PROCEEDINGS, Seventeenth Workshop on Geothermal Reservoir Engineering
Stanford University, Stanford, California, January 29-31, 1992
SGP-TR-141

A CUBIC MATRIX-FRACTURE GEOMETRY MODEL FOR RADIAL TRACER FLOW IN NATURALLY FRACTURED RESERVOIRS

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ABSTRACT

This study presents a general solution for the radial flow of tracers in naturally fractured reservoirs, with cubic blocks matrix-fracture geometry. Continuous and finite step injection of chemical and radioactive tracers are considered. The reservoir is treated as being composed of two regions: a mobile where dispersion and convection take place and a stagnant where only diffusion and adsorption are allowed. Radioactive decay is considered in both regions. The model of this study is thoroughly compared under proper simplified conditions to those previously presented in the literature. The coupled matrix to fracture solution in the Laplace space is numerically inverted by means of the Crump algorithm. A detailed validation of the model with respect to solutions previously presented and/or simplified physical conditions solutions (i.e., homogeneous case) or limit solutions (i.e., naturally fractured nearly homogeneous) was carried out. The influence of the three of the main dimensionless parameters that enter into the solution was carefully investigated. A comparison of results for three different naturally fractured systems, vertical fractures (linear flow), horizontal fractures (radial flow) and the cubic geometry model of this study, is presented.

INTRODUCTION

It is well known that an important fraction of worldwide reservoirs (geothermal and petroleum) are found in naturally fractured reservoirs (Aguilera,1980; van Golft-Rach, 1982; Saidi, 1987). Fluid and heat flow in these systems are more complex than in conventional non-fractured formations for which the theory of fluid and heat flow is at a more advanced level. Determination of an optimal exploitation scheme for a fractured reservoir rest on the ability to adequately characterize this heterogeneous system, that will be extremely important for performance predictions and for enhanced heat recovery studies. The effort needed to accomplish this task is bigger than that required for conventional reservoirs, since two parts compose the porous medium, and are to be properly studied, the fractured and the matrix system. A very important contribution toward accomplishing the characterization of these reservoirs is provided by the interpretation of tracer flow tests.

There are several papers that deal with the flow of tracers in naturally fractured reservoirs, for a review see the papers by Ramírez et al.(1990 and 1991), These papers are concerned with vertical (linear flow) and horizontal (radial flow) fractures. A more realistic visualization of naturally fractured systems has been that of Barenblatt (1960), further discussed by Warren and Root (1963), which presents the naturally fractured reservoir as an idealized system formed by identical cubes, separated by an orthogonal network of fractures. The flow is considered to take place in the fractured network, while the matrix continuously feeds the system of fractures. Assuming transient fluid flow from the matrix to the fractures and cubic geometry of the matrix blocks, Lai et al. (1983) presented a flow model and its correspondent solution for the previously mentioned flow problem. A review of the literature does not show a solution for the radial flow of tracers considering the cubic matrix-fracture geometry model used in the pressure transient analysis theory of Lai et al.

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The purpose of this study is to present a general solution for the radial flow of tracers in naturally fractured reservoirs, considering cubic matrix-fracture geometry. Continuous and finite step injection of chemical and radioactive tracers are considered. This solution takes into account all the important mechanisms that affect tracer flow: diffusion, convection, adsorption and radioactive decay.

MATHEMATICAL MODEL

The model considered in this study is shown in Fig. 1. The naturally fractured medium is represented by means of a system of identical cubic blocks separated by and orthogonal network of fractures. The system shown in this figure consists of two flow regions: 1) a mobile region constituted by the network of fractures and 2) a stagnant or immobile region. Both regions are interconnected by means of a thin fluid layer contained within the immobile region, which controls the fluid and mass transfer between the regions. This type of visualization of the problem has been used previously by other authors (Deans, 1963; Walkup and Horne, 1985; Maloszewski and Zuber, 1985; Chen, 1986; Rivera et al., 1987; Ramírez et al., 1988 and 1991).

In the mobile region 1, the following effects are considered:

a) Longitudinal dispersion that includes molecular diffusion:

$$D_r = \alpha v + D_m \tag{1}$$

The perpendicular z or r directions, depending on the faces of the cubic block considered, horizontal or verticals (Fig. 1), is not dealt with because it is assumed that fracture width 2w is small and, consequently there is no concentration gradient in this direction.

b) Convection, Based upon the discussion presented in a), flow velocity in the perpendicular directions is assumed to be uniform and only its variation is considered along the r direction. For the case of this study of radial flow under constant rate injection, velocity is defined as

$$v = \frac{a}{r} \tag{2}$$

where

$$a = \frac{Q}{2\pi H \phi_1} \tag{3}$$

c) Decay, This condition is considered for the case of a radioactive tracer of decay time less than the transit (travel) time.

For the immobile region the following effects are studied:

a) Diffusion. This effect is only considered in the perpendicular directions because the longitudinal component is assumed to be negligible

b) Adsorption
c) Decay

Based upon the above mentioned assumptions, considering an incompressible fluid, the governing equations for tracer concentrations in the fracture and in the porous matrix can be stated in dimensionless form as follows (Fig. 2):

a) Fractures:

$$\frac{1}{r_D}\frac{\partial^2 C_{D21}}{\partial r_D{}^2}\ \frac{1}{r_D}\frac{\partial C_{Dr1}}{\partial r_D} - \gamma\, C_{Dr_1} + \frac{6}{d_D}\frac{\phi_2}{\phi_1}\cdot$$

$$D_{D2}\left(\frac{\partial C_{Dr2}}{\partial z_D}\right)_{z_{Do}} = \frac{\partial C_{Dr1}}{\partial t_D} \tag{4}$$

b) Matrix:

$$D_{D2}\, R\, \frac{\partial^2 C_{Dr2}}{\partial z_D{}^2} + \frac{2}{z_D} D_{D2}\, R\, \frac{\partial C_{Dr2}}{\partial z_D}$$

$$-\gamma\, C_{Dr2} = \frac{\partial C_{Dr2}}{\partial t_D} \tag{5}$$

where the definitions of the dimensionless groups used in Eqs. 4 and 5 are

$$t_D = \frac{Qt}{2\pi H\phi_1\alpha^2} = \frac{at}{\alpha^2} \tag{6}$$

$$C_{Dr1} = \frac{C_{r1} - C_i}{C_o - C_i} \tag{7}$$

$$C_{Dr2} = \frac{C_{r2} - C_i}{C_o - C_i} \tag{8}$$

$$r_D = \frac{r}{\alpha} \tag{9}$$

$$z_D = \frac{z}{\alpha} \tag{10}$$

$$d_D = \frac{d}{\alpha} \tag{11}$$

$$z_{Do} = \frac{d}{2\alpha} + \frac{\delta}{\alpha} \tag{12}$$

$$D_{D2} = \frac{D_2}{a} \tag{13}$$

$$\gamma = \frac{\lambda\alpha^2}{a} \tag{14}$$

$$R = \frac{\phi_2}{\phi_2 + \rho k(1-\phi_2)} \tag{15}$$

The last term of the left hand side of Eq. 4 considers the interaction between the fractured and the matrix systems, representing a diffusion mass transfer from the fractures to the matrix at z_{Do}, Eq. 12.

Tha equations that complete the continuons injection tracer flow problem are given by Eqs. 16-21:

Boundary conditions

$$C_{Dr1}(r_{Do}, t_D) = 1 \tag{16}$$

$$C_{Dr1}(\infty, t_D) = 0 \tag{17}$$

$$C_{Dr2}(r_D, z_{Do}, t_D) = C_{Dr1}(r_D, t_D) \tag{18}$$

$$\left(\frac{\partial C_{Dr2}}{\partial z_D}\right)_{(r_D,\ 0,\ t_D)} = 0 \tag{19}$$

Initial conditions

$$C_{Dr1}(r_{Do}, 0) = 0 \tag{20}$$

$$C_{Dr2}(r_D, z_D, 0) = 0 \tag{21}$$

To find a solution to this problem the Laplace transformation method was used. The resulting equation after the application of this method to Eqs. 4 and 5, and 16-21, coupling the differential equations yields:

$$\frac{d^2\bar{C}_{Dr1}}{dr_D^2} - \frac{d\bar{C}_{Dr1}}{dr_D} - r_D\,\xi_{r2}\,\bar{C}_{Dr1} = 0 \tag{22}$$

where

$$\xi_{rc} = s + \gamma + \varepsilon\left\{\beta_{rc}\coth(\beta z_{Do}) - 1/z_{Do}\right\} \tag{23}$$

$$\varepsilon = \frac{6}{d_D}\,\frac{\phi_2}{\phi_1}\,D_{D2} \tag{24}$$

$$\beta_{rc} = \sqrt{\frac{s+\gamma}{RD_{D2}}} \tag{25}$$

with the following boundary conditions:

$$\bar{C}_{Dr1}(r_{Do}, s) = \frac{1}{s} \tag{26}$$

$$\bar{C}_{Dr1}(\infty, s) = 0 \tag{27}$$

Eq. 22 with boundary conditions given by Eqs. 26 and 27, define the mathematical model in Laplace space, for the radial continuous flow of tracers in naturally fractured reservoirs, with cubic blocks matrix-fracture geometry. It can be observed that it is similar to the stratified naturally fractured model of Ramírez et al. (1991). Thus, following the results of these authors, substituting the definitions for the Airy functions given in their paper (Abramowitz and Stegun, 1970, p. 448 Eq. 10.4.59), we get the following Laplace space continuous solution:

$$\bar{C}_{Dr1} = \frac{1}{s}\exp\left(\frac{Y - Y_o}{2}\right)\left\{\frac{A_i(\xi_{rc}^{1/3}Y)}{A_i(\xi^{1/3}Y_o)}\right\} \tag{28}$$

A solution for the finite step injection case may be obtained through the use of Eq. 28 and the principle of superposition.

In some field operations, the tracer is injected for a short period and are referred to as "spike" injection tests (Walkup, 1984). It has been stated (Walkup, 1984; Walkup and Horne, 1985; Ramírez et al.,1991) that the solution for a spike injection test can be derived through the time derivative of the finite step solution.

VALIDATION OF THE MODEL

The solution to the continuous tracer injection given by Eq. 28 can be simplified to particular cases, such as a naturally fractured system with horizontal fractures and a homogeneous system.

First, we will deal with the homogeneous simplification. If matrix porosity ϕ_2 and the matrix diffusion coefficient D_2 be small, the matrix (immobile) region will behave as it were impermeable and tracer flow will occur only through the fractures, resulting in negligible values of the third term of the right hand side of Eq. 23, $\varepsilon\ (\beta_{rc} \cdot \coth\ (\beta_{rc} z_{Do}) - 1/z_{Do}) \cong o$. For this case the model given by Eqs.22-27 is identical to the simplified model, for the above mentioned conditions, for horizontal fractures presented by Ramírez et al. (1991), and their discussion in relation to the comparison with respect to previous works (Moench and Ogata, 1981; Hsieh, 1986) holds.

With regard to the naturally fractured case, the model of this study can also be compared to the horizontal fractures model of Ramírez et al. (1991). It can be concluded that the differential coupled equations that describes these two tracer flow problems differ only in the definitions of the ξ_r,s parameters, essentially in relation to the third term that represents the matrix-fracture interaction, due to the differences in matrix-fracture geometry involved. A particular solution (at least theoretically) can be presented for the case of cubic blocks size d equal to the reservoir thickness H, which allows a direct comparison between the models. Before doing this, the influence of the hyperbolic functions that enter into both models, $\tanh\ (\sqrt{\beta_{rh}}\ (E/\alpha\ -\ 2z_{Do}))$ for the horizontal fracture model and $\coth\ (\beta_{rc}\ z_{Do})$ for this cubic matrix-fracture model, was thoroughly investigated for practical ranges of the parameters involved, concluding that essentially the results of their evaluation give unit values. Thus the ξ_r,s parameters are expressed by the following equations:

$$\xi_{rh} = s + \frac{D_{D2}\ \phi_2}{z_{Do}} \sqrt{\beta_{rh}} \tag{29}$$

for the horizontal fractures model, and

$$\xi_{rc} = s + \varepsilon \left\{\beta_{rc} - \frac{1}{z_{Do}}\right\} \tag{30}$$

for the cubic matrix-fracture model. Substituting the definitions of the dimensionless parameters that enter into these equation, given by Eqs. 12, 13, 24 and 25, considering that $\delta/\alpha <<< 1$, and that d=H:

$$\xi_{rh} = s + 2\phi_2\alpha \sqrt{\frac{\pi D_2 s}{(w - s)Q}} \tag{31}$$

$$\xi_{rc} = s + 6\phi_2\alpha \left\{ \sqrt{\frac{\pi D_2 s}{3(w - s)Q}} - \frac{4\pi D_{2\alpha}}{Q\ \ H} \right\} \tag{32}$$

The second term in the brackets of this last expression for ξ_{rc} was evaluated similarly to the hyperbolic functions previously discussed, concluding that it is at its biggest value three orders of magnitude smaller than the first term. Thus, it can be neglected. With this result in mind, a further look at Eqs. 31 and 32 indicates that they are very similar, and as expecteddue to the fact that the cubic matrix-fracture model presents a contact area three times that of the horizontal fractures model, the numerical constant of the second right hand side term changes from 2 to 6. Again, under these conditions all the previous validation for the horizontal fractures model of Ramírez et al. (1991), holds for the subject cubic matrix-fracture geometry model

DISCUSSION OF RESULTS

The results of this discussion were generated using whenever possible the data of Chen (1986).

First, a comparison of results for the radial horizontal fractures model of Ramírez et al. (1991) and results of the radial cubic matrix-fracture geometry model of this study is shown in Fig. 3. The function $F(t_D)$ of the horizontal axis stands for the different definitions of the dimensionless time. For the results of this work, $F(t_D)$ is given by Eq. 6 and for the previous work, is given by Eq. 6 of that paper. Also shown is the curve corresponding to the homogeneous case. We observe that, as expected due to our validation discussion related to the bigger contact area, that results for the cubic model, fall below the results for the horizontal fractures model. Unfortunately, for characterization purposes, the shape of the curves is similar, concluding that a uniqueness problem may arise if a test interpretation were to be conducted without additional information coming from other sources (i.e., geological, core analysis, well logs, etc.).

It is of interest to compare results of the two radial models already discussed to the vertical fractures model of Ramírez et al. (1990). For simplification purposes, we follow the discussion of the model validation section, that considered a cubic block size equal to the thickness H. Similar simplifications were made for the other two models. The tracer continuous injection concentration results are shown in Fig. 4. Once again, as we expect based on the physical principles of these problems, the vertical fractures model tracer response is the biggest of three (see also Fig. 3). Fig. 5 shows similar results for a finite step injection $t_D = 2.01$. Essentially the conclusions reached in relation to the results of Fig. 4 holds for this case. Additionally, we notice from results of these Figs. 4 and 5 that our main conclusion with regard to the characterization problem in naturally fractured reservoirs through tracer test is confirmed.

Next, the influence of the matrix system on continuous tracer injection results, represented by the product $\phi_2\sqrt{D_{D2}}$ that enter into Eqs. 29 and 30 (best seen in their simplifications given by Eqs. 31 and 32), is shown in Fig. 6. The maximum values of this group used, for the horizontal and cubic block models, were 1.13×10^{-4} and 7.2×10^{-4}, respectively. We can observe from these results that for values of $\phi_2\sqrt{D_{D2}} < 1\times10^{-6}$, the matrix system behaves as it were impermeable.

The influence of the cubic block size in the continuous injection tracer response is shown in Fig. 7. The upper most curve correspond to the homogeneous case, continuing downward with curves for different block numbers, up to a maximum value of 100. These results quite logically indicate that as the area of contact increases, the tracer concentration decreases.

Last, we wanted to quantify the effects of fracture aperture in tracer response. The results obtained for the horizontal and cubic block models, are shown in Fig. 8, for an aperture range of 1.0E-04 to 1.0E-0 2 m. The effect of this parameter is quite important, showing an increasing tracer response as fracture aperture w increases.

CONCLUSIONS

The main aim of this study has been to present a solution for the radial flow of a tracer in a cubic matrix-fracture geometry naturally fractured reservoir.

The main conclusions of this investigation are as follows:

1. A model is presented for the radial flow of a tracer in a cubic matrix-fracture geometry naturally fractured reservoir. It considers all the important mechanisms that affect tracer flow: diffusion, convection, adsorption and radioactive decay.

2. The Crump numerical Laplace inversion algorithm was found again to be highly reliable.

3. A thorough validation of the present model against simplified and particular solutions published in the literature was carried out published in the literature was carried out.

4. Solutions are presented for the continuous and finite step injection cases.

5. A comparison of the horizontal and cubic block radial solutions with the vertical fracture solution, for continuous and finite step injection, indicates that a uniqueness problem may arise in the interpretation of a test, especially to distinguish between the radial flow cases.

6. It was found that for the radial flow cases discussed in this study, the matrix system be haves as it were impermeable for values of the group $\phi_2\sqrt{D_{D2}}$ less that $1x10^{-6}$

NOMENCLATURE

a = advection parameter, Eq. 3 L^2/T
$A_i(x)$ = Airy function
C = tracer concentration
C_D = dimensionless tracer concentration, Eqs. 7 and 8
$\bar{C}_D$ = Laplace space dimensionless tracer concentration
D_{D2} = dimensionless matrix diffusion coefficient, Eq. 13
D_m = molecular diffusion coefficient, L^2/T
D_r = longitudinal dispersion coefficient, Eq. 1, L^2/T
d = matrix block size, L
d_D = dimensionless matrix block size, Eq. 11
$F(t_D)$ = dimensionless time for a specific flow geometry (linear flow or radial flow)
H = reservoir thickness, L
k = adsorption constant, L^3/M
N = number of cubic blocks in reservoir thickness H
Q = constant injection rate, L^3/T
r = radial distance, L
r_D = dimensionless radial distance
R = dimensionless parameter, Eq. 15
s = Laplace space parameter
t = time, T
v = fluid velocity, Eq. 2, L/T
w = fracture half width, L
Y = variable transformation, $r_D + 1/4\xi_{rc}$
z = vertical coordinate, L
z_{Do} = dimensionless vertical distance of the fluid film exterior boundary, Eq. 12 (Fig. 2)

Greek symbols

α = longitudinal fracture dispersivity, L
β_{rc} = parameter, Eq. 25
γ = dimensionless parameter, Eq. 14
δ = stagnant fluid film thickness, L
ε = dimensionless parameter, Eq. 24
ξ_{rc} = dimensionless parameter, Eq. 23
λ = radioactive decay constant, T^{-1}
ρ = fluid density, M/L^3

Subscripts

c = cubic
D = dimensionless
h = horizontal
i = initial
o = origin or related to tracer injection concentration
r = radial
1 = mobile or fractured region
2 = immobile (fluid layer and porous matrix) region

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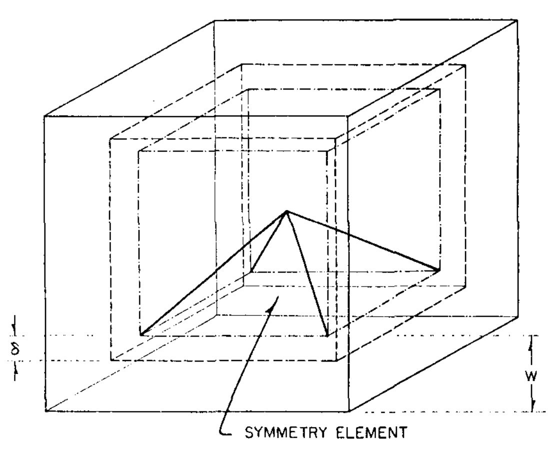

Fig 2 Proposed model for representation of the flow of a tracer in a naturally fractured medium. One dimensional fluid flow approximation

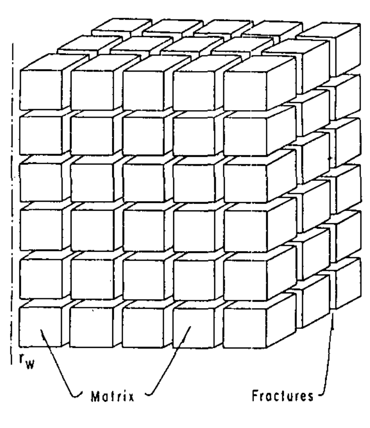

Fig 1 Idealized proposed model of naturally fractured reservoirs

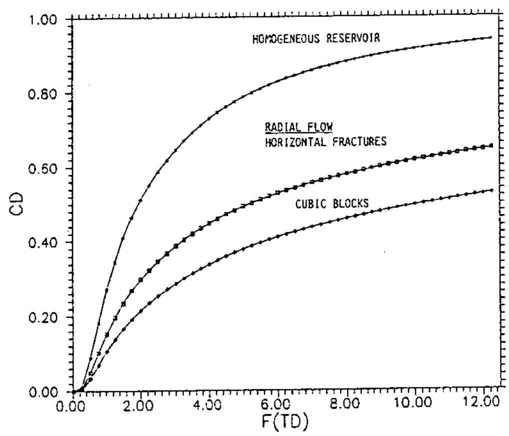

FIG. 3 COMPARISON OF HORIZONTAL AND CUBIC BLOCKS CONTINUOUS TRACER CONCENTRATION SOLUTIONS.

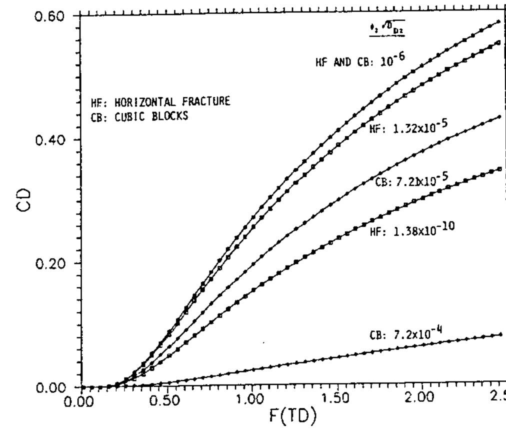

FIG. 6 INFLUENCE OF THE MATRIX CHARACTERISTICS ON THE HORIZONTAL AND CUBIC BLOCKS CONTINUOUS TRACER CONCENTRATION RESULTS.

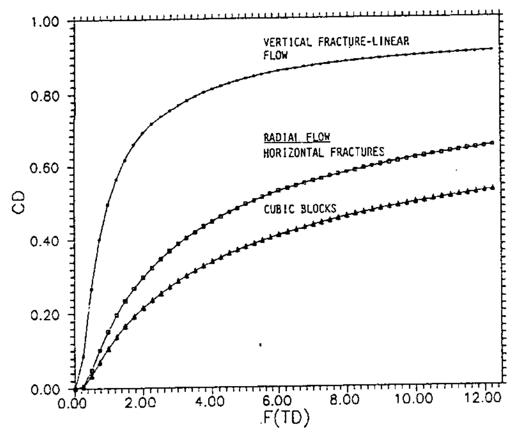

FIG. 4 COMPARISON OF THE VERTICAL FRACTURES (LINEAR) AND RADIALS CONTINUOUS TRACER CONCENTRATION SOLUTIONS.

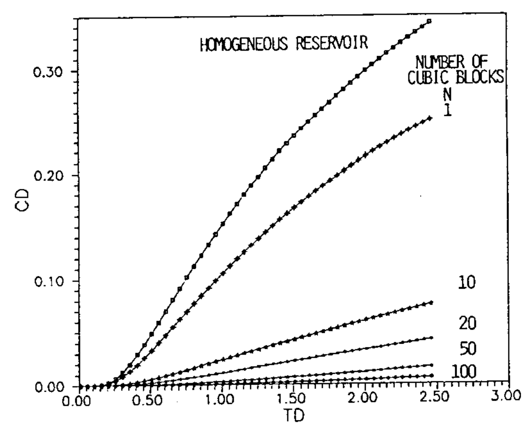

FIG. 7 INFLUENCE OF THE CUBIC BLOCK SIZE ON THE CONTINUOUS TRACER CONCENTRATION RESULTS.

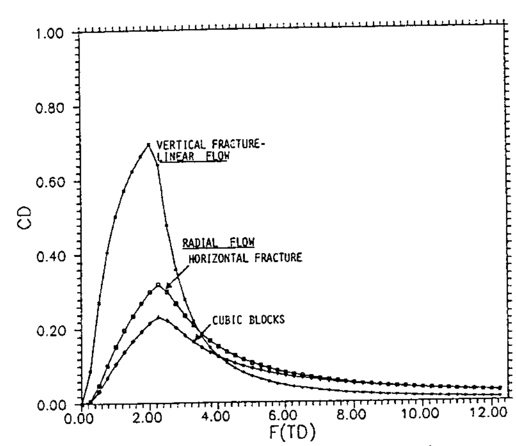

FIG. 5 COMPARISON OF THE VERTICAL FRACTURES (LINEAR) AND RADIALS FINITE STEP TRACER CONCENTRATION SOLUTIONS.

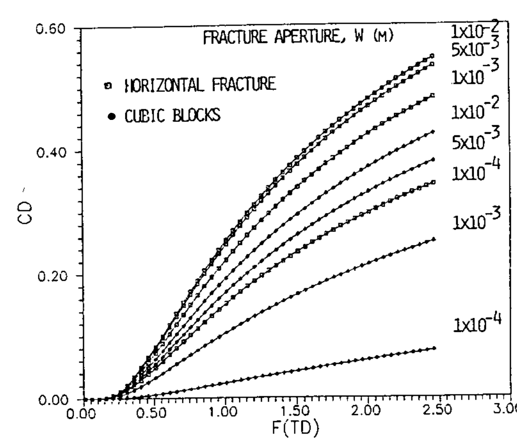

FIG. 8 INFLUENCE OF FRACTURE APERTURE ON THE HORIZONTAL AND CUBIC BLOCKS CONTINUOUS TRACER CONCENTRATION RESULTS.