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FRACTAL ANALYSIS OF PRESSURE TRANSIENTS IN THE GEYSERS GEOTHERMAL FIELD

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ABSTRACT

The conventionally accepted models for the interpretation of pressure transient tests in naturally fractured reservoirs usually involve simplistic assumptions regarding the geometry and transport properties of the fractured medium. Many single well tests in this type of reservoirs fail to show the predicted behavior for dual or triple porosity or permeability systems and cannot be explained by these models. This paper describes the application of a new model based on a fractal interpretation of the fractured medium. The approach, discussed elsewhere [2], [6], is applied to field data from The Geysers Geothermal Field. The objective is to present an alternative interpretation to well tests that characterizes the fractured medium in a manner more consistent with other field evidence. The novel insight gained from fractal geometry allows the identification of important characteristics of the fracture structure that feeds a particular well. Some simple models are also presented that match the field transient results.

INTRODUCTION

Pressure transient responses predicted by the dual or triple porosity models [1], [18], [19] are sometimes not observed in actual transient tests in naturally fractured reservoirs. In some cases the observed behavior is similar to that of a single fracture cutting the wellbore. The well response is characterized by parallel linear plots of pressure and pressure derivative vs time on a log-log scale with a slope m between 0 and 0.5 and a separation equal to $\log(\frac{1}{m})$ as shown in Figure 1. Such behavior is ordinarily explained by assuming the existence of a single fracture of finite conductivity intersecting the well, [3], [8]. This type of response has also been explained as a transition between single infinite conductivity fracture flow ($\frac{1}{2}$ log-log slope) and radial flow (semilog straight line) in the parallelepiped reservoir model [9], [11]. Another case often observed involves pressure derivative plot with a negative slope not large enough to be interpreted by a spherical flow regime. In systems where individual wells are connected to networks of fractures, alternative conceptual models need to be proposed. The purpose of this investigation was to examine the feasibility of using fractal geometry

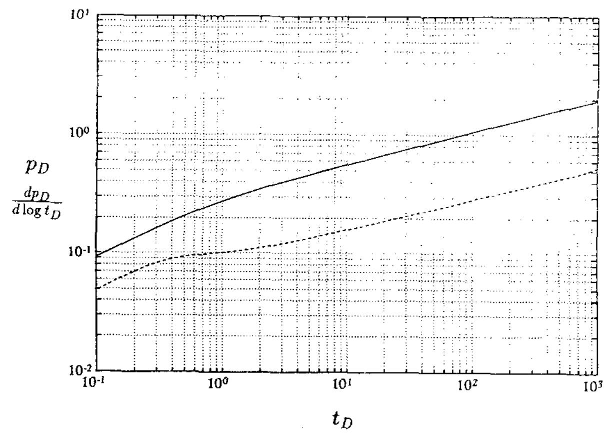

Figure 1: Pressure (solid) and pressure derivative (dashed) for a fractal fracture network $m = 0.27$ ($\delta = 0.73$)

to interpret the above responses in a manner consistent with the expectation that networks of fractures dominate the flow behavior.

The application of fractal geometry to the analysis of pressure transient tests resulted from a direct extension of novel discoveries in diffusion in disordered media and fractal objects [13], [15]. It has been found that the diffusion process, which also governs pressure transient tests, occurs in a unexpected fashion when the medium is highly disordered or fractal. Such phenomena, classified in general as "anomalous diffusion" [13], have helped in explaining many slow diffusion processes poorly understood only a few years ago.

The application of these concepts to pressure transient testing in fractal fractured systems was proposed by Chang and Yortsos [6], who described the general theoretical formalism. The theory was tested using numerical models of fractured networks in the recent work of Acuna and Yortsos [2]. A key feature of a fractal transient response is that the log-log plot of pressure derivative versus time is linear. The observed slope depends on the dimensionality and ranges between -0.5 and 0.5. Where -0.5, 0 and 0.5 correspond to spherical, radial and linear flow respectively. When the slope is between 0 and 0.5, the pressure curve is parallel to the pressure derivative curve, making the identification easier. Regardless of dimensionality, all responses can be considered as particular cases of a general solution.

Although theory and numerical examples have been presented elsewhere [2], [6], [10], practical applications to real well tests have been limited [5]. The Geysers Geothermal Field, whose characterization is still elusive, represents an excellent test case to explore the feasibility of a fractal structure. In particular, the nature of drilling fluid, air, used in the bulk of the fractured system helps in examining fractures relatively free of near-wellbore damage.

THEORETICAL BACKGROUND

The finite conductivity single fracture model [3], [8] predicts that at early times the pressure behavior can be approximated by the expression

$$p \approx At^m \tag{1}$$

where A is a constant. From (1) follows that the log-log plots of pressure and pressure derivative vs time will be parallel with slope m and separated by a distance equal to $\log(\frac{1}{m})$ (or $\frac{p}{t\frac{\partial p}{\partial t}} = \frac{1}{m}$). Many wells in naturally fractured reservoirs behave in this fashion although the existence of only a single fracture as cause of that behavior is not supported by other wellbore or reservoir measurements. The fractal model discussed below shows how a particular fracture network can also be responsible for that behavior.

When a fractured medium is highly disordered and fractal, the single-phase pressure transient of individual wells differ significantly from the homogeneous radial flow case [6]. The theoretical, ideal response would be described as follows: In a perfect fractal object of infinitely many generations of fractures, the mass density of any arbitrary cluster of radius r around an arbitrary point decreases in a power law fashion with respect to the distance r. The exponent of the power law is $D - d$ where D is the mass fractal dimension of the object and d the embedding dimension (2 for two-dimensional case). However, when the object has finite size, deviations with respect to this behavior are expected and will occur, although the average over many origins is expected to give the same power law [12], [14], [15]. Consider a fractal network of fractures. The "mass density" at any given radius corresponds to the average porosity at that radius r, defined as the total void volume divided by the total volume at radius r. This porosity will therefore change in a power law fashion with respect to r.

$$\phi(r) = \phi_0(\frac{r}{r_0})^{D-d} \tag{2}$$

where ϕ_0 is a constant.

Using the same reasoning, we could expect the sample permeability to also vary with r. For example, if steady-state flow across a sphere of radius r occurs, the corresponding single-phase permeability can be expressed as

$$K(r) = K_0(\frac{r}{r_0})^{D-d-\theta} \tag{3}$$

where D and θ are fractal parameters, r_0 is the minimum size considered in the network (smallest fracture) and K_0 is a constant.

These values of porosity and permeability are not point values, as traditionally interpreted, but sample (macroscopic) values over that radius r. For instance, the point value for porosity is either 0 or 1 depending on the point being on the matrix or the fracture. Here, we are concerned with the macroscopic values of these properties. The conductivity and storativity terms KA and ϕA are obtained by multiplying equations (2) and (3) by Br^{d-1}, where B is a constant. As can be observed, conductivity and storativity are power law functions of radius with different exponents. The diffusivity η is, therefore, dependent on the radius as $\eta \propto r^{-\theta}$. This variation of diffusivity with radius gives rise to several phenomena referred in general as "anomalous diffusion" [13], [15]. Diffusion over fractal objects is "anomalous" in that the standard diffusivity equation may not be used. The most rigorous alternative is to consider a Green's function approach, based on which solutions can be readily constructed [7]. A simplification of that approach at late times is a diffusivity equation, but with properties that vary spatially according to (2) and (3). Then, as shown in [6] (see also [7]), one obtains

$$\frac{\partial^2 p}{\partial r^2} + \frac{D-\theta-1}{r}\frac{\partial p}{\partial r} - r^{\theta}\frac{\partial p}{\partial t} = 0 \tag{1}$$

with boundary conditions that p vanishes as $r \to \infty$, and that constant flow rate applies at the wellbore

$$\lim_{r\to 0}(r^{D-\theta-1}\frac{\partial p}{\partial r}) = -1 \tag{2}$$

For this problem a similarity solution was obtained [6], based on which the pressure at the well p_w, after a short time, obeys the power law behavior

$$p_w(t) = C + \frac{(2+\theta)^{2(1-\delta)}}{\Gamma(\delta)(2+\theta-D)}t^{1-\delta} \tag{3}$$

where $C = \frac{\Gamma(\delta-1)}{\Gamma(\delta)(2+\theta)}$ is constant with respect to time and $\delta = \frac{D}{2+\theta}$. The constant term C becomes negligible with respect to the time dependent term when $\delta < 1$. The log-log plots of pressure and pressure derivative versus time appear as two straight parallel lines after sufficient time. When $\delta > 1$ (dimensionality greater than 2), however, the constant term becomes the asymptotic value of the pressure. In such cases only the derivative plot will show the log-log linear behavior with the pressure curve approaching asymptotically a constant value at late times. Equation (3) is valid for a single well test and it cannot be used for multiple well tests, where recourse to the Green's function formalism must be made [7]. For $\delta = 1$ the traditional exponential integral solution arises.

An alternative derivation for a similar problem was presented by Barker [4]. In his derivation conductivity and storativity were assumed to be power law-dependent on the radius with the same exponent. Theoretically, this may only result if we assume that the area perpendicular to flow changes in a power law fashion. Doe [10] attempted to rationalize such a variation in flow area by proposing drainage volumes with unusual shapes. In the Chang and Yortsos [6] notation, this assumption is equivalent to setting the parameter $\theta = 0$, a value characteristic of perfectly connected networks. The diffusivity η is therefore constant and normal diffusion should be expected. According to Barker's solution, the slope of the log-log plot of pressure versus time must approach a constant value equal to $1 - \frac{D}{2}$. This was not observed in our numerical simulations, neither in the simulations by Polek et al. [16], instead, the value obtained from flow simulations is consistently lower than the geometric fractal dimension. The results are, however, entirely consistent with Chang and Yortsos derivation. Although Barker's theory may be appropriate for certain objects (yet to be precisely described), it is certain that it cannot be applied to describe flow in fractals.

According to Chang and Yortsos, for a two-dimensional embedding medium, the slope m of both the pressure and the pressure derivative in log-log plots is given by $m = 1 - \delta = 1 - \frac{D}{2+\theta}$. The two parameters D and θ suffice to formulate the power-law variation. A type of system that gives rise to the above variation is a network of fractures with fractal properties. Indeed, it was shown numerically [2] that in such a fractal network of fractures, the two parameters D and θ have real physical meaning and that parameters, such as permeability and fracture porosity are power-law dependent on the radius.

Parameter D is the mass fractal dimension of the network, a strictly geometric property. It is measured by plotting the cumulative "mass" or length of fractures (for fractures of the same width) contained inside a circle of radius r versus r. A linear log-log plot of mass versus radius indicates the existence of a fractal structure with an exponent identical to D. Parameter θ, on the other hand, is a property depending on both the geometry and the transport properties of the medium. In a given network, it can be measured by taking a large number of random walks of t steps each and by calculating the average square distance of the walker from the origin. Contrary to the normal situation of Euclidean geometry networks, where $< r^2 > \propto t$, fractal systems exhibit slowdown, $< r^2 > \propto t^{\frac{2}{2+\theta}}$ [13], [15], a key feature in the process of anomalous diffusion.

When dealing with transient tests from a single well only, the information at the producing well can only determine the ratio $\delta = \frac{D}{2+\theta}$. In other words, any combination of D and θ that results in the same value of δ, will produce the same slope in the pressure derivative curve. To complete the reservoir characterization, therefore, additional information is needed.

APPLICATION TO THE GEYSERS DATA

To demonstrate the feasibility of such a model we ideally need a naturally fractured reservoir, where drilling circulation losses and fracture damage are minimal. The Geysers Geothermal Field with its air drilled wells fit those requirements quite adequately.

According to [6], given sufficient time in a fractal reservoir with dimensionality D less than 2, the log-log plots of pressure and pressure derivative versus time should appear as two straight, parallel lines. Equation (6) as well as our experience with synthetic numerical networks [2] has shown that the linearity sets in earlier in the pressure derivative curve (In a way, the same is true for a homogeneous radial system ($D = 2$ and $\theta = 0$) where the asymptotic slope of $1 - \delta = 0$ is achieved faster by the pressure derivative curve). When dealing with real systems, however, finite size effects are always important. These effects are of course absent from the theoretical analysis. Thus, at early times, a real system would respond only to a few fractures of a certain finite size around the test well. The effect is mathematically equivalent to a skin factor in the pressure curve. This skin factor together with any constant contribution to the pressure response would be filtered out by the process of differentiation, thus, rendering the slope of the derivative curve much more useful than that of the pressure curve. The noise in the derivative curve is however a problem. The separation $\log(\frac{1}{m}) = \log(\frac{1}{1-\delta})$ of the two lines is invaluable in helping to identify the correct value of the slope m for dimensionality less than 2. At late times, however, boundary conditions begin to influence the data and the curves deviate from the asymptotic trend. If enough time is available in the "infinite acting" period, the pressure curve reaches a slope equal to that of the pressure derivative. For dimensionality greater than 2, the parallel slope behavior is observed only if the pressure is replaced by the term $C - p_w(t)$ where C is the asymptotic constant value of the pressure at very long time.

If additional information suggest that a network of fractures is connected to the well, the approaching of the asymptotic straight lines should be good evidence to consider power-law variation in porosity and permeability

These concepts are illustrated in Figure 2 which shows a buildup test for well A at The Geysers. Both pressure and pressure derivative plots are linear and parallel for a certain period of time, before boundary effects become significant. The slope measured from the derivative curve is $1 - \delta = 0.40$, suggesting a ratio $\delta = \frac{D}{2+\theta} = 0.60$. From our experience with synthetic fractal fracture networks, we found that the parameter θ ranges between 0 and 0.5 for various networks. If we accept that the real fracture network above behaves similarly to our artificial networks [2], possible values of D predicted are in the range $1.20 < D < 1.50$, indicating a sparse network of fractures with radial fractal dimension D in the specified range. A network with

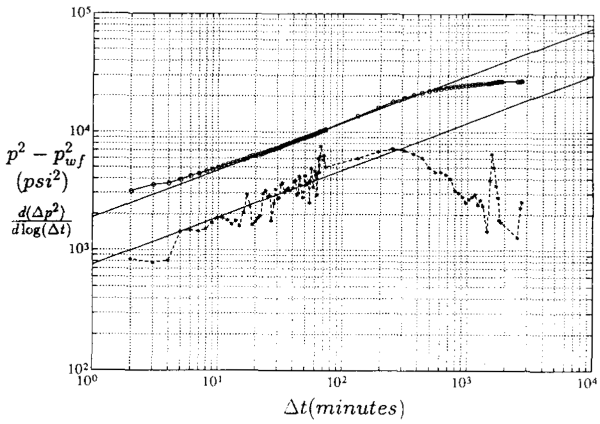

Figure 2: Pressure (o-solid), pressure derivative (*-dashed) and best fit power law ($\delta = 0.60$) at Well A.

such characteristics will be presented below.

Figure 3 shows build-up test data for another field test at location B. Pressure and pressure derivative curves are shown, the associated best fit value of δ being equal to 0.84 for the early part and it changes slightly to 0.87 for the later data. This change may be due to variations in the fractal structure or to boundary effects. Again using the previous estimates, the expected values for the mass fractal dimension D lie in the interval $1.68 < D < 2.0$. Although Figures 2 and 3 could be explained by the response of a single finite conductivity fracture model, all other available evidence such as steam entries, outcrops mapping [17], mud logs, etc, points out to the existence of a network of fractures,

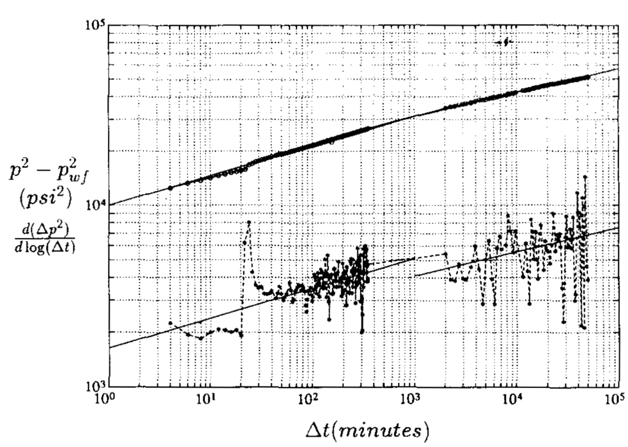

Figure 3: Pressure (o-solid) and pressure derivative (*-dashed) and best fit power law ($\delta = 0.84$ and 0.87) at Well B.

rather than one single fracture feeding the well. The fractal model proposed above gives the most plausible explanation of why a pressure transient response of wells in The Geysers resemble those of hydraulically fractured wells. We should point out that the significance of the need for a proper value of D cannot be overemphasized. For example, the resulting fracture porosity volume in a drainage volume of a disk of radius r_e and constant thickness would scale as $\phi_0 r_e^D$ which is a factor r_e^{D-2} smaller than if estimated at constant porosity.

Values $\delta = 1$ are indicative of a homogeneous radial system, as in the case of well C presented in Figure 4. Of course, there is always the possibility of D being different than 2, implying a power-law varying porosity. For example this may happen when $D = 2 + \theta$ as can be corroborated from equation (3). Because the value of θ is always positive, the "homogeneous response" would

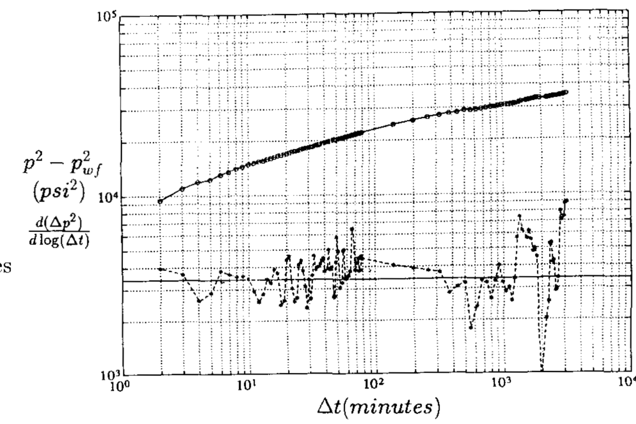

Figure 4: Pressure(o-solid), pressure derivative (*-dashed) and best fit power law ($\delta = 1.0$) at Well C.

require $D > 2$. This is not possible for a fractal embedded in a two-dimensional space, but it is for a finite size network as explained below.

The case $\delta > 1$ (negative pressure derivative slope) such as shown for well D in Figure 5 deserves particular attention. Because $\theta > 0$, values of $\delta > 1$ imply $D > 2$, namely the fracture structure is much more dense, the system features being intermediate between two and three-dimensional structure. The parallel plots will not be observed but the slope of the pressure derivative equals $1 - \delta$. The pressure curve itself equals a constant C minus a power-law term with the same exponent than the derivative curve. To improve the estimation of the slope m we draw a straight line $(C - p_w(t))$ parallel to the derivative plot separated a distance equal to $\log\left(\frac{-1}{m}\right)$ $(m < 0)$, as shown in Figure 5. The value of C can be calculated as twice the value of the pressure at the intersection between the pressure curve and this parallel line. This simple geometric construction can be verified using equation (6). A trial an error approach may be necessary. In this case, the flow can be interpreted as intermediate between radial and spherical types.

We should point out that a value of the mass exponent $D > 2$ implies that the fracture mass increases with the radius with an exponent greater than 2. A network with dimensionality greater than 2 has this kind of variation. A finite 2-D network, however, can also give this type of response if the porosity increases with

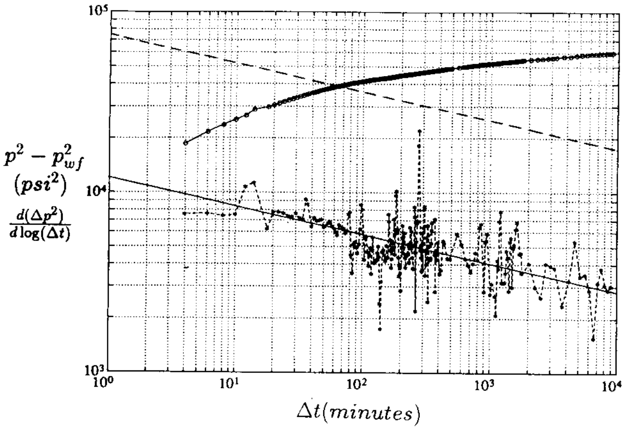

Figure 5: Pressure (o-solid), pressure derivative (*-dashed) , best fit power law for derivative (solid) and $C - p_w(t)$ (dashed) at Well D. ($\delta = 1.16$)

radius. Although this is not possible for infinite fractal objects, it is, nonetheless, conceivable for finite size networks.

For the separate estimation of D and θ, additional information is needed. This information could be the response at observation wells located at the Euclidean distance r. The parameters could be estimated using the results of [6] and [7]. An important practical problem, however, is that a given point at a distance r is usually not representative of the behavior of every point at that same distance. The fractal parameters reflect an overall behavior, therefore, for a good estimate in a particular well, finite size effects must be minimal.

As in every inverse problem of this type, including problems in Euclidean homogeneous media, there is an inherent non-uniqueness associated with single point measurements. Thus, even though the values of D and θ may be available, the precise structure of the reservoir is unavailable, different relations giving rise to the same single-well result. Nevertheless, by applying the numerical techniques developed in [2], we may get a qualitative understanding of the structure of the network that can give rise to the responses shown above (e.g. for wells A and B).

Although we have the capability of creating more natural-looking networks, such as the ones presented in [2], there is no particular reason to include additional geometric characteristics. In the absence of other information, therefore, these models have a regular, somewhat unrealistic appearance. The networks presented do not contain many generations of fractures, therefore, the fractal mass variation with respect to radius is not expected to be observed in every point. To overcome this problem, these network are constructed in such a way that those power law variations hold for the point where the well is. This type of behavior would appear for any point of the network if the number of generations is substantially increased, something we are not able to do due to computer limitations.

Figure 6 shows a realization corresponding to a radial fractal dimension equal to 1.26. This is one of infinitely many networks that can be generated with the same radial fractal dimension. It is the presence of gaps of a given size at a given distance that controls the value of D. Because we do not have very good control on the

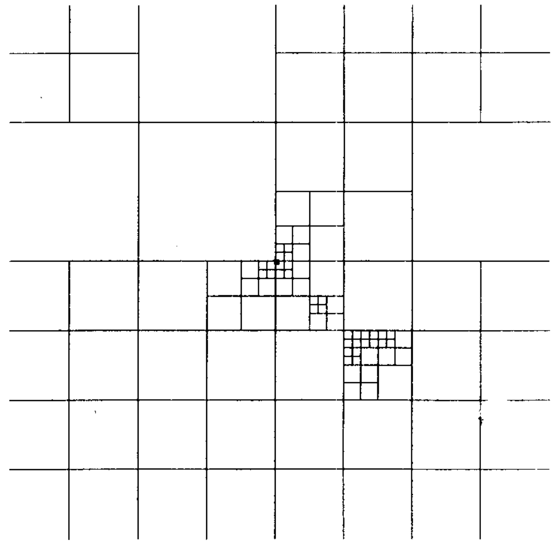

Figure 6: Synthetic network with the pressure transient response similar to that of Well A ($D = 1.26$).

value of θ, except for the fact that its range is fairly limited, a trial and error approach is necessary in order to obtain the desired value of δ (here equal to 0.60). Likewise, Figure 7 is a synthetic network whose transient response is similar to Well B. Again, this one of many possible realizations

Acceptance of a fractal model in the particular drainage area of a well implies a porosity distribution that is power-law dependent on the distance from the well, as well as within any other test volume where the fractal description applies. In such systems, significant consequences on the estimation of the pore volume associated with the well and on the expected behavior of heat recovery by cold water injection can be expected.

CONCLUDING REMARKS

We have presented an interpretation of pressure transient tests of certain wells in The Geysers Geothermal Field that behave similarly to a single fracture of finite conductivity or that tend to resemble spherical flow. Our fractal model shows how a network of fractures with some special characteristics can explain such response. This explanation appears more consistent with other field evidence [17] as well as with the concept of fracturing in naturally fractured systems.

Several wells analyzed in The Geysers Field show characteristics of fractal behavior, although the power-

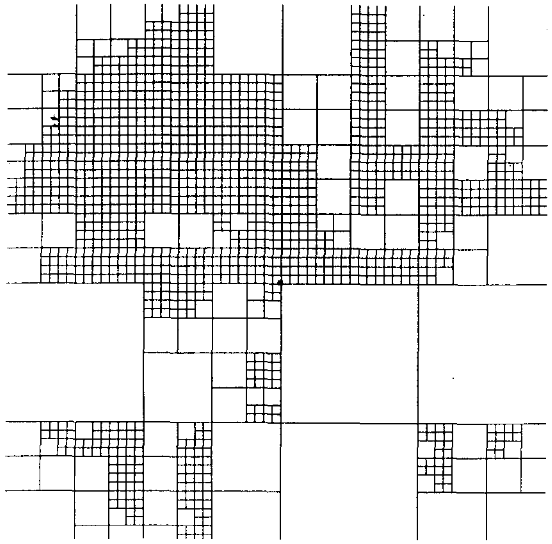

Figure 7: Synthetic network with the pressure transient response similar to that of Well B ($D = 1.84$).

law exponents are different at different locations. It was found in our previous sensitivity studies [2], and it is also evident by a casual inspection of Figures 6 and 7 that the position of the well may affect substantially the characteristics of the transient response, provided that finite size effects are important. Such would not have been the case if the upper and lower cutoffs (which correspond to the largest and smallest sizes for Figures 6 and 7, for instance),were greatly separated. Real systems, however, involve cutoffs of finite values, thus making the pressure response site specific.

This explanation can be offered to interpret why every well in The Geysers Geothermal Field does not respond in the same way. Another distinct possibility is that the true fractal parameters do vary in space as a result of different structure. Having different fractal parameters for individual wells does not invalidate the approach. On the contrary, it allows the determination of the individual parameters for each well. A true fractal reservoir would be one of infinite size in which well defined power laws for the permeability and porosity would be asymptotically approached, once the local variations at small radii have dissipated. From the limited pressure transient tests available to us we do not observe a tendency towards a single set of fractal parameters for The Geysers Field. Therefore, a single model cannot be proposed in which the entire reservoir is a single fractal object. On the other hand, the local fractal information derived from each well can be used to improve the global description of the reservoir. Future progress of this research is aimed at extracting the appropriate power-law variations for each well and to arrive at a unified model.

NOMENCLATURE

d = Euclidean dimension of medium in which fractal object is embedded
D = mass fractal dimension
K = permeability at radius r. $[L^2]$
m = slope in a log-log plot
p = pressure. $[ML^{-1}T^{-2}]$
r = radial distance. $[L]$
t = time. $[T]$
η = hydraulic diffusivity $[L^2T^{-1}]$
$\Gamma(x)$ = Gamma function
ϕ = porosity
θ = fractal exponent

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