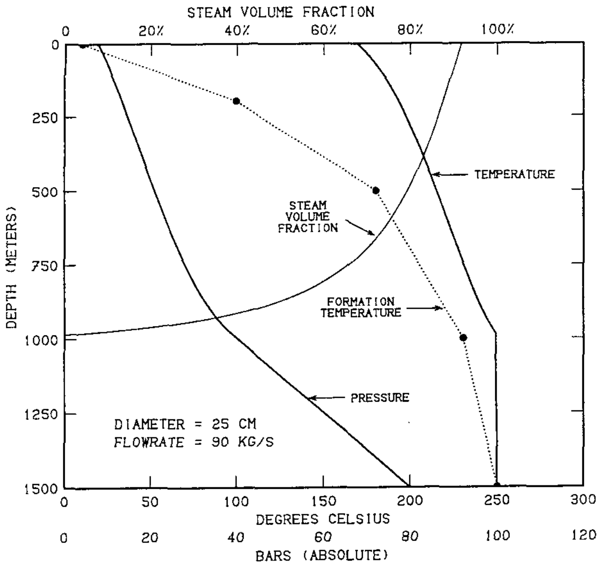

Figure 1. Typical flowing conditions in production well.

## 2. RESULTS NEGLECTING RESERVOIR FLOW RESISTANCE

It is first instructive to examine the fluid carrying capacity of the well as a function of borehole diameter without consideration of the resistance of the reservoir to fluid discharge. Therefore, for the first series of calculations, it was assumed that the bottomhole flowing pressure is the same as the reservoir pressure (80 bars). In reality, of course, the bottomhole flowing pressure will be somewhat lower than the stable reservoir pressure—the effects of finite reservoir flow resistance are taken up later in this paper.

The simplest approximation for scaling fluid carrying capacity between boreholes of different diameters is to assume that the fluid flow rate (for a given pressure difference between wellhead and bottomhole) will vary in proportion to the cross-section area of the interior of the borehole. Therefore, we define the "area-scaled borehole discharge rate" $M^*$ by:

$$M^* = M \times (25 \text{ cm}/D)^2 \qquad (1)$$

where $M$ is the actual borehole discharge rate. Thus, if the borehole diameter $D$ is equal to 25 centimeters (that of our "typical" production well), the "area-scaled" discharge rate will be equal to the actual discharge rate. If $D > 25$ cm, $M^* < M$ and if $D < 25$ cm, $M^* > M$. In particular, if $D = 10$ cm (our "slim hole"), then

$$M^*(D = 10 \text{ cm}) = 6.25M \qquad (2)$$

Calculations were carried out for a variety of borehole diameters ($D$); Figure 2 shows how the stable flowing wellhead pressure varies with area-scaled discharge rate ($M^*$). In each case, a "minimum" and "maximum" value of the discharge rate is present. For flowrates outside these bounds, the borehole cannot spontaneously discharge (with wellhead pressure above one bar absolute pressure). At some intermediate flowrate, the wellhead flowing pressure takes on a maximum value. The flowrate/wellhead pressure relationships for the various borehole diameters do not collapse to a single curve, even when flow rates are adjusted to account for differences in cross-sectional area. As borehole diameter increases, (1) the maximum area-scaled discharge rate increases, (2) the minimum area-scaled discharge rate decreases, and (3) the maximum attainable flowing wellhead pressure increases, and occurs at a higher area-scaled discharge rate.

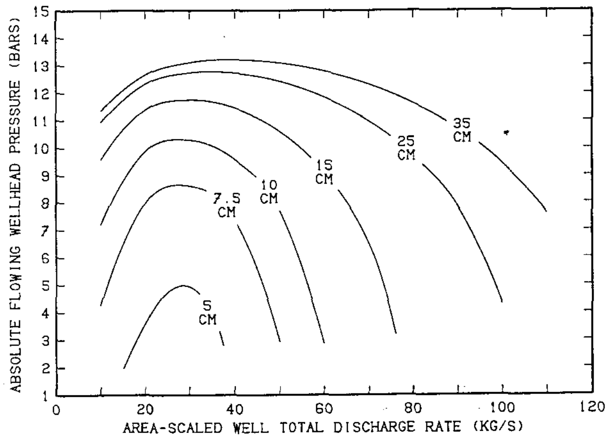

Figure 2. Influence of borehole diameter on borehole performance characteristics (no reservoir flow resistance).

## 3. THE INFLUENCE OF PIPE FRICTION

One reason why the discharge characteristics (scaled by cross-section area) depend upon borehole diameter is the influence of frictional forces in the pipe. The effects of friction are most important at high flow rates. The pressure gradient within the borehole consists of three components: (1) the hydrostatic pressure gradient due to gravity, (2) an acceleration term associated with fluid expansion, and (3) pipe friction. In the special case of single-phase flow, the pressure gradient due to pipe friction may be written (in the pertinent high Reynolds number range):

$$\left(\frac{dP}{dz}\right)_{friction} = 2f\left(\frac{M}{A}\right)^2 \Big/ \rho D \qquad (3)$$

where $f$ is a "friction factor" which depends weakly on Reynolds number, $A$ is the cross-section area of the pipe, $D$ is pipe diameter and $\rho$ is fluid density. For two-phase flow, Dukler, *et al.*, (1964) provide modifications to this formula, but the general form is the same. If two wells of different diameters flow at the same "area-scaled" flow rate $M^*$ (so that $M/A$ is the same for both), the frictional pressure gradient will be more important for the smaller-diameter hole. Since the frictional pressure gradient increases with the square of the discharge rate, this effect is most important at high flow rates. Figure 3a shows how the maximum attainable flow rate depends upon borehole diameter. Except for very small diameters, the maximum flow rate ($M_{\max}$) increases with borehole diameter ($D$) raised to the power ~2.5. Therefore, the "area-scaled" maximum attainable flow rate ($M^*_{\max}$) increases approximately with the square root of borehole diameter.

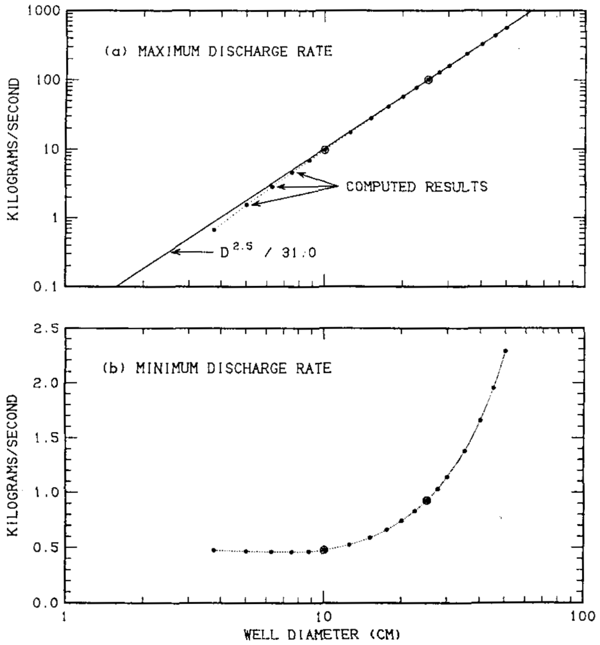

Figure 3. Maximum and minimum fluid discharge rates as functions of borehole diameter in the absence of reservoir flow resistance.

## 4. THE INFLUENCE OF HEAT LOSSES

At relatively low flow rates, the effect of frictional pressure gradient is less important. We note, however, that the discharge performance curves shown in Figure 2 do not converge at low area-scaled flow rates either. For any value of the borehole diameter $D$, the discharge rate must lie between a particular maximum value and a particular minimum value for spontaneous flow to be possible. The variation of the minimum permissible flow rate with wellbore diameter is indicated in Figure 3b. For small borehole diameters (less than 10 cm or so) the minimum permissible discharge rate is nearly independent of diameter and equal to about 0.5 kilogram per second. For larger pipe diameters, the minimum flow rate is greater, but the "area-scaled" minimum flow rate decreases monotonically with increasing borehole diameter.

This behavior results from heat losses from the rising fluid within the borehole to the formation outside; Figure 1 shows that the formation temperature is lower than the interior temperature, particularly at shallow depths. For these calculations, the feedpoint flowing enthalpy is 1086 Joules per gram, but the enthalpies at the wellhead are lower. Figure 4 shows the difference between bottomhole and wellhead enthalpies as functions of well diameter and of area-scaled discharge rate $M^*$.

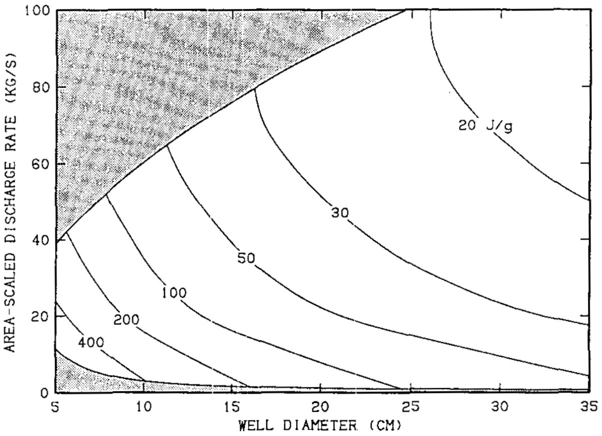

Figure 4. Wellhead fluid enthalpy decrease (relative to bottomhole flowing enthalpy) as a function of borehole diameter ($D$) and area-scaled discharge rate ($M^*$) neglecting reservoir flow resistance.

Wellhead fluid enthalpies are lower than bottomhole enthalpies due to three effects: (1) increases in kinetic energy due to fluid expansion, (2) work done against gravity raising the fluid from the feedpoint level (1500 meters deep) to the wellhead, and (3) lateral conductive heat losses to the formation. The work done against gravity is the same in all cases, and is equal to 14.7 Joules per gram. The effects of kinetic energy changes are even smaller. The enthalpy changes illustrated in Figure 4 are, therefore, mainly due to lateral heat loss.

In these calculations, heat losses to the formation are treated by assuming an outward-directed heat flux from each element of the well surface given by:

$$\text{Heat flux} = U\left(T_{fluid} - T_{rock}\right) \qquad (4)$$

where $T_{fluid}$ is the local temperature of the fluid within the well, $T_{rock}$ is the undisturbed rock formation temperature distant from the borehole at the same depth, and $U$ is a heat transfer coefficient (i.e., Watts per square meter per degree Celsius). This heat transfer coefficient, in turn, may be approximated by:

$$U = 0.6K/D \tag{5}$$

(where $D$ is borehole diameter and $K$ is the effective thermal conductivity of the rock formations outside the borehole—taken as 4 W/m/°C for the present calculations) as shown by Pritchett (1981) based upon the work of Minkowycz and Cheng (1976). In effect, heat conduction takes place through a "thermal boundary layer", with temperature equal to the borehole temperature at the inner edge of the boundary layer and equal to the reservoir temperature outside the boundary layer. Beyond the conductive boundary layer, heat transfer is dominated by convection in the porous reservoir rock. Minkowycz and Cheng (1976) showed that the thickness of the boundary layer, at equilibrium, will be proportional to the borehole diameter. Based upon the above expression, it may easily be shown that the total power (i.e., Watts) lost by the fluid within the borehole is given by:

$$\text{Total Power} = 0.6\pi KZ\overline{\Delta T} \tag{6}$$

where $Z$ is the depth of the borehole and $\overline{\Delta T}$ is the vertically-averaged temperature difference between the borehole and the surrounding formation. Dividing the heat loss rate (from Eq. 6) by the rate of fluid production ($M$) yields the specific fluid enthalpy decrease due to heat losses:

$$\Delta H = 0.6\pi KZ\overline{\Delta T}/M \tag{7}$$

Note that heat losses tend to increase with increasing well depth. For two boreholes of equal depths but of different diameters ($D_1$, $D_2$), the ratio of the enthalpy change due to heat losses at the same area-scaled discharge rate ($M^*$) will be:

$$\frac{\Delta H_1}{\Delta H_2} = \frac{\overline{\Delta T_1}}{\overline{\Delta T_2}}\left(\frac{D_2}{D_1}\right)^2 \tag{8}$$

In other words, heat losses will always be more significant for the *slimmer* of the two boreholes; the enthalpy loss will vary inversely with "area-scaled" discharge rate and also inversely with the square of the borehole diameter. As shown in Figure 4, the greatest enthalpy declines occur for relatively small-diameter boreholes operating at low discharge rates.

This difference in relative heat loss is responsible for the fact that "area-scaling" fails to produce the same discharge characteristics for production-size wells and for slim holes, even at low flow rates. The computed wellhead pressure is shown in Figure 5 as a function of area-scaled flow rate ($M^*$) for 10-cm slim holes and for 25-cm production wells both for the calculations described above (solid lines) and for a special series of calculations in which the heat-transfer coefficient was set to zero (dotted lines). The following are noteworthy: (1) for all flow rates, the wellhead pressure is larger if heat losses are neglected, (2) the difference in wellhead pressure between the "no-heat-loss" and "heat-loss" cases (for a fixed borehole diameter) is greatest for the lowest discharge rates, (3) the influence of heat losses upon wellhead pressure is much greater for the 10-cm slim hole than for the 25-cm production well, and (4) in the absence of heat losses (compare the two dotted curves) the wellhead pressure becomes independent of borehole diameter and depends only on area-scaled discharge rate for low flow rates (where the effects of pipe friction are relatively unimportant).

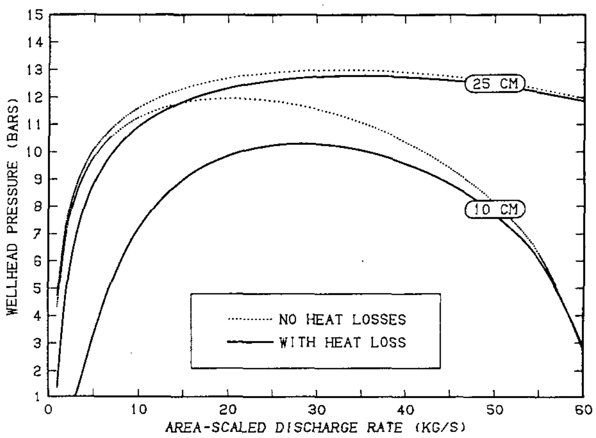

Figure 5. Relative effects of heat losses to the formation on flowing characteristics of production wells (25 cm diameter) and slim holes (10 cm).

The mechanism by which heat loss influences wellhead pressure may be understood by consideration of the flowing pressure distribution indicated in Figure 1. Note that, below the boiling interface, the vertical flowing pressure gradient is approximately hydrostatic (~80 bars per kilometer) but that within the two-phase zone above the boiling level the flowing pressure gradient is significantly lower (~35 bars per kilometer in Figure 1). If heat losses increase, then the liquid temperature below the boiling surface will decline. As a result, the boiling level will occur at a shallower depth within the borehole, at the boiling-point pressure for the lower temperature. Therefore, a greater portion of the borehole will contain the high-pressure-gradient single-phase liquid region and a smaller portion will contain the low-pressure-gradient two-phase region. As a consequence, the average pressure gradient for the borehole as a whole will increase. Since the bottomhole flowing pressure is fixed, this means that the wellhead pressure must decline with increasing heat loss.

## 5. THE DIFFICULTY OF INITIATING SLIM-HOLE DISCHARGE

Field experience has shown that it is frequently difficult (and sometimes impossible) to induce stable self-sustained discharge from deep slim holes drilled into a field from which spontaneous discharge is readily obtained using wells of conventional diameter. While the greater importance of pipe friction in slim holes may play a limited role in this problem, the main reason appears to be the relatively larger heat losses to the surrounding formation for boreholes of small diameter.

As noted above, heat losses from the borehole to the formation under stable steady-flow conditions may be adequately represented using a heat transfer coefficient of the form:

$$U = 0.6 K / D \tag{9}$$

The problem is that the above formula is only applicable after a stable condition has been reached. The stable heat-transfer model discussed above involves a conductive boundary layer around the borehole of which the thickness is proportional to the borehole diameter. Prior to discharge initiation, however (assuming that the borehole has previously been left in a shut-in condition for a long period of time), the temperature distribution within the borehole will closely resemble that in the formation outside. When discharge begins, temperatures within the borehole change almost discontinuously to higher values. This induces the formation of a thermal boundary layer around the borehole of which the thickness increases with time, eventually reaching the asymptotic value described by Minkowycz and Cheng (1976). At early times, however, the boundary layer will be thinner than at steady-state, so that conductive heat transfer will be augmented. In short, the transient case may be regarded as a succession of states in which the heat transfer coefficient is a decreasing function of time which reaches the above asymptotic value only at infinite time. This situation may be described by defining a "heat loss multiplier" which declines toward unity as time goes on. Computed values for the heat transfer multiplier as a function of time after flow startup for various borehole diameters are illustrated in Figure 6.

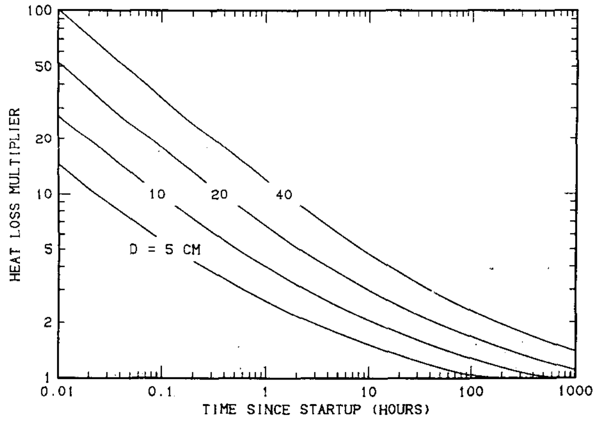

Figure 6. Effective heat transfer coefficient multiplier as a function of time for various borehole diameters.

Therefore, under transient conditions, the results computed above for the discharge capacities of boreholes of various diameters are overly optimistic since these calculations, by assuming steady heat transfer, underestimate the effects of heat losses at early times. To investigate this issue, a series of calculations was performed involving boreholes of both 10 cm and 25 cm diameters, with the "heat transfer coefficient" augmented by various factors relative to the stable steady value. Results are indicated in Figure 7, which shows the ranges of area-scaled discharge rates within which flow may occur as functions of this heat loss multiplier factor. The maximum flow rate is only weakly influenced by increases in heat loss (as expected in light of the above discussion), but the minimum possible discharge rate rises significantly with increases in heat loss. For each borehole diameter, the "critical" heat loss multiplier is reached when the borehole can no longer discharge at any rate. This critical value is much larger (~148) for the 25-cm production well than for the 10-cm slim hole (critical multiplier value ~16.2).

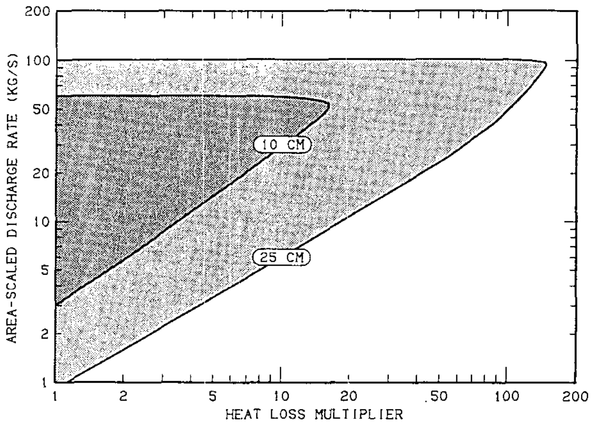

Figure 7. Regions of possible spontaneous discharge for 10-cm slim holes and 25-cm productivity wells as functions of heat loss multipler (no reservoir flow resistance).

This difference in heat loss effects is probably responsible for the difficulty often encountered in inducing deep slim holes (depth >> 300 meters) to discharge (heat losses are relatively unimportant for shallow wells). For example, if we assume that for stable discharge to occur the heat-loss multiplier must drop below one-fourth of the above "critical value", as shown in Figure 8, it should be possible to sustain stable discharge in a 25-cm production well after a transient period of about two minutes. To reach a comparable state for the 10-cm diameter slim hole requires nearly one hour. Since transient processes associated with conventional flow-initiation techniques (swabbing, pressurization, gas injection etc.) usually involve time-scales of only a few minutes, this may explain why deep slim holes often fail to sustain discharge even after several initiation attempts when larger-diameter wells start flowing without difficulty. This implies that to induce deep slim holes to discharge, it may be necessary to employ unusual techniques such as pre-heating the borehole prior to startup.

## 6. INCORPORATION OF RESERVOIR FLOW RESISTANCE

Up to this point, all calculations presented have assumed that the resistance of the reservoir itself to fluid flow may be neglected compared to the resistance imposed by the borehole; in other words, all calculations have assumed a bottomhole flowing pressure of 80 bars absolute at the feedpoint (at a depth of 1500 meters in the borehole). For extremely permeable geothermal reservoirs, this may be an appropriate approximation. Under many realistic circumstances, however, the finite permeability of the reservoir causes the bottomhole flowing pressure to be appreciably less than the stable reservoir pressure, and the amount of such pressure decline increases with increasing discharge rate.

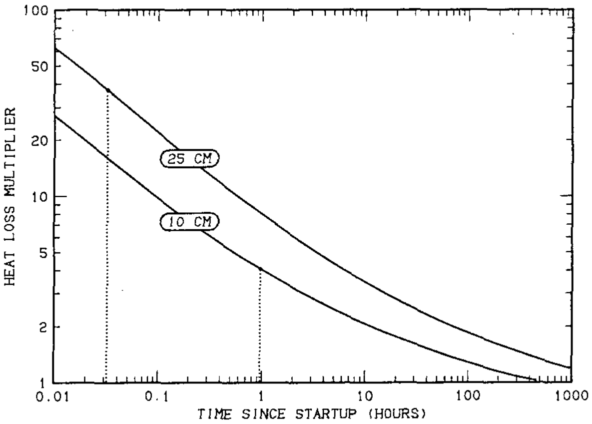

Figure 8. Time required for heat loss multiplier to drop to 1/4 of critical value for 10-cm diameter slim holes and 25-cm diameter production wells.

In light of the purpose of this study, it is appropriate to ignore pressure-transient effects and to assume that the bottomhole flowing pressure may be related to the reservoir pressure and the discharge rate by a "productivity index" ($I$) so that:

$$P_{bottomhole} = P_{reservoir} - (M/I) \qquad (10)$$

where $M$ is the discharge rate and the productivity index ($I$) may be regarded as a constant for each borehole. For the remaining calculations, this general form was assumed, and it was further assumed that the fluid flows isenthalpically from "reservoir" conditions (at $P = P_{reservoir} = 80$ bars, $T =$ 250°C) to "bottomhole" conditions (at $P = P_{bottomhole}$), then up the borehole.

The essential difficulty involved in performing calculations with non-zero reservoir flow resistance (that is, values of the productivity index $I$ which are less than infinity) is that of estimating the proper value to use for $I$ for various borehole diameters. It is frequently observed that, if several production wells (of about the same physical characteristics) are drilled into the same reservoir, they are found to be characterized by a wide variety of values of $I$. This is particularly true in fractured reservoirs, in which the productivity index of a particular well depends mainly on the degree to which it happens to intersect productive fractures. For purposes of this study, the essential question is: if a "slim hole" is drilled into a particular location, what is the relationship likely to be between the productivity index of the slim hole ($I_{slim}$) and the productivity index that would have been obtained if instead a production-size well had been drilled ($I_{prod}$)? It is useful to adopt the following mathematical form for the productivity index ratio of two boreholes of different diameters ($D_{slim}$, $D_{prod}$):

$$\frac{I_{slim}}{I_{prod}} = \left( \frac{D_{slim}}{D_{prod}} \right)^{p} \qquad (11)$$

where the appropriate (presumably non-negative) value for the exponent $p$ remains to be established.

Eventually, it is hoped that the appropriate value for $p$ will be determined empirically by analyses of actual field measurements. One approach would be to examine data sets from geothermal fields in which a sufficient number of both slim holes and production wells have been drilled and tested to provide a statistically significant sample. If sufficient productivity-index determinations are not available, the same $p$ exponent should, in principle, govern the ratio of "injectivity indices", so that injection-test data could be substituted.

For the present, lacking such empirical evidence, we are forced to try to estimate the value of the exponent $p$. For example, it may be shown that the steady line-source solution results in a value for $p$ which asymptotically approaches zero as the "drainage radius" becomes large. This suggests that the productivity index should be independent of well diameter, all else being equal. In geothermal reservoirs, fluid often enters wells through discrete feedpoints, rather than a long continuous section of hole. In such cases, the point-source (or spherical-source) solution is more appropriate. Evaluation of such solutions suggests that the proper value for $p$ lies somewhere between zero and unity.

Therefore, a final series of calculations was performed involving three well configurations:

(1) a 25-cm diameter production well with productivity index $I_{prod}$,

(2) a 10-cm diameter slim hole using $I_{slim} = I_{prod}$ (that is, $p = 0$ scaling for productivity index), and

(3) a 10-cm diameter slim hole using $I_{slim} = 0.4 \times I_{prod}$ ($p = 1$ productivity index scaling).

Numerous values for $I_{prod}$ were considered, from 1 kg/second/bar upwards. Computed results are indicated in Figure 9, which shows both the maximum attainable wellhead pressure and the maximum area-scaled total mass discharge rate $M^*$ (at one bar wellhead pressure) as functions of the reciprocal of the production-well productivity index ($1/I_{prod}$).

It is noteworthy that changing the reservoir flow resistance has a greater influence upon the discharge characteristics of the production size (25 cm) well than upon those of the slim hole (10 cm). The reason is simply that the total resistance to flow is the sum of the effects of the reservoir and of the borehole itself. As discussed above (see Figure 3), the capacity of the borehole to deliver fluid to the wellhead varies approximately as $D^{2.5}$, whereas the capacity of the reservoir to deliver fluid to the feedpoint varies as $D^p$, with $0 < p < 1$. Consequently, the reservoir resistance comprises a greater proportion of the total resistance for larger diameter holes, and therefore large holes are more sensitive to variations in reservoir flow resistance.

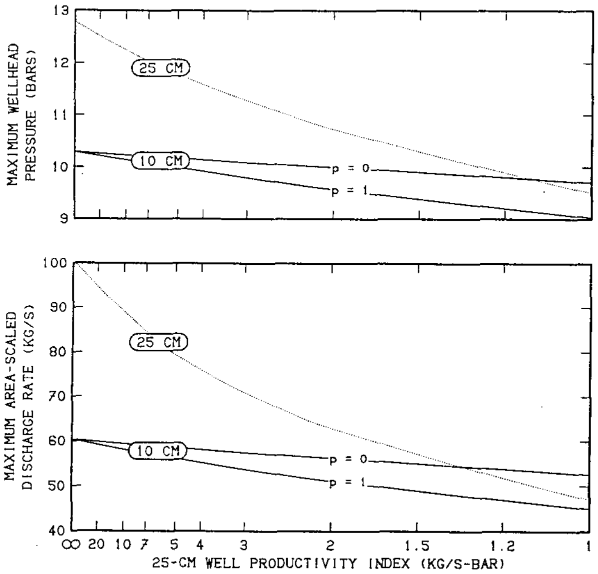

Figure 9. Effects of reservoir flow resistance on performance characteristics of 10-cm diameter slim holes and 25-cm diameter production wells.

It is also noteworthy that, despite the shortcomings of "area-scaling" of discharge rates (arising from the failure of the effects of pipe friction and of heat losses to scale in this manner), scaling-up the discharge characteristics of the 10-cm slim hole using cross-section area provides a conservative estimate of the probable productivity of larger diameter wells so long as the reservoir flow resistance is not too large (productivity index > 1 kg/second/bar or so), at least for conditions similar to those considered in this study.

## 10. ACKNOWLEDGMENT

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