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AN INVESTIGATION OF RADIAL TRACER FLOW IN NATURALLY FRACTURED RESERVOIRS

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ABSTRACT

This study presents a general solution for the radial flow of tracers in naturally fractured reservoirs. Continuous and finite step injection of chemical and radioactive tracers are considered. The reservoir is treated as being composed of two regions: a mobile region where longitudinal dispersion and convection take place and a stagnant region where only diffusion and adsorption are allowed. Radioactive decay is considered in both regions. The model of this study is thoroughly compared to those previously presented in literature by Moench and Ogata, Tang et al., Chen et al., and Hsieh et al. The solution is numerically inverted by means of the Crump algorithm. A detailed validation of the model with respect to solutions previously presented and/or simplified physical conditions solutions (i.e., homogeneous case) or limit solutions (i.e., for short times) was carried out.
The influence of various dimensionless parameters that enter into the solution was investigated. A discussion of results obtained through the Crump and Stehfest algorithm is presented, concluding that the Crump method provides more reliable tracer concentrations.

INTRODUCTION

Reservoir characterization plays a very important role in the optimization plan of a reservoir. Among the tools available to acomplish this task, the injection of tracers and proper analysis has shown through the years to be a useful mean. Naturally fractured reservoirs are complex systems that require careful studies for optimal exploitation conditions. In aiming toward this goal, tracer flow test provide estimation of basic reservoir parameters, in addition to the connectivity of the reservoir and the transit time of injected fluids. This latter information is very important in fluid reinjection projects.

There are several papers that deal with the flow of tracers in naturally fractured reservoirs (for a review, see the paper by Ramírez et al., 1990). The existing literature with regard to the radial flow of tracers appears to be mostly oriented to the homogeneous case (Bailey and Gogarty, 1961; Brigham and Smith, 1966; Moench and Ogata, 1981; Pickens, et al., 1981; Güven, et al., 1985; Hsieh, 1986; Guvansen and Guvansen, 1987; Chen, 1987; Falade and Brigham, 1989). A review indicates only a few papers that discuss the radial flow of tracers in naturally fractured systems (Chen, 1985 and 1986; Stephenson et al., 1989). The purpose of this study is to present a general solution for the radial flow of tracers in naturally fractured reservoirs. Continuous and finite step injection of chemical and radioactive tracers are considered. This solution considers all the important mechanisms that affect tracer flow: diffussion, convection, adsorption, and radioactive decay.

MATHEMATICAL MODEL

The model considered in this study is shown in Fig. 1. The naturally fractured medium is represented by means of a system of horizontal equally spaced parallel fractures, alternated with matrix blocks. The system shown in this figure consists of two flow regions: a) a mobile

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region constituted by the fracture network and b) a stagnant or immobile region. Both regions are interconnected by means of a thin fluid layer contained within the immobile region, which controls the fluid and mass transfer between the regions. This type of visualization of the problem by means of two regions has been used by other authors in the past (Deans, 1963; Walkup and Horne, 1985; Maloszewski and Zuber, 1985; Chen, 1986; Rivera et al., 1987; Ramírez et al., 1988 and 1990).

In the mobile region the following effects are considered:

a) Longitudinal dispersion that includes molecular difussion:

$$D_r = \alpha v + D_{mr} \tag{1}$$

The Z direction is not considered because it is assumed that fracture with 2w is small and, consequently there is no concentration gradient in this direction.

b) Convection. Based upon the discussion presented in a), flow velocity in the Z direction is assumed to be uniform and only its variation is considered along the r direction. For the case of this study of radial flow under constant rate injection, velocity is defined as

$$v = \frac{a}{r} \tag{2}$$

where

$$a = \frac{Q}{4\pi(w-\delta)} \tag{3}$$

c) Decay. This condition is considered for the case of a radioactive tracer of decay time less than the transit (travel) time.

For the immobile region the following effects are considered:

a) Difussion. This effect is only considered in the Z direction, because the longitudinal component is assumed to be negligible.
b) Adsorption
c) Decay

Based upon the above mentioned assumptions, considering an incompressible fluid, the governing equations for tracer concentrations in the fracture and in the porous matrix can be stated as follows:

a) Fractures:

$$\frac{\partial C_{1D}}{\partial t_D} = \frac{1}{r_D}\frac{\partial^2 C_{1D}}{\partial r_D^2} - \frac{1}{r_D}\frac{\partial C_{1D}}{\partial r_D} - \gamma C_{1D} + \frac{D_{2D}}{Z_{0D}}\left.\frac{\partial C_{2D}}{\partial Z_D}\right|_{Z_{0D}} \tag{4}$$

b) Matrix

$$\frac{\partial C_{2D}}{\partial t_D} - RD_{2D}\frac{\partial^2 C_{2D}}{\partial Z_D^2} + \gamma C_{2D} = 0 \tag{5}$$

where the definitions for the dimensionless groups that enter into these equations are

$$t_D = \frac{at}{\alpha^2} \tag{6}$$

$$C_{1D} = \frac{C_1 - C_1}{C_0 - C_1} \tag{7}$$

$$C_{2D} = \frac{C_2 - C_1}{C_0 - C_1} \tag{8}$$

$$Z_D = \frac{Z}{\alpha} \tag{9}$$

$$r_D = \frac{r}{\alpha} \tag{10}$$

$$D_{2D} = \frac{D_2}{a} \tag{11}$$

$$\gamma = \frac{\lambda\alpha^2}{a} \tag{12}$$

$$R = \frac{\phi_2}{\phi_2 + \rho K(1-\phi_2)} \tag{13}$$

The last term of Eq. 4 considers the interaction between the fractures and matrix systems, representing a difussion mass transfer from the fractures to the matrix at $Z_{0D} = (w-\delta)/\alpha$.

The equations that complete the model are given by Eqs. 14-19.

Boundary conditions

$$C_{1D}(r_{0D}, t_D) = 1 \tag{14}$$

$$C_{2D}(r_D, Z_{0D}, t_D) = C_{1D}(r_D, t_D) \tag{15}$$

$$C_{1D}(\ , t_D) = 0 \tag{16}$$

$$\left.\frac{\partial C_{2D}}{\partial Z_D}\right|_{(r_D, E/2\alpha, t_D)} = 0 \tag{17}$$

Initial conditions

$$C_{1D}(r_D,0) = 0 \qquad (18)$$

$$C_{2D}(r_D,Z_D,0) = 0 \qquad (19)$$

To find the solution to this problem, the Laplace transformation method was used. The resulting equations after the application of this method to Eqs. 4 and 5 were coupled, yielding:

$$\frac{d^2\bar{C}_{1D}}{dr_D^2} - \frac{d\bar{C}_{1D}}{dr_D} - \xi r_D \bar{C}_{1D} = 0 \qquad (20)$$

where

$$\xi = S+\gamma+\frac{D_{2D}}{Z_{OD}}\sqrt{\beta}\tanh\left\{\sqrt{\beta}\left(\frac{E}{\alpha}-2Z_{OD}\right)\right\} \qquad (21)$$

$$\beta = \frac{S+\gamma}{RD_{2D}} \qquad (22)$$

with the following boundary conditions:

$$\bar{C}_{1D}(r_{OD},S) = \frac{1}{S} \qquad (23)$$

$$\bar{C}_{1D}(\ ,S) = 0 \qquad (24)$$

To express Eq. 20 in an easier form, the following expressions can be used (Tang and Babu, 1979):

$$y = r_D + \frac{1}{4\xi} \qquad (25)$$

$$\bar{C} = \bar{C}_{1D}\exp(-y/2) \qquad (26)$$

$$x = \xi^{1/3}\, y \qquad (27)$$

The result is the standard Airy equation (Abramowitz and Stegun, 1970):

$$\frac{d^2\bar{C}}{dx^2} = x\bar{C} \qquad (28)$$

The general solution of Eqs. 20 and 21-28 is

$$\bar{C}_{1D} = \frac{1}{S}\exp\left(\frac{y-y_0}{2}\right)\frac{A_i(\xi^{1/3}\,y)}{A_i(\xi^{1/3}\,y_0)} \qquad (29)$$

where

$$y_0 = r_{OD} + \frac{1}{4\xi}$$

Eq. 29 gives the tracer concentration, chemical or radioactive, in the fractured region for the case of continuous injection.

A solution for the finite step injection case, may be obtained through the use of Eq. 29 and the principle of superposition.

METHOD OF SOLUTION

The Airy functions $A_i(x)$ that enter into Eq. 29 were computed according to (Abramowitz and Stegun, 1970, p. 448, Eq. 10.4.59).

For small times, Z>4.8:

$$A_i(Z) = \frac{1}{2}\pi^{-1/2} Z^{-1/4} e^{-\xi}\sum_{K=1}(-1)^K C_K \xi^{-K} \qquad (30)$$

where

$$\xi = \frac{2}{3}Z^{3/2}$$

$$C_0 = 1$$

$$C_K = \frac{\Gamma(3K+1/2)}{54^K K!\,\Gamma(K+1/2)}$$

For $-5.0 \le Z \le 4.8$, $A_i(x)$ can be computed according to (Abramowitz and Stegun, 1970, p. 446, Eq. 10.4.2)

$$A_i(Z) = C_1 f(Z) - C_2 g(Z) \qquad (31)$$

where

$$f(Z) = \sum_{K=0} 3^K\left(\frac{1}{3}\right)_K \frac{Z^{3K}}{(3K)!}$$

$$g(Z) = \sum_{K=0} 3^K\left(\frac{2}{3}\right)_K \frac{Z^{3K+1}}{(3K+1)!}$$

$$(\alpha+1/3)_0 = 1$$

$$3^K(\alpha+1/3)_K = (3\alpha+1)(3\alpha+4)\dots(3\alpha+3K-2)$$

and $C_1 = 3^{-2/3}/\Gamma(2/3) = 0.3550280538$,

$$C_2 = 3^{-1/3}\,\Gamma(1/3) = 0.2588194037$$

where α for this problems is equal to 0.

An excellent comparison with the tabulated values of the Airy function of Abramowitz and Stegun (1970, p.475, Table 10.11) was observed keeping 20 terms each of f(Z) and g(Z).

For the inversion of Eq. 29 the algorithm of Crump (1976) was used, obtaining excellent results for the range of parameters used; this will be discussed later in this paper. The Stehfest numerical inverter algorithm (1970) was also used for the inversion of Eq. 29, finding important numerical dispersion problems for large t_D's or large r_D's .

VALIDATION OF MODEL

The solution of the model of this study was compared in its homogeneous porous media version with those of Moench and Ogata (1981) and Hsieh (1986). It is important to keep in mind that the differential equation that describes the tracer flow problem of these two papers and the one used in this work are the same, the only change being the method of solution. The continuous tracer concentration solutions of these authors are expressed in the Laplace space by means of Airy functions. Moench and Ogata numerically invert their solution using the Stehfest algorithm and Hsieh's solution is of integral type. It can be demonstrated that these solutions correspond to particular cases of the model of this study. For the case of small diffusion coefficient and porosity of the matrix (Stagnant) region, $(D_{2D}/Z_o)\sqrt{(S+\gamma)/(RD_2)} \cong 0$, the naturally fractured porous media approaches homogeneous behavior and, the proposed model simplifies to a homogeneous model. In other words, for these conditions Eq. 21 reduces to = S.

Table 1 shows a comparison of the results presented by Hsieh and those of this study, for dimensionless times t_D equal to 50 and 100, r_{wD}= 10 and different dimensionless radial distances r_D. It can be observed, in light of a comparison with the analytical solution of Hsieh that the accuracy of the results of this model is very good.

With regard to the naturally fractured case, the solution of this study will correspond to that of Chen (1985), provided that the matrix region behaves under transient conditions (the matrix region be of infinite extent for the times of interest).

For these conditions, Eq. 21 becomes,

$$\xi = S + \left(\frac{SD_{2D}}{Z_{oD}^2 R} \right)^{1/2}$$

Fig. 2 presents a comparison of results obtained by means of the continuous tracer concentration analytic and approximate solutions of Chen and those of this study. It can be observed that agreement is excellent.

DISCUSSION OF RESULTS

Next, a presentation of results for the continuous tracer concentration solution will follow. The data used to get the results are those of Chen (1986), where the constant injection rate Q is equal to 0.01 m^3/d.

Fig. 3 shows results of tracer concentration versus radial distance for dimensionless times of 1, 5 and 10. The concentration level gradually decreases with radial distance.

Fig. 4 presents results of tracer concentration versus radial distance, for values of the dimensionless well radius of 0.5, 1.0, 1.5, and 2.0, for a dimensionless time of 1. These results are similar to those presented by Hsieh (1986) for the homogeneous case, showing that tracer concentration response is strongly affected by the wellbore radius.

Fig. 5 shows a graph of tracer concentration versus time for a wellbore radius of 0 and for a radial distance of 1. This graph can be compared to the tracer concentration results of Ramírez et al. (1990) for linear flow, not shown in this paper, concluding that a uniqueness problem may arise if the test interpretation were to be conducted without additional information coming from other sources (i.e., geological, core analysis, well logs, etc.).

Fig. 6 presents results of tracer concentration for the finite-step case, for injection times of 0.3 and 0.5. As expected, it is observed from results of this figure that as the injection time increases, so does the maximum tracer concentration and, that the time at which this maximum is attained is larger than the injection period, due to the dispersion effects. Again, as previously discussed with regard to Fig. 5, a uniqueness problem may arise if the test interpretation were to be conducted without additional information coming from other sources (i.e., geological, core analysis, well logs, etc.).

CONCLUSIONS

The main objective of this work has been to present a solution for the radial flow of a tracer in a naturally fractured reservoir.

The main conclusions of this study are as follows:

1. A model is presented for the radial flow of a tracer in naturally fractured reservoirs. It considers the following mechanisms: diffusion, convection, adsorption, and radioactive decay.

2. The Crump algorithm was found to be superior for the numerical inversion of the solution of this study to that provided by the Stehfest algorithm.

3. A thorough validation of the model against simplified and similar solutions published in the literature was carried out.

4. Solutions are presented for the continuous and finite step injection cases.
5. A comparison of radial and linear tracer flow continuous and finite-step injection solutions indicates that a uniqueness problem may arise in the interpretation of a test.

NOMENCLATURE

a = advection parameter, Eq. 3
$A_i(x)$ = Airy function

C = tracer concentration
$\bar{C}$ = parameter, Eq. 26
C_D = dimensionless tracer concentration Eqs. 7 and 8
D_{mr} = molecular diffusion coefficient, L^2/T
D_r = longitudinal dispersion coefficient, Eq. 1, L^2/T
E = thickness of the symmetry element, L
K = adsorption constant, L^3/M
Q = constant injection rate, L^3/T
r = radial distance, L

R = dimensionless parameter, Eq. 13
t = time, T
v = fluid velocity, L/T
w = fracture half-width, L
x = parameter, Eq. 27
y = parameter, Eq. 25
Z = vertical coordinate, L
Z_{OD} = dimensionless effective fracture half-width

Greek simbols

α = dispersivity of fracture, L
β = parameter, Eq. 22
γ = dimensionless parameter, Eq. 12
δ = Stagnant fluid film thickness, L
= parameter, Eq. 21

Subscripts

D = dimensionless
i = initial
1 = mobile or fractured region
2 = immobile (fluid layer and porous matrix) region.

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TABLE 1. COMPARISON OF THE HOMOGENEOUS RESULTS OF HSIEH (1986), G. AND THOSE OF THIS STUDY, C_D.

$t_D = 50$			$t_D = 100$		
r_D	G	C_D	r_D	G	C_D
11.0	0.964	0.96412	11.0	0.993	0.99301
12.0	0.892	0.89231	13.0	0.949	0.94875
13.0	0.775	0.77523	14.0	0.900	0.90010
14.0	0.617	0.61714	16.0	0.724	0.72402
15.0	0.439	0.43925	18.0	0.463	0.46298
16.0	0.273	0.27294	20.0	0.213	0.21295
17.0	0.145	0.14493	22.0	0.065	0.06501
18.0	0.645	0.64501	23.0	0.030	0.03005
19.0	0.023	0.02207	25.0	0.004	0.00397

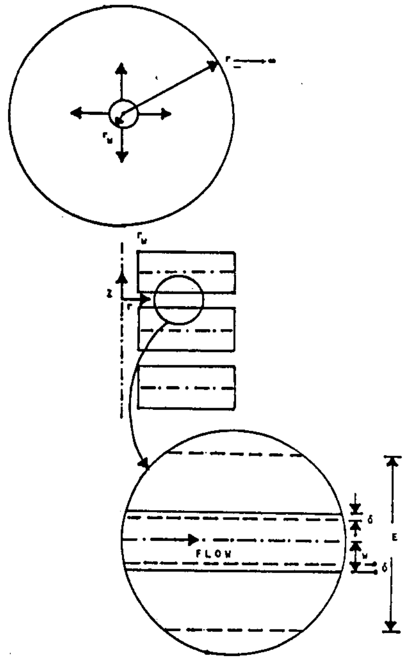

Fig. 1. Proposed Model for representation of the naturally fractured medium.

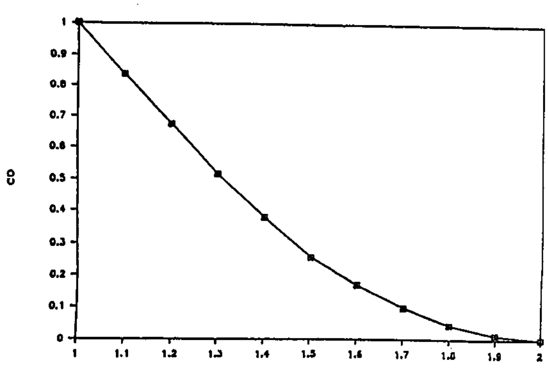

Fig. 2. Comparison of the continuous tracer concentration solutions, for naturally fractured reservoirs, of Chen (1986) and of this study.

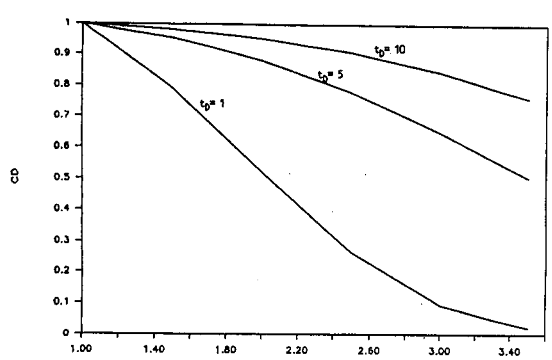

Fig. 3. Continuous injection tracer concentration vs radial distance for various values of t_D.

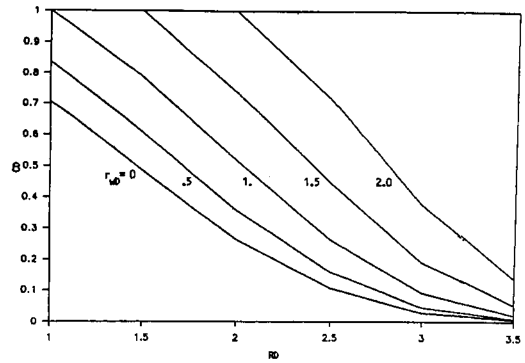

Fig. 4. Continuous injection tracer concentration vs radial distance for various values of the dimensionless wellbore radius, for t_D=1.

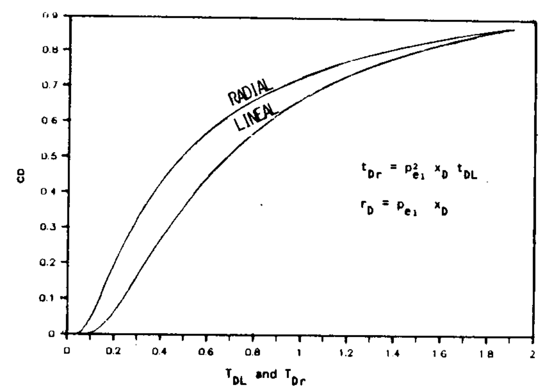

Fig. 5. Comparison of the radial and linear continuous tracer concentration responses for equivalent matrix-fracture system characteristics.

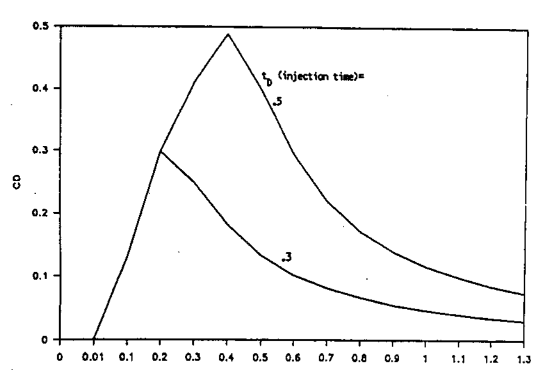

Fig. 6. Influence of the injection period on the finite-step injection tracer concentration.