

VAPOR-LIQUID COUNTERFLOW IN HETEROGENEOUS POROUS MEDIA

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ABSTRACT

Based on a continuum description, the effects of permeability heterogeneity on steady state, countercurrent, vapor-liquid flow in porous media are analyzed. It is shown that the capillary heterogeneity induced acts as a body force, that enhances or diminishes gravity effects on heat pipes. Selection rules that determine the particular steady states reached in homogeneous, gravity-driven heat pipes are formulated. It is shown that the "infinite" two-phase zone terminates only if a substantial change in permeability occurs somewhere in the medium. The two possible sequences that result, namely liquid - liquid dominated - dry or liquid - vapor dominated - dry find applications in geothermal systems. Finally, it is shown that weak heterogeneity affects only gravity-driven flows, but stronger variations in permeability give rise to significant capillary effects.

INTRODUCTION

Countercurrent vapor-liquid flows in porous media have been the subject of many recent studies due to their relevance in geothermal processes, boiling, thermal methods for oil recovery and nuclear waste disposal [1-5]. Of particular interest are steady state heat pipes driven by gravity. In theory, when the system is homogeneous, an infinitely long two-phase zone of nearly constant saturation develops if the heating rate is low enough (below a critical value). Two such states are predicted, one corresponding to low liquid saturation (vapor-dominated, VD) and one corresponding to high liquid saturation (liquid-dominated, LD). In a recent note [6], using the detailed analysis described in a previous paper [7], we have proposed that the selection of the particular solution only depends on the past history of the system. For instance, in boiling applications which involve bottom heating, it is the LD branch that is followed. While, in the case of condensation of a superheated vapor (top cooling), it is the VD branch that is selected. In either of the two cases, capillarity is necessary to connect the constant saturation profiles to the subcooled or dry regions, respectively.

In practice, because of finite size, the two-phase zone must terminate at a finite location, thus, the "infinite" extent predictions of the heat pipe formalism cannot hold indefinitely. Termination of the two-phase zone must be obtained by smoothly merging the two-phase region with a subcooled liquid or a dry region, in the LD or VD cases, respectively (otherwise, non-zero vapor and liquid fluxes would exist at the impermeable boundary of the medium [6]). Analogous problems arise in the gravitational stability of counterflow vapor-liquid systems, when a vapor-rich region underlies a subcooled liquid layer. Even if we accept the suitability of the various base states used in the analysis [8,9] (the validity of which is questionable), the present consensus is that unconditional stability is possible only if a heterogeneity is present somewhere in the two-phase region.

Heat pipe instability, but in the different context of the sensitivity of steady, 1D profiles to the boundary conditions, was considered in a recent study [10]. It was suggested that, under certain conditions, a VD solution is unconditionally "unstable" and must revert to a "stable" LD configuration, or vice versa, if the boundary conditions are reversed. While not immediately apparent, this problem is really related to heterogeneity, as shown below. One should recall that boundary end effects in immiscible displacement can be considered as special cases of heterogeneity, when the change in permeability is abrupt and very large [11].

Effects of heterogeneity on vapor-liquid concurrent flow were studied in [12] where previous works on the steady state, two-phase flow of non-condensing fluids [11] were generalized. With the exception of a rather incomplete study [13], however, heterogeneity effects on countercurrent vapor-liquid flows have not been addressed in much detail and they are currently poorly understood. Cases in point are the previously mentioned questions on the termination of the theoretically "infinite" two-phase zone, on the gravitational instability and on the sensitivity to boundary conditions. This paper aims at resolving some of these issues. We investigate effects of heterogeneity under various configurations. We find that permeability (capillary) heterogeneity acts in reality as a body force (e.g. gravity), with the

important additional property that it is also spatially varying. Heterogeneity may thus enhance or counterbalance gravity effects, depending on magnitude, form and direction of change.

This paper is a brief summary of results contained in [14]. It is organized as follows: We first study the simpler, but quite useful, horizontal case. Then, we apply the results obtained to interpret effects of gravity when the heat flux is below critical. Next, we address problems involving both heterogeneity and gravity at conditions of slow and fast permeability variation. Throughout the paper, the formalism of [7] is followed.

FORMULATION

We proceed with the assumption that the main heterogeneous variable is permeability [11,12]. An important parameter which is affected in the present case of 1D counterflow is capillary pressure. This is a result of the Leverett J -function representation:

$$P_c = \frac{\sigma J(S)}{\sqrt{k}}$$

The function J (as well as the relative permeabilities) may also be taken as weakly varying, although it is the dimensional \sqrt{k} -dependence that basically controls the capillary variation (see also [11] for a more detailed discussion).

In the absence of heat conduction, we can formulate the problem in a straightforward fashion, because saturation and temperature are decoupled from each other and the solution is obtained by simple means. Following [7], a straightforward manipulation of mass, momentum and energy balances yields the simple equation:

$$\tau J' \frac{dS}{d\xi} - J \frac{d\tau}{d\xi} = \omega \frac{(k_{r\ell} + \beta k_{rv})}{k_{r\ell} k_{rv}} + \sin\theta \tau^2 \quad (1)$$

Here $\tau \equiv \sqrt{k/k^*}$ is the heterogeneity variable which is spatially varying, k^* denotes a constant reference permeability and superscript ' indicates derivatives with respect to S . The notation is identical to [7] except for τ , which here does not denote temperature. The coordinate ξ increases in the direction from the liquid to the vapor, such that the liquid velocity is positive, while the dimensionless heat flux $\omega = q\mu_v/k^* L_{vg}\Delta\rho\rho_v$ is normalized with a reference permeability. In this notation, therefore, different permeability regions have the same value of ω , but not the same critical values (see also below). Equation (1) must be generally solved numerically. Preliminary insight can be obtained by an analytical solution, which is possible for a special case in horizontal counterflow.

A. Horizontal Counterflow

In a horizontal system ($\theta = 0$) counterflow is driven by capillarity only [5]. It is shown in [14] that useful re-

sults are obtained in the special case when τ is piecewise linear

$$\tau = \begin{cases} 1 & ; \xi < 0 \\ a\xi + 1 & ; 0 < \xi < d \\ \tau_+ & ; d < \xi \end{cases} \quad (2)$$

where $\tau_+ \equiv ad + 1$. Two different cases are considered:

1. $a > 0$ (Figure 1a)

Here, the permeability is increasing, and we obtain:

$$\int_{S_0}^S \frac{k_{r\ell} k_{rv} J' dS}{\omega(k_{r\ell} + \beta k_{rv}) + k_{r\ell} k_{rv} a J} = \frac{1}{a} \ln(a\xi + 1) \quad (3)$$

where S_0 is the saturation at 0 (presently unknown). Because of $a > 0$, the saturation decreases steadily also within the region of heterogeneity (Figure 1). The particular saturation profile depends on the conditions imposed away from the heterogeneity. If the location of the subcooled liquid boundary on the left is known, then integration occurs from left to right, and S_0, S_1 etc. can be determined sequentially (see [14]). The reverse applies if it is the location of the dry region on the right which is known.

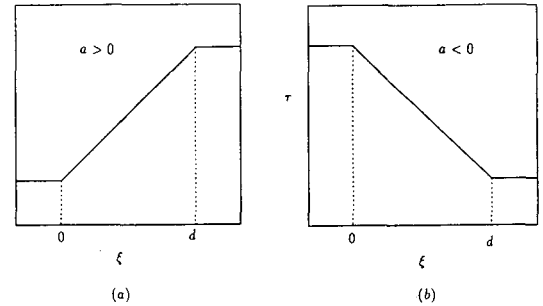


Fig. 1. Schematic of a special heterogeneity profile: (a) Permeability increase; (b) Permeability decrease.

Of special interest is the case of a sharp discontinuity ($a \gg 1$). Then (3) yields

$$\frac{J(S_1)}{J(S_0)} = \tau_+ = \sqrt{\frac{k_1}{k_0}} \quad (4)$$

which is nothing else but the condition of constant capillary pressure, implying a saturation jump across the discontinuity. This is the static (no flow) condition, which differs from the case $a < 0$, as shown below, as well as from the case of concurrent flow [11]. In the latter, a build-up of the wetting phase saturation is necessary before a high permeability region is entered.

2. $a < 0$ (Figure 1b)

While the previous are straightforward, non-trivial effects arise in the case of a permeability decrease. When $a < 0$, the denominator in (3) may vanish, if ω is small enough. For this to occur, the following equation must

admit a real solution:

$$\omega = -a J \frac{k_{r\ell} k_{rv}}{k_{r\ell} + \beta k_{rv}} \quad (5)$$

The *RHS* of (5) is schematically plotted in Figure 2. We note that there exists a critical value

$$\omega_{cr,H} = (-a) \max_S \left(\frac{J k_{r\ell} k_{rv}}{k_{r\ell} + \beta k_{rv}} \right) \quad (6)$$

above which a real solution to (5) does not exist. In dimensional notation we obtain:

$$q_h \propto \frac{\sigma L_v \rho_v}{\mu_v} \left(-\frac{d\sqrt{k}}{dx} \right) \quad (7)$$

Thus, sharper changes in permeability result into larger critical flux values. The system response depends on the relative value of ω :

(i) For $\omega > \omega_{cr,H}$, equation (5) has no solution. Then, the effect of heterogeneity is identical to the previous ($a > 0$), as schematically plotted in Figure 2.

(ii) For $\omega < \omega_{cr,H}$, on the other hand, equation (5) has two roots, denoted by S_{VH} and S_{LH} ($0 < S_{VH} < S_{LH} < 1$), in very close analogy with the vapor-dominated and liquid-dominated regimes, respectively, of gravity-driven heat pipes. The similarity with the latter is very interesting. Indeed, as in heat pipes, the saturation integral diverges at the two saturations, thus nearly flat saturation profiles (either VD or LD) develop to span the region of heterogeneity. Here, however, it is capillary heterogeneity, with the permeability decreasing in the direction of liquid flow, and not gravity, that sustains the constant saturation profiles.

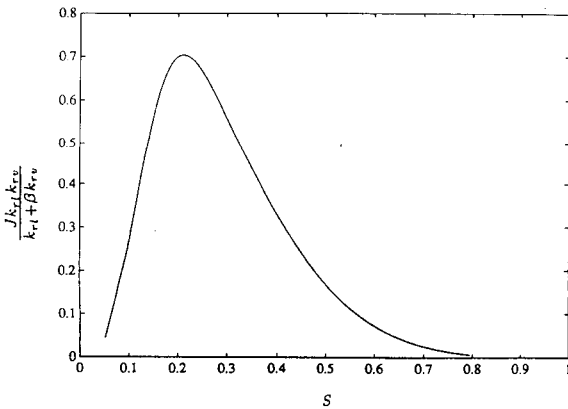


Fig. 2. The capillary heterogeneity function $\omega(S)$.

Consider, integration from the vapor side (Figure 3). This requires that superheated vapor exists somewhere on the right so that we may start integrating from the location $S = 0$ in the negative ξ direction. The saturation, S_1 , reached when the heterogeneity is entered, $\xi = d$, dictates how the solution behaves inside the heterogeneity:

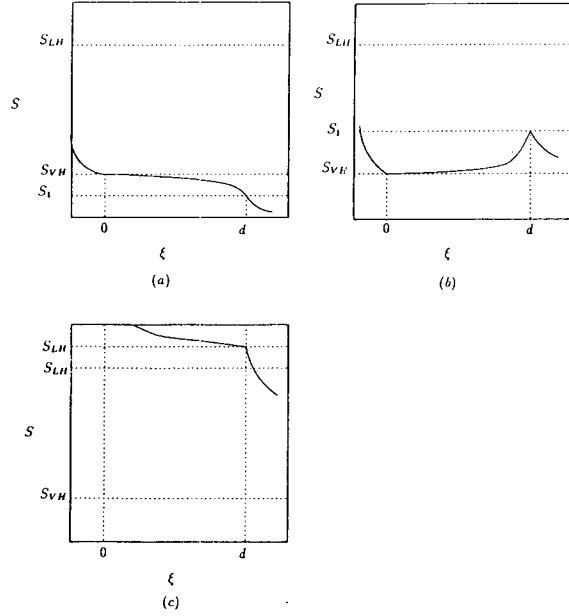


Fig. 3. Saturation profiles for integration from right-to-left: (a) $S_1 < S_{VH}$; (b) $S_{VH} < S_1 < S_{LH}$; (c) $S_{LH} < S_1$

- If $S_1 < S_{VH}$, then $dS/d\xi < 0$, and the solution is rapidly attracted to the asymptotic value S_{VH} as shown in Figure 3a. This is a vapor-dominated regime as in gravity-driven heat pipes. Outside the heterogeneity, $\xi < 0$, the integration is straightforward:

$$\int_{S_{VH}}^S \frac{k_{r\ell} k_{rv} J' dS}{(k_{r\ell} + \beta k_{rv})} = \omega \xi \quad (8)$$

This solution applies until conditions of subcooled liquid are reached ($S = 1$).

- If $S_{VH} < S_1 < S_{LH}$, then $dS/d\xi > 0$, and the solution is again asymptotic to S_{VH} , except that the saturation is now decreasing in the short region before the asymptote is reached (Figure 3b).

- Finally, if $S_{LH} < S_1$, then $dS/d\xi < 0$, but the solution cannot be now attracted to a flat profile. The latter does not develop, instead the saturation is described with the previous equations, much like the case A.1 (Figure 3c).

Consider, next, integration from the liquid side. We assume that subcooled liquid exists somewhere on the left, such that we can proceed integrating from the location $S = 1$ in the positive ξ direction. If we denote by S_0 the saturation at $\xi = 0$, the following options are possible:

- If $S_{LH} < S_0$, then $dS/d\xi < 0$, and the solution is attracted to the liquid-dominated regime with value S_{LH} (Figure 4a). After exiting the heterogeneity, further integration proceeds normally, until superheated vapor conditions are eventually reached ($S = 0$).

- If $S_{VH} < S_0 < S_{LH}$, then $dS/d\xi > 0$, and the solution is attracted to the same liquid-dominated asymptote,

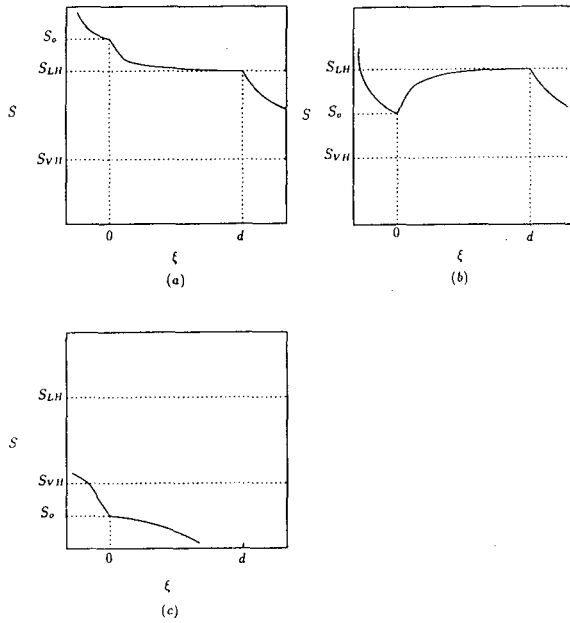


Fig. 4. Saturation profiles for integration from left-to-right: (a) $S_{LH} < S_o$; (b) $S_{VH} < S_o < S_{LH}$; (c) $S_o < S_{VH}$

except that now the saturation increases in the short region before this asymptote is reached (Figure 4b).

- Finally, if $S_o < S_{VH}$, then $dS/d\xi < 0$, but the solution is not attracted to a flat profile. Instead, it decreases relatively fast, much like in the homogeneous case (Figure 4c).

Thus, depending on the direction of integration, two different solutions (a VD and an LD) are found to satisfy the system. This feature is particular to vapor-liquid steady state counterflow. The selection of the particular paths (for example, whether it is the profiles in Figure 3 or those in Figure 4) is strictly determined from the past history of the system, which therefore attributes a hysteresis effect, albeit on a large scale [6]. The VD solutions of Figure 3 correspond to steady states reached by a system which is initially vapor-occupied and subsequently cooled from the left, while superheated conditions are maintained somewhere on the right. This is a condensation process (akin to imbibition). The LD solutions of Figure 4, on the other hand, correspond to steady states reached by an initially liquid-occupied system which is subsequently heated from the right, while subcooled conditions are maintained somewhere on the left. This corresponds to a boiling process (akin to drainage).

B. Vertical Counterflow

The above pertained to counterflow in the absence of gravity. We consider, next, the case of vertical counterflow (Figure 5). Here, two generic configurations are possible, heating from the top ($\theta = \pi/2$, $\sin\theta > 0$), and heating from the bottom ($\theta = 3\pi/2$, $\sin\theta < 0$).

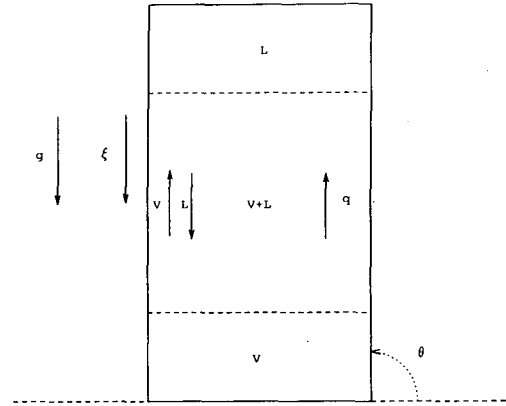


Fig. 5. Schematic of vertical counterflow.

We refer to [14] for the general case. In the first case, equation (1) yields:

$$\tau J' \frac{dS}{d\xi} = \omega \frac{(k_{r\ell} + \beta k_{rv})}{k_{r\ell} k_{rv}} - (\tau^2 - Ja) \quad (9)$$

where $a(\xi)$ is the heterogeneity gradient, $a \equiv d\tau/d\xi$. In the homogeneous case ($\tau \equiv const.$, $a \equiv 0$), the *RHS* above vanishes for the two saturation values S_{VG} and S_{LG} that solve the equation:

$$\omega = \frac{\tau^2 k_{r\ell} k_{rv}}{k_{r\ell} + \beta k_{rv}} \quad (10)$$

provided that $\omega < \omega_{cr,G}$. The critical value $\omega_{cr,G}$ is constant for a homogeneous system of a given permeability (e.g. equal to 0.3063 for $\tau = 1$, [3]). The two steady states of gravity-driven homogeneous heat pipes were investigated in [10], where a theory of heat pipe instability was developed. We recall that an identical multiplicity was also encountered in heterogeneous, horizontal counterflow. Because of this similarity, we contend below that, in a strict sense, instability is not really relevant and that the selection mechanisms of the horizontal case are much more appropriate.

(i) Homogeneous Systems: Steady State Selection

When the integration proceeds from the bottom (the "vapor side") upwards, it is the VD branch S_{VG} which is selected, if the starting saturation S_1 lies to the left of S_{LG} , $S_1 < S_{LG}$ (Figure 6a). This would be the case if superheated vapor existed somewhere below, as in the bottom curve of Figure 6a. In the interpretation of [6] this case could result from an initially superheated system that partly condenses due to top cooling. If $S_1 > S_{LG}$, on the other hand, a flat profile does not develop and the saturation rapidly converges to $S = 1$ (Figure 6b).

By contrast, when the integration proceeds from the top (the "liquid side") downwards, it is the LD branch, S_{LG} , which is selected, if the starting saturation S_o lies

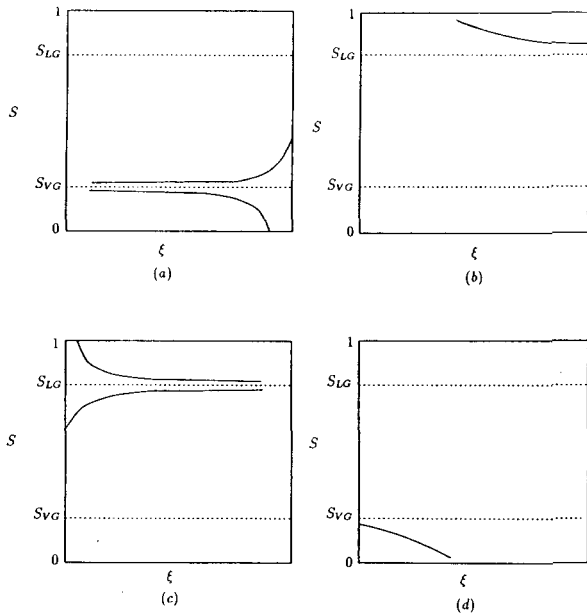


Fig. 6. Steady-state selection in homogeneous heat pipes: (a) Integration upwards, $S_1 < S_{1,G}$, (b) Integration upwards, $S_1 < S_{1,G}$, (c) Integration downwards, $S_o < S_{V,G}$, (d) Integration downwards, $S_o < S_{V,G}$.

to the right of $S_{V,G}$, $S_o > S_{V,G}$ (Figure 6c). This is the case of subcooled liquid somewhere at the top, a typical application being boiling [6]. If $S_o < S_{V,G}$, a flat profile does not develop, the saturation rapidly approaching the dry regime, $S = 0$ (Figure 6d). We readily conclude that it is the past history of the system that determines the steady state solution. Evidently, all such saturation profiles are intrinsically stable.

(ii) Sharp Discontinuity: Termination of an "Infinite" Two-Phase Zone

Next, we consider the special case of an abrupt discontinuity ($|a| \gg 1$). This analysis is necessary to explain how VD or LD saturation profiles can merge with subcooled liquid or superheated vapor, respectively, thus how an "infinite" two-phase zone can terminate for $\theta = 3\pi/2$. In the case of large $|a|$, heterogeneity is much stronger than gravity and controls the saturation profile much like the horizontal counterflow of section A.

Consider, first, integration from the bottom within a constant permeability region (where $\omega < \omega_{cr,G}$). Then, a VD regime is rapidly reached. In a homogeneous medium this regime is predicted to continue indefinitely (although see [6] and [7]). Can this profile merge with another LD regime or with a region of subcooled liquid? The answer is negative to the first part, but not to the second. In either case, for a change in the saturation state, a region of low permeability k_t must exist somewhere at the top. Then, because a is positive and large, the response is much like in the horizontal case

and capillary pressure continuity applies. If k_t is such that ω remains below critical in the top (recall that $\omega_{cr,G}$ is proportional to τ^2 or k), the previous scenario (pertaining to Figures 6a-6b) is in effect and the solution is either another VD region or a rapid approach to subcooled liquid, depending on the particular conditions. On the other hand, if $\omega > \omega_{cr,G}$ at the top, only a finite two-phase zone develops that rapidly ends by merging with a subcooled liquid region.

If integration proceeds from the top (where $\omega < \omega_{cr,G}$), an LD region is rapidly approached. Again, for this flat profile to eventually change, and for a dry region to be eventually encountered, the bottom must be at a higher permeability. Since for this case we also have $a > 0$, we can employ the same reasoning as before to reach the conclusion that it is the scenario of Figures 6c-6d that is followed, namely there will be either an attraction to another LD solution or a relatively fast approach to superheated (dry) conditions. However, in the present case, the approach to superheated conditions can also be accomplished if the bottom is at a (much) lower permeability.

A somewhat different way of stating the above is that in order to terminate a steady state vertical counterflow with $\theta = 3\pi/2$, when a VD region exists at the bottom, it is necessary that the permeability increases somewhere in the downwards direction. Subcooled liquid dominates the top. If an LD region lies at the top, the two phase zone terminates if a sharp change in permeability occurs. Superheated vapor must exist at the bottom. Significantly, LD and VD branches never merge with each other, regardless of the position or form of heterogeneity. This contrasts some of the arguments of [10] in which an "unstable" VD regime becomes connected to a "stable" LD regime, and vice versa.

(iii) General Heterogeneity Effects

Consider, next, the more general case of heterogeneity, with normal variations in τ . Equation (9) suggests that heterogeneity enhances (makes more vapor-rich or liquid-rich) the respective VD or LD regimes when $a < 0$, and acts to diminish them in the opposite case.

For a numerical example we used the profile of Figure 7a. Because here the combination $\tau^2 - aJ$ is always positive, it is possible for the RHS of (9) to vanish for all τ provided that ω is low enough ($\omega < \omega_{cr,min}$, where $\omega_{cr,min}$ must be obtained numerically). According to [14], the solution must follow closely the variation of τ^2 , resulting into either an LD or a VD branch, depending on the direction of integration. Numerical results shown in Figures 7b and 7c for the respective regimes verify the theoretical predictions. After a short interval, the profiles are attracted to this asymptotic state and, with

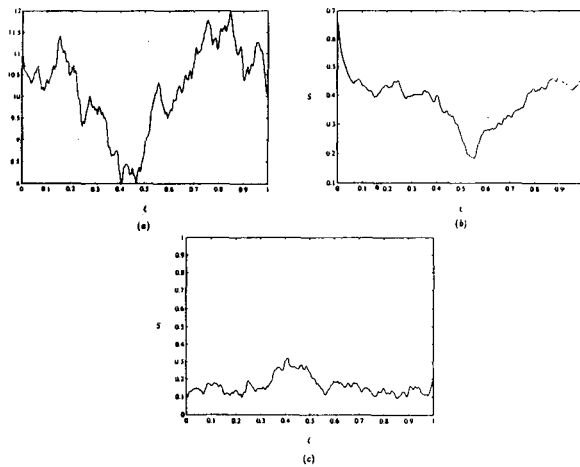


Fig. 7. Weak heterogeneity (a) τ profile, (b) LD regime, $\omega = 23.0$, (c) VD regime, $\omega = 25.0$.

a small spatial delay of about 0.05, mimic the variation of τ^2 . The VD solution shows a weaker sensitivity due to the relatively narrower range of saturation values allowed. As predicted theoretically, saturations in the LD regime increase or decrease as τ increases or decreases, respectively, while the saturations in the VD regime follow opposite trends. Capillary effects are significant only near the initial boundary.

When the heat flux acquires larger values ($\omega > \omega_{cr,min}$), there are spatial locations where the local critical values may be exceeded ($\omega > \omega_{cr}(\xi)$). Then, the saturation departs from the corresponding regimes and becomes rapidly attracted to a single phase region (dry-out in the case of an LD state, as in Figure 8a, or subcooled liquid in the case of a VD state, as in Figure 8b). On the opposite side, for very low values of ω , as is typical in geothermal reservoirs, all saturation values in the VD regime are very low, hence the profile is very nearly flat, (Figure 9), despite the variation in permeability. It is clear that the existence of a flat profile should not be taken to imply a homogeneous medium.

The second case investigated corresponds to a normal variation of τ (Figure 10a). Here, the combination $\tau^2 - aJ$ changes sign often within the interval. The solution displays hysteresis again, depending on the direction of integration. However, now capillary effects are quite significant. For an LD state this is contrary to the gravity effect noted in the previous. Consider, for instance, integration from the left, where an LD regime is obtained provided that ω is low enough (Figure 10b). As long as the τ variations are not too great, the saturation values are relatively constant (early part of Figure 10b). The saturation variation is mild even though regions of relative large increase in τ are traversed. This behavior is similar to the horizontal counterflow for a negative and large. At the point where a sharp increase

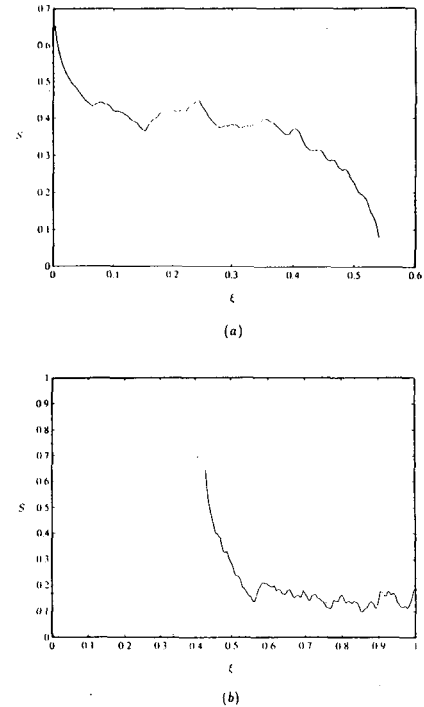


Fig. 8. Weak heterogeneity for larger heat fluxes: (a) LD regime, $\omega = 24.0$, (b) VD regime, $\omega = 28.0$.

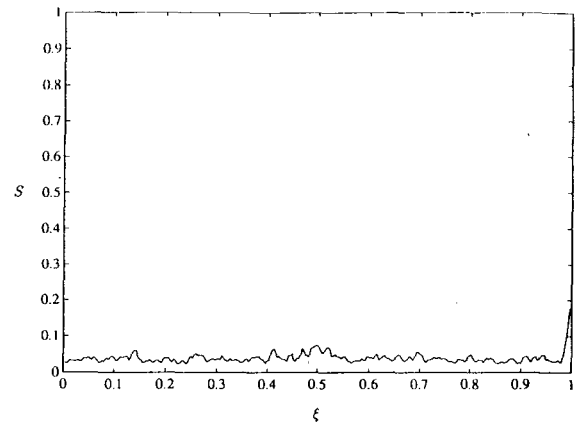


Fig. 9. The VD regime corresponding to Figure 7a for $\omega = 0.6$.

is encountered and a becomes large (around the midpoint of Figure 10a), capillarity dominates, capillary pressure continuity is enforced and the saturation falls significantly. If the drop is not too high, a lower saturation state, still of the VD type, will be followed in the remaining part.

Under the same conditions in ω , a VD regime arises, when the integration is from the right (Figure 10c). The first part of the profile (for ξ roughly between 0.5 and 1) corresponds to heterogeneity with generally positive slope ($a > 0$), thus capillary pressure continuity applies,

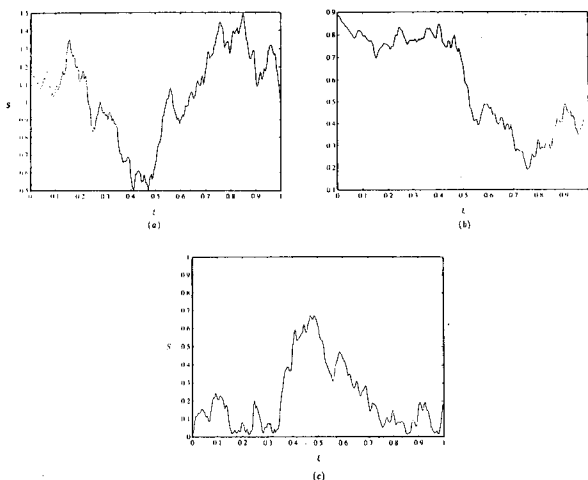


Fig. 10. Normal heterogeneity: (a) τ profile, (b) LD regime, $\omega = 0.02$, (c) VD regime, $\omega = 0.02$

the saturation rising as lower permeabilities are encountered. The second part of the heterogeneity, however, involves a rather steep negative slope (between 0.15 and 0.4). As pointed out previously, the saturation response may not be given by capillary pressure continuity alone. Indeed, after the saturation falls rapidly (for ξ between 0.3 and 0.4), further large changes in permeability do not induce significant saturation response. For further details see [14].

CONCLUSIONS

Within the framework of a continuum description, effects of permeability heterogeneity on steady state, vapor-liquid counterflow in porous media were examined. Permeability variations affect mainly two processes, gravity-driven flow and capillarity. The variations of the latter can be significant. It was shown that, as in similar previous flows [11], capillary heterogeneity acts like an external body force (such as gravity), with the additional property that it also varies spatially. For example, a multiplicity of steady states similar to gravity-driven heat pipes was found for decreasing permeabilities in horizontal counterflow and for heat fluxes lower than a critical value. Vapor-dominated and liquid-dominated regimes were obtained using selection rules that were postulated to depend on the past history (transient state) of the system. The analysis was aided by an exact solution obtained for a special heterogeneity profile [14].

The selection rules were next applied to determine the steady state regimes in gravity-driven heat pipes in homogeneous systems. It was shown that VD regimes originate from underlying dry regions, while LD regimes are extensions of overlying subcooled liquid regions. Significantly, the different regimes may never connect with each other, thus retaining their identity as long as the system remains in a two-phase state. The issue of the

termination of the infinite two-phase zone was next analyzed. It was determined that termination requires that a sharp change in the permeability occurs somewhere in the medium. Across this discontinuity it was shown that, depending on past history, either the overlying LD state rapidly connects with a dry region below, or the underlying VD state rapidly converts to a subcooled liquid above. The emerging picture (from top-to-bottom) is thus, subcooled liquid - LD - (discontinuity) - dry region, or subcooled liquid - (discontinuity) - VD - dry region, in the respective cases. This ordering may be helpful in the interpretation of the nature and origin of geothermal systems. Unfortunately, this argument cannot apply for homogeneous systems, the termination of an "infinite" two-phase zone within which remains an unresolved question.

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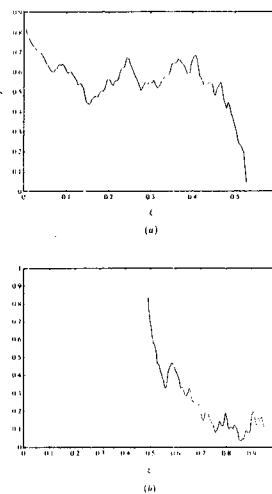


Fig. 11. Normal heterogeneity for larger heat fluxes: (a) LD regime, $\omega = 0.2$, (b) VD regime, $\omega = 0.2$.

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