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SOME APPROACHES TO ROCK MASS HYDROFRACTURE THEORY

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ABSTRACT

A new engineering method has been developed at the Leningrad Mining Institute for defining hot dry rock hydrofracturing parameters. It reflects the structural features of a real jointed rock mass, its gravity-tectonic components of the stress tensor and volume character of deformations, taking into account the inertial effects of hydrodynamics in the non-Darcy zone of radial fluid flow near the injection well, and conversion of the heat energy extracted from hot rock by circulating water partly into filtration-flow additional pressure. Results of calculations are compared to field experiments at Fenton Hill, NM, and are used for the first HDR circulation systems in the USSR.

INTRODUCTION

The study of hydrofracture mechanics became popular in the 1950-60's when hydrofracturing was widely used in the oil and gas industry to improve reservoir permeability. By this time, several theories of hydrofracture essentials had been formed; some are found in the investigations by S.A. Khristianovich, G.I. Barenblatt and Yu.P. Zheltov (1975), by C. Fairhurst and B. Haimson, and others by A. Inglend and A. Green, T. Perkins and L. Kern, P.Fox and A. MacDonald given by Alexeev (1987) and Haimson (1978).

Because of the great difficulties entailed in the attendanthydro-geomechanical problem, the first solutions were obtained for an idealized rock mass as a homogeneous linear elastic medium loaded by constant liquid pressure distributed from a flat fracture and with assumed absence of resistance of the above medium to destruction (e.g., Dyadkin, 1989; Alexeev, 1987; Zheltov, 1975). Application of available calculation methods appeared difficult as noted by Usachev (1986).

Renewed interest in hydrofracture theory occurred in connection with the creation of artificial circulation systems for geothermal energy extraction from hot rock in the USA, Great Britain, Japan, Germany, and France (Dash and Murphy, 1985; Garnish 1987; Tester, et al, 1989). Table 1 lists some of the available water-pressure data from measurement after the pumping unit, P_p, and water flow rate, Q, at various depths, H, and initial rock temperatures, T. The following data from Experiment 2032 by the Los Alamos National Laboratory on hydrofracturing of granodiorite mass by water at H = 3550 m and T = 240 °C at Fenton Hill area in New Mexico were used for the illustration described later (all dimensions in SI): rock density $\rho = 2700$, Young's modulus $E = 5x10^{10}$, shear modulus $G = 2.65x10^{10}$, Poisson ratio $\upsilon = 0.257$, permeability $k = 10^{-18}$, total rock pressure $q = 91.7x10^{6}$, pore liquid pressure $P_o = 35.06x10^{6}$, vertical stress component $\sigma(z) = q-P_o$, minimum horizontal component of stress tensor $\sigma(x) = 32.15x10^{6}$, maximum component of horizontal stress $\sigma(y) = 48.2x10^{6}$, compressive rock strength $[\sigma(c)] = 175x10^{6}$, uniaxial tension strength $[\sigma(t)] = 15x10^{6}$, structural weakness coefficient $\phi = 0.5$, water density $\rho(w) = 1032$, water viscosity $\mu(w) = 0.95x10^{-3}$, temperature extension coefficient $d_w = 1.5x10^{-4}$, water elasticity modulus $E_w = 0.22x10^{10}$, injection rate $Q_w = 0.111$ m/s, length of well open interval h = 22, radius of well cross-section $R_w = 0.125$, water pressure in well open interval at hydrofracture $P_w = 83x10^{6}$, water pressure after pumping unit $P_p = 42x10^{6}$, total hydrofracturing duration $\Sigma(t_f) = 62$ hr, size of sub-vertical fractured zone ≈ 1150 x 800 x 150, and fracture opening (aperture) $\delta_o = 2.3$ mm.

In the next approach, Haimson (1978) recognized the need to take rock strength into account. This was regarded as a hydro-mechanical task, i.e., the rock mass elasticity deformation and friction forces at viscous liquid flow in fractures were considered simultaneously (Geerstma and Klerk, 1969). Satisfactory solutions for determining filtration leakage to not appear to exist, although this effect caused great difficulties in the cited geothermal experiments. Investigations by Dyadkin (1985, 1989) showed that several other factors must be taken into account: filtration leakage at fissure development of hydrofracturing in a permeable rock mass overcoming the pore pressure; inertia effects of turbulent flow in the near well zone; rock strength energy criteria; and thermal expansion of the fluid in the rock mass. These factors were not taken into account by other investigators, although they are very important for geothermal hydrofractures.

PROCEDURE DEVELOPMENT

The practicality of the proposed calculation procedure is shown by comparison of the estimated design values with the observed results of field experiments (as listed in Table 1). To avoid repetition of previously published sections of the calculation procedure given in Dyadkin (1985, 1989) for developing a Physical Model for hydrofracturing of the jointed rock mass, it is necessary only to define the procedure for two main parameters: (1) critical pressure at the fracture border P_b and (2) maximum fracture opening δ_o near the well for typical component relations of gravity-tectonic stress field (Haimson, 1978):

$$\sigma_x < \sigma_z < \sigma_y \qquad (1)$$

when the stress difference $\sigma_y - \sigma_x$ "supplies" the difference in anisotropy rock mass destruction resistance in various directions and the hydrofracture develops in vertical plane "yz" on the normal to minimum stress σ_x. Evidently, the critical liquid pressure P_b at the border of the growing vertical fracture totally discharges the rock mass element from compressive stress and at same time overcomes not only pore liquid pressure P_o, but also the rock mass resistivity to breakage ΔP_r under the condition of volume deformation. At the first stage of this investigation, it was assumed that the uniaxial tension strength $[\sigma_t]$ is an indicator of this resistivity. This may result in too large an increase in pressure P_b as the action of compression stress σ_y and σ_z reduces rock resistivity. The greatest slacking effect is considered by the Griffith energy criterium of strength $[\sigma_t]^*$ for plane deformation conditions of "tension- compression". The condition under which this slackening effect is insignificant may be taken as:

$$\sigma_y < 3\,[\sigma_t]_x \qquad (2)$$

The above condition being formulated as $[\sigma_t]^*_x \approx [\sigma_t]_x$. Rock of great strength at small depth corresponds to condition (2). For instance, frozen-rock hydrofractures at depths of 7-10 m are described in Dyadkin (1989). In jointed rock masses at deep horizons and at intensive tectonic stress, condition (2) is not observed and Griffith's criterion value may be found in his equation relating σ_y with $[\sigma_t]_x$, indicated by Baklashov (1988). The solution obtained for the equation is:

$$[\sigma_t]^*_x = \sqrt{a_x^2 + \sigma_y(8[\sigma_t]\phi - \sigma_y)} - a_x \qquad (3)$$

where

$$a_x = 4\,[\sigma_t]_x\phi + \sigma_y \qquad (4)$$

ϕ = coefficient of rock mass structural weakness, which for hard rock, may range from 0.5 to 0.03 (Baklashov,1988; Turchaninov, et al, 1989).

The slackening effect of the second compression stress σ_z may be considered by the VNIMI correction coefficient (Petukhov and Linkov, 1986), illustrating the peculiarities of volume development of deformation:

$$k_v = 1 - \sigma_z / [\sigma_c]\,\phi \qquad (5)$$

The part of liquid pressure to overcome the resistance of rock mass is expressed as

$$\Delta P_{r,v} = [\sigma_t]_x^* \{1 - \sigma_z/[\sigma_c]\phi\} \qquad (6)$$

and condition of hydrofracture at the border of the growing vertical fracture may be shown as

$$P_{b,v} = P_o + \sigma_x + \Delta P_{r,v} \qquad (7)$$

The important influence of the relaxing intensities of the slacking effect under the above conditions is illustrated by LANL Experiment 2032. For these values, it is possible to get $[\sigma_t]^* = 3.55\text{x}10^6$ and $k_v = 0.372$. That is, instead of 15 MPa for monolith rock specimen at uniaxial tension strength, the resistance of weakened rock mass under the conditions of vertical hydrofracture deformation will be only $\Delta P_r = 1.32\text{x}10^6$.

Figure 1 shows the jointed mass structure for granite, typical for granodiorite and other hard rocks. The main joints system with their azimuth α_n (the angle to maximum stress direction σ_y) at sloping angle β_n. The second joints system crossing of the first and has angles α_s and β_s. These systems, together with sub-horizontal joints, separate the structural blocks with length l_b, width a_b, and thickness t_b. Joints of the first system are partially filled by different second

TABLE 1

Results of LANL field experiments on hydrofracturing of hot granodiorite at Fenton Hill for comparison with LMI calculated data for designing USSR Geothermal Circulation Systems

Number and Date of Experiment Type of rock	Depth (m)	Temp. (°C)	Pump unit parameters Pressure (Mpa)	Flowrate (m^3/s)	Time (hr)	Total Water Volume (m^3)
2016 19 Jun 82 granodiorite	4252	282	46.6	0.093		4883
2020 6-7 Oct 82 granodiorite	3656	245	46.9	0.092		3090
2023 8 Nov 82 granodiorite	3162-4084	270	12.9	0.017		150
2032 11-12 Dec 83	3528-	240	48.0	0.114	61	21200
2042 15-19 May 84	3588	245	41.4	0.026		7600
2061 29 Jun 85	3827-4017	290	31.0	0.011		5230
2066 30Jan-2Feb 86	3914	260	46.2	0.019		3770
Russkie Komarovtsy, Uhzgorod, Ukraine 1988 granodiorite	2240	124	19.0	0.44	19	10500
Cholpon-Ata, Lake Issik-Kul 1988 granite	2400	91	11.0	0.022	50	12000
Tirniaus, Elbrus Mtns 1989 granite	3300	185	28.9	0.044	88	18000
Palanga, Lithuania 1990	4300	141	25.7	0.07	15	3870

mineralizations and define the rock mass fracture permeability k, the greatest structural weakness coefficient ϕ_n; they also define the rock mass fracture porosity m and natural porous liquid pressure P_o. The joints of the second system are considered closed and provide only a small contribution to the rock mass characteristics given above.

In a rock mass containing a vertical hydrofracture, there is the possibility of shear deformation development and displacement of neighboring structural blocks along joints α_s β_s of the second system under irregular loads by injecting liquid pressure from permeable joints α_n β_n of the first system. This kind of hydro-fracturing mechanism is possible even without complete discharge of joints α_s β_s from the normal compressive pressure $\sigma_{n,s}$ if the active pressure loading along the block width a_6 is enough for Coloun's shear condition and block displacement on the long side l_b along second joints α_s β_c:

$$P_{b,s} = 2\frac{a_b}{l_b}([\tau_o]\varphi_s + \sigma_{n,s}tgf) \qquad (8)$$

where tgf, $[\tau_o]$ and ϕ_s are friction coefficient, pure shear resistivity, and the coefficient of structural weakness along the second joints system, the values of which may be approximately estimated by the experimental data of Turchaninov, et al (1989), and $\sigma_{n,s}$ is the compressive stress by normal of secant joints:

$$\sigma_{n,s} = \sin(\beta_s)(\sigma_x \cos(\alpha_s) + \sigma_y \sin(\alpha_s)) + \sigma_z \cos(\beta_s) \quad (9)$$

Evidently, in the case of complete discharge of joints of the main system from normal compressive stress σ_n by injecting liquid pressure P_B, the third version of hydrofracturing mechanics is possible, when the main joints are reopened after overcoming of the weakened contact resistivity :

$$P_{b,n} = P_o + \sigma_{n,t} + [\sigma_t]_n^* (1 - \sigma_a/[\sigma_c]) \Phi \quad (10)$$

where

$$[\sigma_t]_n^* = \sqrt{a_n^2 + \sigma_h (8 [\sigma_t] \varphi_n - \sigma_h} - a_n \quad (11)$$

$$a_n = 4[\sigma_t] \varphi + \sigma_h \quad (12)$$

The values σ_n, σ_a, and σ_h are connected with components of the stress field and the orientation of the joint contacts:

$$\sigma_n = \sin(\beta_n)(\sigma_x \cos(\alpha_n) + \sigma_y \sin(\alpha_n)) + \sigma_z \cos(\beta_n) \quad (13)$$

$$\sigma_a = \cos\beta_n \sqrt{(\sigma_x \sin\alpha_n)^2 + (\sigma_y \cos\alpha_n)^2} - \sigma_z si \quad (14)$$

$$\sigma_n = \sqrt{(\sigma_x (\cos\alpha_n \cos\beta_n)^2 + (\sigma_y \cos\alpha_n \sin\beta_n)^2} \quad (15)$$

Like a filtration wedge advancing the hydrofracture border in sedimentary strata with constant (independent of direction) average permeability, injected liquid will flow along the permeable main joints advancing the breaking border and partially discharging rock mass from normal compressive stress. The length of this advancing discharging filtration zone depends on liquid viscosity μ_w and flowrate inside fracture Q_f, its average radius R_f, aperture (opening) near border δ_b, joint permeability (much greater than value of rock mass average level) and pressure difference:

$$l_f = 0.5\, \delta_b \exp((\pi R_f k_n / Q_f \mu_w)(P_{b,n} - P_o)) \quad (16)$$

A specific zone of complete discharge from normal compressive stress exists ahead of the fracture border:

$$l_t = 0.5\, \delta_b \exp((\pi R_f k_n / Q_f \mu_f)(P_{b,n} - P_o - \sigma_n)) \quad (17)$$

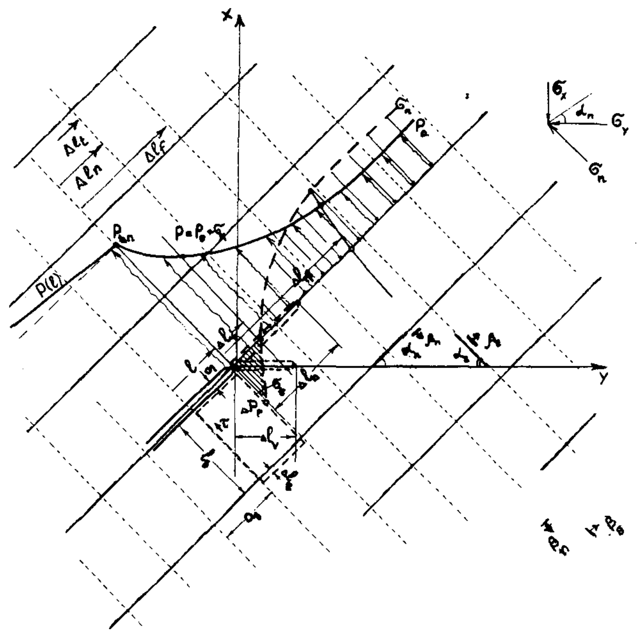

Fig. 1 Distribution of liquid pressure in fracture and in advanced filtration wedge from $P_{b,n}$ to P_o and stress level from δ_n to ΔP_r during hydrofracture of jointed rock mass in version of vertical crack propogation Δl_v, joint reopening Δl_n, or structual block shear displacement Δl_s.

Section l_t is a zone of tensile stress development, increasing together with injection liquid pressure, up to the level of $k_v[\sigma_t]_n^*$ before a jump-like break and hydrofracture border moves at value Δl_n.

The most likely hydrofracturing mechanics version corresponds to the minimum value among possible calculated levels: $P_{b,v}$, $P_{b,s}$, and $P_{b,n}$. For instance, in the cases of Table 1, it is noted that for the conditions of the Russkie Komarovtsy project, separate vertical tension fractures seem to be more likely, while at the Cholpon-Ata site, sub-vertical volume shear-fracturing zone is expected, and in the case of the Tirniaus project, the initial data for both of these versions appear to be equally likely.

At the Leningrad Mining Institute, it has been recommended (Dyadkin, 1985) that the definition of the very important parameter, fracture opening (aperture), near the well be adopted from Geerstma and Klerk (1969):

$$\delta_w = 2 \sqrt[4]{\frac{(1-\nu)}{G} (\mu_w Q_w R_f)} \quad (18)$$

For the conditions of LANL experiment 2032, equation (18) yields $\delta_o = 2.27$ mm, compared to the value of $\delta_o = 2.3$ mm reported for the experiment by Dash and Murphy (1985) using several different calculating methods. But the regularity of such coincidence is dubious, because in the LANL experiments subvertical fractured zones were formed around the well, cutting across its inclined axis. Equation (18) appears only for the case of hydrofracture along the vertical well axis with rectangular uni-dimensional filtration flow. The same principles of hydrofracturing geometry were solved by Perkins and Kern (1961), Inglend and Green (Alexeev,1987), and Barenblatt (Zheltov,1975), Nordgren (1972), and Geerstma and Haafkens (1979).

The structure of equation (18) does not take into account either of two factors which acquire a special meaning under hydrofracturing of hot hard rock at considerable depth. These factors are: (1) rock strength (or more precisely, their resistivity to break, ΔP_r), and (2) liquid pressure increase ΔP_T as its temperature T_w rises. It is noted that equation (18) considers the filtration process as laminar unidimensional flow. However, for HDR systems, the more typical case is the creation of sub-vertical hydrofractured zones from inclined wells with radially diverging flow of injected liquid as shown from Dash and Murphy (1985), Dyadkin (1989), Garnish (1987) and Tester, et al (1989). For this case and laminar flow Q around the well with cross-sectional radius R, the liquid pressure drop in the fracture aperture. The average radius R can be written as

$$\Delta P_{H,D} = 6\,(Q_w \mu_w / \pi \delta^3)\,\ln(R_f/R_w) \qquad (19)$$

Assuming the suggestion in Geerstma and Klerk (1969) and Geerstma and Haafkens (1979) on aperture opening being constant for the main part of the fracture (its change in the zone near a well is discussed later), we obtain a value for rock mass elastic deformation as being the fracture aperture near the well, δ_{HD}, within the laminar regime of radial flow Q for liquid viscosity μ_w:

$$\delta_{HD} = 1.175 \sqrt[4]{\frac{(1-\nu)}{G} \mu_w Q_w R_f \ln\frac{R_f}{R_w}} \qquad (20)$$

Under conditions of the discussed example, we obtain $\Delta P_{HD} = 0.11 \times 10^6$ and $\delta_{HD} = 2.46$ mm. Note that for well radius $R_w = 0.125$ m , the referred fracture opening and liquid injection flow rate $Q_w = 0.111$ m/s, the radial speed at the fracture entrance $R = R_w$ and at distances of 1 m and 10 m from the axis will be 57.7, 7.2 and 0.72 m/s ,respectively. At such high speeds near the well, forces of inertia play an important role, Darcy's law is irrelevant, a turbulent flow zone occurs, as fixed, for instance, in experiments on hydrofracture of granites at Falkenberg (Garnish, 1987). The radius of inertia effects zone is calculated by the following equation:

$$R_{in} = \rho_w Q_w^2 / \pi \mu_w Re^* \qquad (21)$$

where the critical Reynold's number $Re^* = 1,500$ is taken from Pavlov's experimental data (Dyadkin, 1985,1989). Pressure losses in this zone (in our case $R_{in} = 25.6$ m) can be writen as $k_{in}\,\Delta P_{HD}$, where the coefficient, accounting for the influence of inertia effects, cannot be less than 1:

$$k_{in} = 0.286 \sqrt{\frac{R_f Re^* \mu_w}{\rho_w Q_w}} \sqrt[4]{\frac{R_f Re^*}{R_w \ln\frac{R_f}{R_w}}} \geq 1 \qquad (22)$$

and value of rock mass elasticity deformation from fracture that provides the turbulent flow in zone of inertia effects can be defined as

$$\delta_{in} = 0.128 \sqrt[4]{\frac{(1-\nu)\,\rho_w^2 Q_w^3}{G R_w \mu_w Re^*}} \qquad (23)$$

Under the discussed conditions we obtain $\delta_{in} = 2.8$ mm and $\Delta P_{in} = 1.95 \times 10^6$, which exceeds ΔP_{HD} by 1.95/0.11 = 17.7 times. This indicates that ignoring flow turbulence in the fracture may lead to serious errors. In our case, with very large value of $Q_w = 0.111$ m/s and $R_f \approx 400$ m, equation (22) gives $k_{in} = 17.7$, which corresponds to the above mentioned correlation. Note that as R_f decreases to the radius of inertia effects zone $R_f = R_{in} = 25.6$ m, the value of K_{in} decreases to 2.45; at $R_f = 10$ m $k_{in} = 1.3$, and for $R_f = 7$ m, the calculated result of equation (22) appears to be less than 1 and it must be assumed that $K_{in} = 1$ and $\Delta P_{in} = \Delta P_{HD}$.

It was already noted that equation (18) does not take into account the rock mass mechanical resistance, although for this case, the value of $\Delta P_b = 1.32 \times 10^6$ is quite comparable with ΔP_{in} and much more than ΔP_{HD}.

It is also necessary to take into account the effect of liquid pressure increasing as a result of its being heated by hot rock in the fracture (Dyadkin,1985,1989). This positive pressure effect may be estimated as

$$\Delta P_T = k_p\, \alpha_w\, E_w\, \Delta T \qquad (24)$$

where the coefficient k_p shows what part of the total heat energy obtained by the liquid flow from rock is being converted into pressure increase, assuming that the major part of this energy is consumed as work of filtration leakage outside the fracture and also goes to elastic and plastic deformation of the rock mass. As a first approach, it is possible to estimate the value of k_p as a ratio of the liquid volume V_w inside the well and fracture V_f to the total volume of injected liquid ΣV_w, including V_w, V_f, and leakage in the advancing filtration wedge ΔV_b and in two equal filtration zones on both sides of fracture $2\Delta V_f$:

$$k_p = \frac{V_w+V_f}{V_w+V_f+\Delta P_b+2\Delta V_f} \qquad (25)$$

where values of ΔV_b and ΔV_f are defined by the method developed in Dyadkin (1985,1989). For conditions of LANL Experiment 2032, it is possible to find for $V_w + V_f \approx 170$ m^3 and $\Sigma V_w = 21600$ m^3, the approximate value of $k_p = 0.034$. Taking for water $\alpha_w = 1.5 \times 10^{-4}$ and $E_w = 0.22 \times 10^{10}$ by equation (24) we obtain for the average temperature difference $\Delta T = 180$ °C, that $\Delta P_T = 2.02 \times 10^6$, which slightly exceeds ΔP_r and is completely comparable with ΔP_{in}.

The total change of liquid pressure in the fracture can be written as

$$\Delta P_o = k_{in}\,\Delta P_{HD} + \Delta P_r - \Delta P_T \qquad (26)$$

The interaction of all discussed factors can be taken into account by using the "strength-thermal" coefficient:

$$k_\delta = 1 + \frac{\Delta P_r - \Delta P_T}{k_{in}\Delta P_{HD}} = 1 + \frac{\Delta P_r - \Delta P_T}{\Delta P_{in}} \qquad (27)$$

In our case, at $\Delta P_r = 1.32 \times 10^6$, $\Delta P_T = 2.02 \times 10^6$, and $\Delta P_{in} = 1.95 \times 10^6$, we obtain $K_\delta = 0.641$. Now correcting equation (23) by function (27), we can finally write:

$$\delta_o = \delta_{in} \sqrt[4]{k_\delta} \qquad (28)$$

For this example, we have $\delta_{in} = 2.8$ mm from equation (23) and function $k_\delta = 0.641$, and we obtain $\delta_o = 2.5$ mm.

SUMMARY

Thus, in consequence of the transfer from laminar uni-dimensional to radially divergent flow resulting in turbulence, the pressure increase caused by heating of the liquid in the fracture, and the influence of jointed rock massive mechanical resistivity under volume deformation, together overcoming the compressive stress of the gravity-tectonic field, the computation result of equation (18) changed somewhat (with mutual compensation of subordinate factors), but the calculated values remained rather close to the experimental values.

Naturally, in the case of a massive hydrofracture forming a vertical crack of constant height along the well axis, the value of δ_o should be estimated by equation (18) and for approximate appreciation of above factors, it is possible to use as the correction coefficient the fourth root of k_δ in (18).

Both equation (23) and the coefficient k_p require careful definition in the approximate solution given in equation (28). Continuous experimental examination of thermal-pressure effects in the Leningrad Mining Institute program yielded recent results of $\Delta P_T = 0.16$ MPa/°C in the experiments with impermeable plexiglass. The resulting coefficient has an approximate value of 0.5 as a result of elasticity deformation only.

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