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ESTIMATION OF SINGLE INJECTOR TRACER TEST IMPULSE RESPONSES

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ABSTRACT

Interpretation of tracer tests presupposes a single tracer injection mass history – ideally a single slug of tracer. Unfortunately, tracer injection functions can be very erratic, leading to variations in the recovered tracer concentrations. Under these circumstances it is essential to correct the data for the erratic tracer injection, thereby recovering the impulse or slug response of the injector-producer system.

We discuss a method for estimating the impulse response using the Wiener-Hopf equation. The estimation technique involves forming and solving a Toeplitz matrix approximation of the Wiener-Hopf equation for the impulse response. The elements of the matrix are autocorrelations of the injection tracer mass with respect to time, while the known vector is the cross-correlation of the injected tracer mass and the produced tracer concentrations. The matrix solution is easy to formulate and computationally rapid.

The Wiener-Hopf method has several advantages over other published techniques for correction for tracer reinjection. First, the method can estimate the impulse response in principle no matter how complicated the initial injection tracer mass history. It is also the least-squares estimator of the impulse response.

We illustrate the impulse estimation technique using reactive and non-reactive tracer data gathered at Dixie Valley.

INTRODUCTION

During a tracer test a quantity of tracer is injected into a well and the outflow at other wells is monitored for the presence of the tracer. The measured concentration of the tracer in the monitoring wells as a function of time can contain information about the hydrologic connection and the thermal regime between wells (Robinson, 1985; Robinson et al., 1988). One potential problem in comparing tracer tests or in interpreting the measured concentrations in terms of a deterministic or stochastic model is that the input concentration of tracer may vary from experiment to experiment. This is especially true since in many cases production water will be re-injected and the injected concentration will partially depend on the concentration of tracer in the production wells. Thus it is desirable to standardize tracer tests to a uniform tracer input function. Since the transfer of tracer from well to well must be a causal linear process in a properly designed tracer test, it is reasonable to use the impulse response of the system as the standard of comparing or modelling a tracer test. In this case, we must find effective means of estimating the impulse response of the system from an arbitrary tracer input and a measured output at the production well.

PROBLEM FORMULATION

Suppose that the impulse response of the tracer transfer system is denoted $h(t)$. Then if the input tracer mass as a function of time is $x(t)$ and the produced concentration is $C(t)$,

$$C(t) = \int_0^\infty x(t-\alpha)h(\alpha)d\alpha. \quad (1)$$

We wish to estimate $h(t)$ from this equation given a finite number of noisy measured values of $C(t)$ and $x(t)$.

Robinson (1985) presents a method for estimating $h(t)$, for the case where produced water is re-injected. Briefly, he supposes that the injection tracer concentration consists of an initial slug and subsequent reinjected waters. Thus for $t \geq 0$,

$$x(t) = \delta(t) + \beta(t)C(t), \quad (2)$$

where $\beta(t)$ is some function which expresses the degree to which the reinjected water is concentrated or rarefied. Thus introducing (2) into (1) gives

$$C(t) = h(t) + \int_0^t \beta(t-\alpha)C(t-\alpha)h(\alpha)d\alpha. \quad (3)$$

or

$$h(t) = C(t) - \int_0^t \beta(t-\alpha)C(t-\alpha)h(\alpha)d\alpha. \quad (4)$$

Thus for the case when the data has been normalized to give a unit input slug, the impulse response can be calculated at each time t by evaluating the convolution integral over past values of h. This scheme can be given a discrete form. Suppose that the input concentration at discrete points is

$$x[n] = \delta[n-1] + \beta C[n]$$

for $n = 1,2,\ldots,N$ and output concentration $C[n]$. In this case we have assumed for convenience that the reinjection fraction β is constant as a function of n. Then the discrete version of equation (3) is

$$C[n] = h[n] + \beta \sum_{k=1}^{n} C[n-k+1]h[k]. \quad (5)$$

Solving this equation for $h[n]$ gives

$$h[n] = \frac{1}{1+\beta C[1]} \{C[n] - \beta \sum_{k=1}^{n-1} C[n-k+1]h[k]\}. \quad (6)$$

As in the case of continuous data, the impulse response at a particular time is a function of the impulse response at previous times.

This formulation is concise and appealing. However, it has several possible drawbacks. First, the technique supposes that each measurement is exact, when in fact noise is always present in the data. Second, it presupposes an initial impulse injection, when in fact the actual initial injection concentration may be unlike an impulse. In this paper we seek a technique of estimating the impulse response which will not have these two failings.

We wish to solve equation (1) in a least squares sense, which will ameliorate the effects of data noise on the estimate of h. Following Papoulis (1977, p. 340), the least-squares solution to (1) is the solution to the Wiener-Hopf equation,

$$R_{cx}(\tau) - \int_0^\infty R_{xx}(\tau-\alpha)h(\alpha)d\alpha = 0, \quad (7)$$

for $\tau \ge 0$, where R_{cx} is the cross-correlation of $C(t)$ and $x(t)$ and R_{xx} is the autocorrelation of $x(t)$. This equation has a formal solution for h, which is discussed by Papoulis.

In an actual tracer experiment, data is collected at discrete time intervals. Thus, the discrete version of the Wiener-Hopf equation is (Papoulis, 1977, eq. 10-115),

$$R_{cx}[m] - \sum_{k=1}^{\infty} R_{xx}[m-k+1]h[k] = 0, \quad (8)$$

where $m \ge 0$. Since data is collected at only N points, equation (8) becomes

$$R_{cx}[m] - \sum_{k=1}^{N} R_{xx}[m-k+1]h[k] = 0, \quad (9)$$

where $m = 0,1,\ldots,N-1$. The values of R_{cx} and R_{xx} can be estimated using the formulas

$$R_{rs}[m] = \frac{1}{N-|m|} \sum_{n=1}^{N-|m|} r[n]s[n+m],$$

or

$$R_{rs}[m] = \frac{1}{N} \sum_{n=1}^{N-|m|} r[n]s[n+m],$$

(Oppenheim and Schafer, 1975). These estimators differ only in the value of the anterior multiplier. The first estimator is unbiased but has large variance as m approaches N. The second estimator has a bias which approaches 0 as N becomes large. It has a smaller variance than the first estimator as m approaches N. Although some study as to the proper estimator to use might be useful we simply use the second following the conjuncture of Jenkins and Watts (1968) that it may in many cases give a smaller mean-square residual.

Equation (9) determines a set of equations which give the matrix form

$$\begin{pmatrix} R_{cx}[1] \\ R_{cx}[2] \\ \cdot \\ \cdot \\ R_{cx}[N] \end{pmatrix} = \begin{pmatrix} R_{xx}[0] & R_{xx}[1] & R_{xx}[2] & \ldots & R_{xx}[N-1] \\ R_{xx}[1] & R_{xx}[0] & R_{xx}[1] & \ldots & R_{xx}[N-2] \\ R_{xx}[2] & R_{xx}[1] & R_{xx}[0] & \ldots & R_{xx}[N-3] \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ R_{xx}[N-1] & R_{xx}[N-2] & R_{xx}[N-3] & \ldots & R_{xx}[0] \end{pmatrix} \cdot \begin{pmatrix} h[1] \\ h[2] \\ h[3] \\ \cdot \\ \cdot \\ h[N] \end{pmatrix}, \quad (10)$$

where we have used the fact that $R_{xx}[-k] = R_{xx}[k]$. The matrix is a Toeplitz matrix, which means that the matrix equation can be rapidly solved for h using the Rybicki algorithm (Press et al., 1986). By design, the solution will be valid in a least-squares sense.

NUMERICAL EXPERIMENTS

As a test of the Wiener-Hopf technique we considered a simple tracer experiment, depicted in Figure 1. The impulse response for this experiment converts a unit impulse into two impulses of amplitude .5 lagged one and two time units respectively, as shown in Figure 1. During the experiment, the produced water-tracer solution is diluted by an equal part of pure water and is reinjected, as in Figure 1.

The sampled values of $x[i]$ and $c[i]$ for this case were used to estimate $h[i]$ using both the Robinson technique and the Toeplitz matrix technique. In the case of noise-free data both techniques did a good job of estimating $h[i]$, giving estimates which were .5 to three significant figures at $i = 2,3$ and were smaller than $.5\times10^{-4}$ for all other $h[i]$ values. In both cases, the estimated impulse responses oscillated about 0 with decreasing envelope as i increased. We then contaminated the tracer data with Gaussian noise having a variance equal to 5% of the data values using an algorithm from Press et al. (1986). Using this contaminated data, the Robinson technique gave $h[2] = .537$ and $h[3] = .478$, while the least-squares technique gave $h[2] = .569$ and $h[3] = .512$. Both techniques gave $h[i]$ less than $.2\times10^{-3}$ for all other

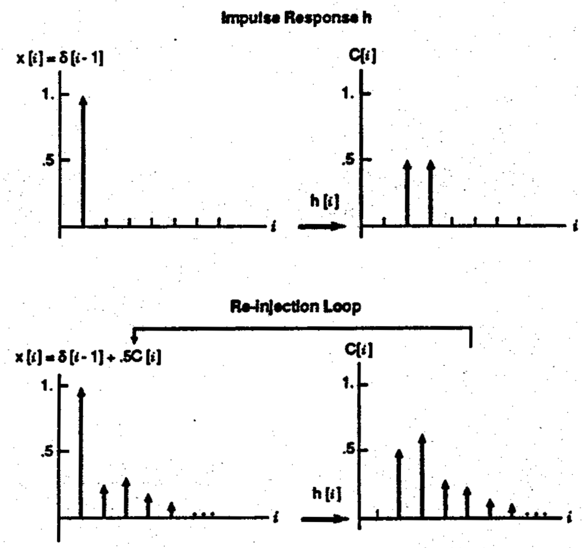

Figure 1. Theoretical impulse response and re-injection loop for the first numerical test case.

h[i] values. As before, both estimates oscillated about 0 with decreasing envelope as i increased.

Although the Robinson technique worked well with noisy data in this particular case, we emphasize that its general response to noisy data in unknown.

In this and the subsequent numerical experiment, we have not examined the variation of the impulse response estimate with respect to amount of data included, autocorrelation estimators used, or pre-filtering of the data. Thus our impulse response estimates may not be the best achievable from a particular data set using the Wiener-Hopf equation.

Figures 2 and 3 give concentration data for a tracer experiment at Dixie Valley (Adams et al., 1989) using an initial 150 Kg slug of Fluorescein dye and a 100 Kg slug of Benzoic acid respectively. We present the tracer responses as concentrations in an attempt to lessen the effects of non-constant production flow rate. The Benzoic acid data is initially smooth but then becomes slightly oscillatory. The Fluorescein data is much smoother than the Benzoic acid data, with oscillations in the curve tail much attenuated compared to those in the Benzoic acid data curve. It is unclear whether these oscillations represent error in chemical analyses, represent actual flow characteristics of the earth or reflect variations in the tracer concentrations of re-injected waters, as shown in Figures 4 and 5. However, if the oscillations reflect variations in the mass of the reinjected tracers, they should not be present in the least-squares estimate impulse of the response.

We used the Wiener-Hopf technique to estimate the least-squares impulse responses for the Fluorescein and the Benzoic acid data. The final impulse-response estimates are shown in Figures 2 and 3. The impulse response agrees well with the data at early times for both cases. At later times the impulse response falls below the data, as is expected. The total percent difference between the data and the impulse response at day 70 for the Fluorescein is 8% of the impulse response, while the discrepancy for the Benzoic acid at day 70 is 10% of the impulse response. For both tracers, the impulse response has oscillations which track the oscillations in the data. This suggests that these high frequency variations in the tracer responses are not caused by variations in the tracer mass of re-injected water.

CONCLUSIONS

We have discussed the estimation of the impulse response of single injector tracer experiments using the Wiener-Hopf equation. This approach is advantageous because it is not restricted to a particular initial tracer injection mass history and it gives the least squares estimation of the impulse response. We have illustrated the use of the technique using tracer data gathered at Dixie Valley. In this application the technique was important in demonstrating the independence of oscillations in the tracer responses from tracer mass variations in the re-injection water. We have not examined the variation of the impulse response estimate with respect to amount of data included, autocorrelation estimators used, or pre-filtering of the data. Thus our impulse response estimates may not be the best achievable from a particular data set using the Wiener-Hopf equation.

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BIBLIOGRAPHY

Adams, M. C., Benoit, W. R., Doughy, C., Bodvarsson, G. S., and Moore, J. N., 1989, Dixie Valley, Nevada Tracer Test, accepted GRC Transaction, 5p.

Jenkins, G. M., and Watts, D. G., 1968, Spectral Analysis and Its Applications: Holdens-Day, Inc., San Francisco,

Oppenheim, A. V., and Schafer, R. W., 1975, Digital Signal Processing: Prentice-Hall, Inc.

Papoulis, A., 1977, Signal Analysis: McGraw-Hill Book Co.

Press, W. H., Flannery, B. P., Teukolsky, S. A., and Vertterling, W. T., 1986, Numerical Recipes - The Art of Scientific Computing: Cambridge University Press.

Robinson, B. A., 1985, Non-reactive and chemically reactive tracers: theory and applications: Ph.D. thesis, Massachusetts Institute of Technology.

Robinson, B. A., Tester, J. W., and Brown, L. F., 1988, Reservoir sizing using inert and chemically reacting tracers: SPE Formation Evaluation, March, pp. 227–234.

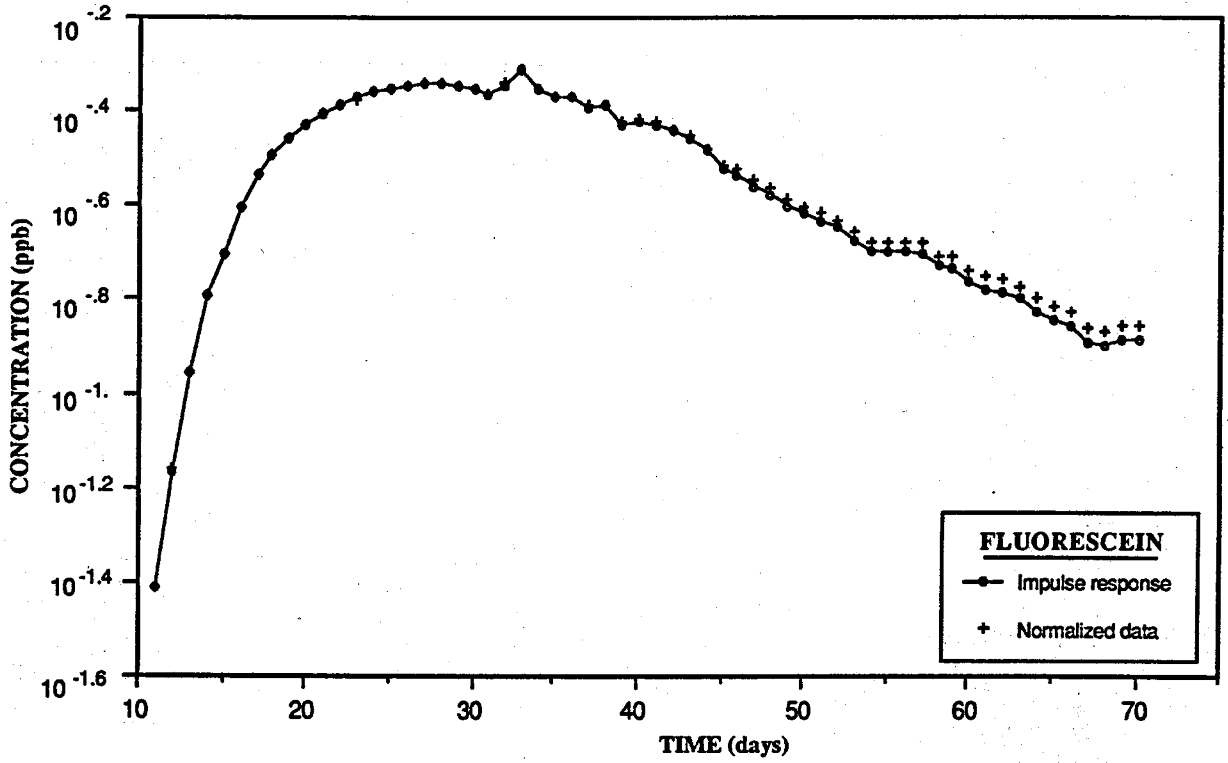

Figure 2. Fluorescein concentration data for the Dixie Valley Test., normalized to a unit mass initial slug, together with the impulse response estimated using the Wiener-Hopf equation.

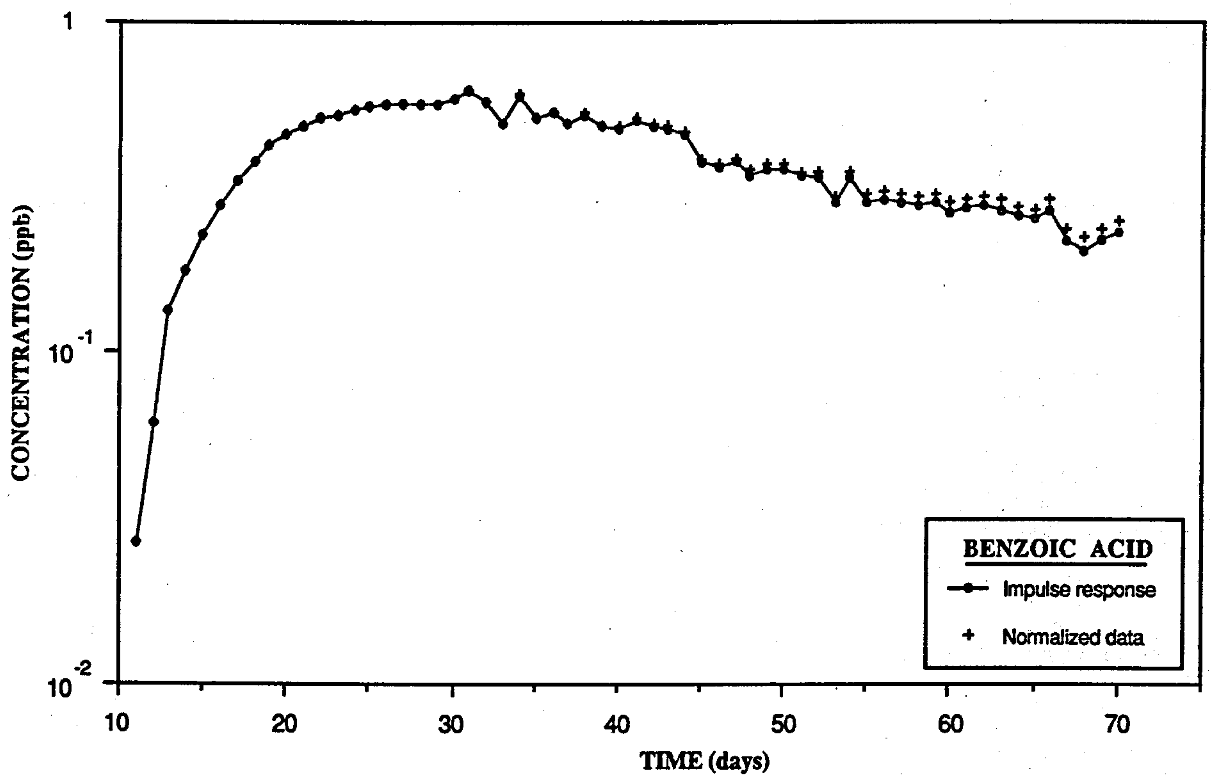

Figure 3. Benzoic acid concentration data for the Dixie Valley test, normalized to a unit mass initial slug, together with the impulse response estimated using the Weiner-Hopf equation.

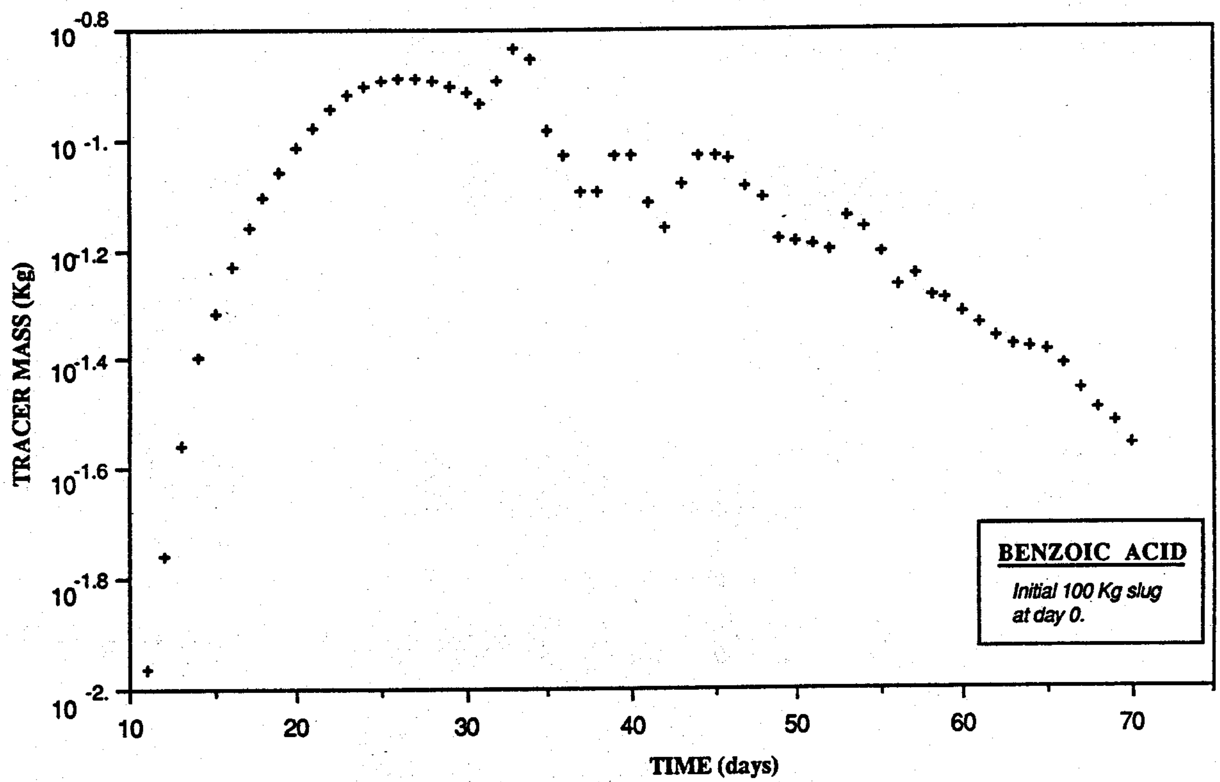

Figure 4. Re-injected Fluorescein as a function of time.

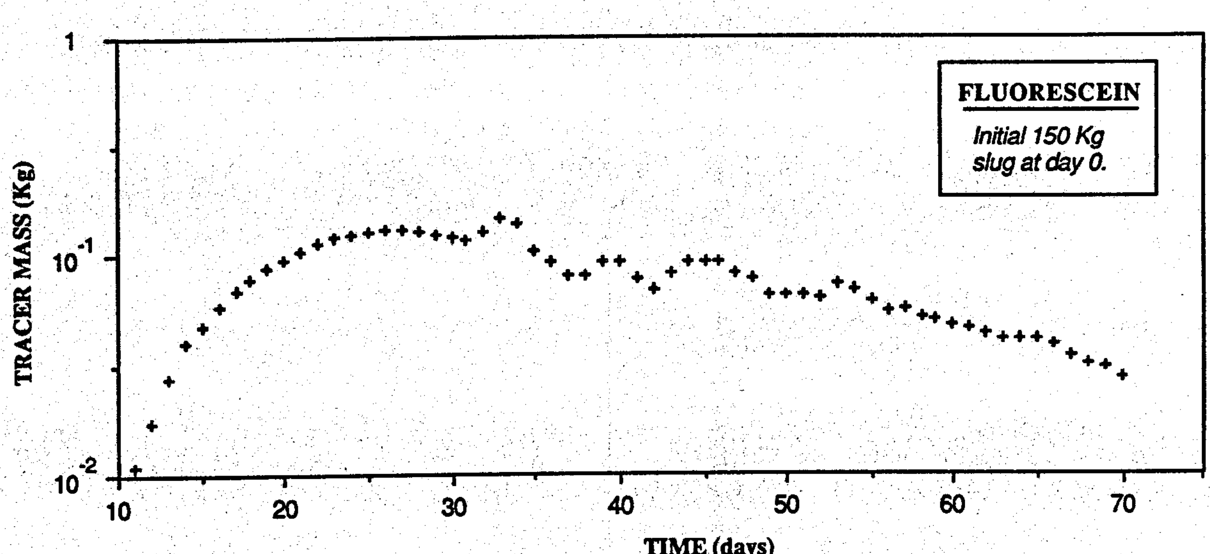

Figure 5. Re-injected Benzoic acid as a function of time.