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A SEMIANALYTICAL SOLUTION FOR TRACER FLOW IN NATURALLY FRACTURED RESERVOIRS

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ABSTRACT

This study presents a semianalytical solution of the integral form for continuous, finite step and spike-injection flow of a tracer in naturally fractured reservoirs. An important advantage of this solution is that the numerical dispersion reported by previous investigators when using the Stehfest inverter Laplace transform algorithm is avoided. The reservoir is treated as being composed of two regions: a mobile region where diffusion and convection take place and a stagnant region where only diffusion and adsorption are allowed. This model can also be used to study the flow of radioactive tracers. The solution of this paper is expressed in terms of two dimensionless parameters: the Peclet number for the fractures, P_{e1}, and a parameter $\alpha = \xi\sqrt{P_{e2}}$, where $\xi = \phi_2 D_2 / v(w-\delta)$ and P_{e2} is the Peclet number for the matrix. A sensitivity study was carried out with regard to the above mentioned main parameters that enter into the continuous, finite step and spike-injection solutions.

INTRODUCTION

The use of tracers has long been used for the characterization of hydrocarbon reservoirs, however, it wasn't until recently that they gained importance as a useful means to study geothermal reservoirs. Geothermal reservoirs are in most of the cases complex naturally fractured systems that require careful studies for optimal exploitation conditions. Tracer flow tests, in addition to well testing, provide an excellent means to obtain estimations for basic reservoir parameters, as well as the connectivity of the reservoir and the transit times of injected fluids. This latter information is of outmost importance in reinjection projects of brine into these reservoirs.

During the last two decades several papers appeared in the literature dealing with the flow of tracers in naturally fractured systems (Grove and Beetem, 1971; Grisak and Pickens, 1980 and 1981; Neretnieks, 1980; Tang et al.,1981; Fossum and Horne, Tester et al., 1983; Horne and Rodríguez, 1983; Jensen, 1983; Hugakorn et al., 1983; Maloszewski et al., 1985; Rasmuson, 1985; Okandan, 1987; Rivera et al., 1987; Ramírez et al., 1988). It is important to know that the number of studies that consider a quantitative determination of reservoir parameters is limited (Grove and Beetem, 1971; Tang et al., 1981; Walkup and Horne, 1985; Rivera et al., 1987; Ramírez et al., 1988)

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The purpose of this study is to present a semianalytic solution of the integral form for the continuous, finite step and spike-input flow of a tracer in naturally fractured reservoirs. This solution considers all the
affect tracer flow:
sorption and radio-

ξ = xi
φ = phi
δ = delta
α = alpha

his study is shown
fractured medium
a system of equally
alternated with
hown in this figure
ns a mobile region
constituted by the fracture network where diffusion and convection take place and a stagnant or immobile region where only diffusion and adsorption are allowed. This model also considers the possibility of a radioactive tracer. Both regions are interconnected by means of a thin fluid layer contained within the immobile region which controls the fluid and mass transfer between these regions. This type of visualization of the problem by means of two regions has been used by other authors in the past (Deans, 1963; Walkup and Horne, 1985; Maloszewski and Zuber, 1985; Rivera et al., 1987; Ramírez et al., 1988). For a detailed description of the model the reader is referred to the work of Ramírez et al. (1988). Appendix A shows the definitions of the dimensionless groups used in this study.

For the case of a continuous tracer injection the governing equation for flow in the fractures is as follows:

$$\frac{1}{P_{e1}}\frac{\partial^2 c_{1D}}{\partial x_D^2} - \frac{\partial c_{1D}}{\partial x_D} - \gamma c_{1D} + \xi \left.\frac{\partial c_{2D}}{\partial y_D}\right|_{\frac{w-\delta}{L}} - \frac{\partial c_{1D}}{\partial t_D} = 0 \quad (1)$$

The solution of this equation in Laplace space considering the proper initial and boundary conditions is given by (Ramírez et al., 1988):

$$\bar{C}_{1D} = \frac{1}{s}\exp\left(\frac{P_{e1}x_D}{2}\right)\exp\left[-x_D\sqrt{\frac{P_{e1}}{4}+P_{e1}\left[s+\gamma+\xi m_1\tanh m_1\left(\frac{E-2w+2\delta}{2L}\right)\right]}\right] \tag{2}$$

where

$$m_1 = \sqrt{\frac{P_{e2}}{R}(s+\gamma)} \tag{3}$$

It has been shown (Ramírez, 1988) that for practical purposes the thickness E of the repetitive element of Fig. 1 does not sensibly affect the tracer concentration results, because for the parameters that enter into the hyperbolic tangent, its evaluation results approximately in a value of 1. Under these conditions, Eq. 2 can be written as follows:

$$\bar{C}_{1D} = \frac{1}{s}\exp\left(\frac{P_{e1}x_D}{2}\right)\exp\left[-x_D\sqrt{\frac{P_{e1}^2}{4}+P_{e1}\left[s+\gamma+\xi\sqrt{\frac{P_{e2}(s+\gamma)}{R}}\right]}\right] \tag{4}$$

It can be mentioned that Tang et al. (1981) have presented an expression similar to Eq. 4.

If Eq. 4 is analyzed, it is possible to define a new dimensionless parameter α that would allow to express this solution in terms of only two dimensionless parameters:

$$\alpha = \xi\sqrt{P_{e2}} = \frac{\phi_2}{v(w-\delta)}\sqrt{P_{e2}} = \frac{\phi_2}{(w-\delta)}\sqrt{\frac{D_2L}{v}} \tag{5}$$

Considering a chemical tracer (γ=0) and that R=1, Eq. 4 can be written as follows:

$$\bar{C}_{1D} = \frac{1}{s}\exp\left(\frac{P_{e1}x_D}{2}\right)\exp\left[-x_D\sqrt{\frac{P_{e1}^2}{4}+P_{e1}(s+\alpha\sqrt{s})}\right] \tag{6}$$

It can be observed from Eq. 6 that through the parameter α the stagnant region affects the tracer concentration of the mobile region. Based upon this observation, for small values of $\alpha\ (\alpha\rightarrow 0)$, the system would behave as if the immobile region were impermeable and that the tracer will only flow through the mobile (fractured) region. This case is equivalent to consider the flow of a tracer through a homogeneous media. Thus, under this condition, Eq. 6 can be expressed:

$$\bar{C}_{1D} = \frac{1}{s}\exp\left(\frac{P_{e1}x_D}{2}\right)\exp\left\{-x_D\sqrt{\frac{P_{e1}^2}{4}+P_{e1}s}\right\} \tag{7}$$

The inversion of this equations yields:

$$C_{1D} = \frac{1}{2}\operatorname{erfc}\left[\frac{\sqrt{P_{e1}}\,x_D}{2\sqrt{t_D}}-\sqrt{\frac{P_{e1}t_D}{4}}\right] + \frac{1}{2}\exp(P_{e1}x_D)\operatorname{erfc}\left[\frac{P_{e1}x_D}{2\ t_D}+\sqrt{\frac{P_{e1}t_D}{4}}\right] \tag{8}$$

Eq. 8 has been presented by Coats and Smith (1964) for homogeneous systems. For the case of this study of a fractured formation, it represents a limit analytic solution due to the fact that maximum tracer concentrations are attained when there is no tracer transfer to the second immobile region, which as mentioned corresponds to impermeable conditions ($\alpha \cong 0$).

In many field situations, the tracer is injected for a short period and are referred to as "spike injection tests" in the literature (Walkup, 1984). It has been stated (Walkup, 1984; Walkup and Horne, 1985) that the solution for an spike test is the time derivative of a step injection solution:

$$(C_{1D})_{spike} = \frac{\partial C_{D1}}{\partial t_D} = \mathcal{L}^{-1}\{s\bar{C}_{1D}\} \tag{9}$$

Substituting Eqs. 6 and 7 into Eq. 9:

$$(C_{1D})_{spike} = \exp\left(\frac{P_{e1}x_D}{2}\right)\exp\left[-x_D\sqrt{\frac{P_{e1}^2}{4}+P_{e1}(s+\alpha\sqrt{s})}\right] \tag{10}$$

$$(C_{1D})_{\substack{spike\\limit}} = \exp\left(\frac{P_{e1}x_D}{2}\right)\exp\left[-x_D\sqrt{\frac{P_{e1}}{4}+P_{e1}s}\right] \tag{11}$$

The solution for a finite step injection for fractured and homogeneous systems can be derived through the application of the principle of superposition and the use of Eqs. 6 and 7.

METHOD OF SOLUTION

First the algorithm for the Laplace transform of Stehfest was used to invert Eq. 6, finding some numerical dispersion of the results, which increased with the dimensionless time. Based on this finding, an alternate approach to solve this problem was sought.

Applying Eq. 7.4.3 of Abramowitz and Stegun (1970) to Eqs. 4, 6 and 7 we obtained the following solutions:

Continuous injection.

Radioactive tracers.

$$C_{1D}(x_D, t_D) = \frac{1}{\sqrt{\pi}} \exp\left(\frac{P_{e1}x_D}{2}\right) \int_0^\infty \exp\left[-\tau^2 - \left(\frac{P_{e1}^2 x_D}{4\tau^2} + \gamma\right)\right] \cdot$$

$$\exp\left(-\frac{P_{e1}\,\alpha\sqrt{\gamma}}{4\tau^2} x_D^2\right) \operatorname{erfc}\left[\frac{P_{e1}\alpha\, x_D^2}{8\tau^2\sqrt{t_D - \frac{P_{e1}x_D}{4\tau^2}}} - \sqrt{\gamma\left(t_D - \frac{P_{e1}x_D^2}{4\tau^2}\right)}\right]$$

$$+\exp\left(\frac{P_{e1}\,\alpha\sqrt{\gamma}\,x_D}{4\tau^2}\right) \operatorname{erfc}\left[\frac{P_{e1}\alpha\, x_D^2}{8\tau^2\sqrt{t_D - \frac{P_{e1}x_D}{4\tau^2}}} + \sqrt{\gamma\left(t_D - \frac{P_{e1}x_D^2}{4\tau^2}\right)}\right]$$

$$\cdot H\left(t_D - \frac{P_{e1}x_D^2}{4\tau^2}\right) d\tau \qquad (12)$$

Continuos injection.

Chemical tracers ($\gamma = 0$).

$$C_{1D}(x_D, t_D) = \frac{2}{\sqrt{\pi}} \exp\left(\frac{P_{e1}x_D}{2}\right) \int_0^\infty \exp\left(-\tau^2 - \frac{P_{e1}^2 x_D}{16\tau}\right) \cdot$$

$$H\left(t_D - \frac{P_{e1}x_D^2}{4\tau^2}\right) \operatorname{erfc}\left[\frac{P_{e1}\alpha\, x_D^2}{8\tau^2\sqrt{t_D - \frac{P_{e1}x_D}{4\tau^2}}}\right] d\tau \qquad (13)$$

spike - injection

$$C_{1D}(x_D, t_D)_{spike} = \frac{P_{e1}\alpha\, x_D^2}{4\pi} \int_0^\infty \exp\left[-\tau^2 - \left(\frac{P_{e1}x_D}{4\tau}\right)^2\right] \cdot$$

$$\left[1 + \left(\frac{\alpha x_D}{2\tau}\right)^2 \frac{1}{\left(t_D - \frac{P_{e1}x_D^2}{4\tau^2}\right)}\right] H\left(t_D - \frac{P_{e1}x_D^2}{4\tau^2}\right) d\tau \qquad (14)$$

VALIDATION OF THE SEMIANALYTICAL (INTEGRAL) SOLUTION.

Equations 12 and 13 were numerically integrated using the algorithm of O'Hara and Smith (1969), and compared to the numerical inversion solutions using the algorithm of Stehfest (1970 a,b) of Eqs. 6 and 10. Figures 2 and 3 show the results obtained of tracer concentration for the continuous injection and Figs. 4 and 5 show the corresponding results for the spike-injection case. It can be observed from these results that the semianalytical (integral) solution is far better than that obtained through numerical Sthefest inversion.

Another step taken toward the validation of the integral solution method was to obtain the limit integral solution for the continuous case of Eq. 7, following the same procedure already outlined in the previous section:

$$C_{1D}(x_D, t_D) = \frac{2x_D}{\sqrt{\pi}} \exp\left(\frac{P_{e1}x_D}{2}\right) \int_0^\infty \exp\left(-x_D^2\tau^2 - \frac{P_{e1}^2}{16\tau^2}\right) \cdot$$

$$H\left(t_D - \frac{P_{e1}}{4\tau^2}\right) d\tau \qquad (15)$$

This equation 15 was numerically integrated by the algorithm of O'Hara and compared to the limit solution for homogeneous systems given by Eq. 8, finding an excellent comparison, as will be shown next. The next step in the validation effort was to compare the continuous solutions obtained through the numerical integration of the limit solution given by Eq. 15 and the limit analytic solution for homogeneous systems given by Eq. 8, with the numerical integration solution of the general Eq. 13 for an α value of 0.01. This value was chosen based on the previous finding (Ramírez, 1988) that for $\alpha \leq 0.01$ tracer response is esentially the same as for $\alpha=0$, which corresponds to the zero diffusion into the matrix (homogeneous case). Table 1 shows the results of this comparison for a value of P_{e2} equal to 2, with a maximum difference in the results of 0.00015 % percent.

Similar validations of the integral solution to the previously discussed for the continuous injection case were carried out for the finite-step and spike injection cases finding an excellent comparison with the limit corresponding solutions.

DISCUSSION OF RESULTS

In this section a discussion is presented of the tracer responses for the continuous, finite-step and spike injection cases given for the first and the last case by Eqs. 13 and 14, respectively. An important part of this discussion will focus on the finite-step injection case that is often used in field applications.

It is important to keep in mind that the integral solution of this study, used to describe the tracer flow in naturally fractured systems, is expressed in terms of only two basic parameters: the α parameter and the Peclet number for the mobile region P_{e1}. Thus, it is easy to check the effect of each of these parameters on the tracer flow response.

Fig. 6 shows the tracer behavior for different distances of the injector to the producing well, for values of $P_{e1}=1$ and $\alpha=0.01$. For practical purposes, tracer concentration is important for a distance $x_D=1$ and, consequently, results of this study will correspond to this distance.

It has been previously discussed that the α parameter determines the influence of the immobile region upon the tracer flow through the fractures (its definition given by Eq. A-11 involves the porosity of the matrix ϕ_2 and the difussion coefficient D_2).

It is useful to further discuss Fig. 3 which shows the influence of the parameter α upon tracer concentration. It can be noticed that as α increases, the tracer concentration at $x_D=1$ decreases and also that the breakthrough time of the tracer is essentially constant for all α values. In addition, the graphical results indicate that the curve for the limit concentration, $\alpha=0.01$, is an upper limit for all solutions and that the tracer responses for the various α values eventually will reach a maximum concentration less than 1. Of course, this is due to the transfer of tracer to the immobile region.

Figs. 7 and 8 illustrate, for the case of continuous injection, the influence of Peclet number of the mobile region upon the tracer concentration. If we analyze the definition of the Peclet number (Eq. A-5), it can be concluded that for practical purposes the variation of P_{e1} is due to diffusion in the mobile region represented by D_1:

$$D_1 = D_L V + D^* \qquad (16)$$

where D_L is the logitudinal dispersion coefficient of the fractures and D^* is the tracer molecular diffusion coefficient. The values of the molecular diffusion coefficient D^* are usually quite smaller than the values of D_L and consequently, the Peclet number P_{e1} is inversely proportional to D_L.

Fig. 7 also indicates that the breakthrough time of the tracer is a function of the Peclet number for the mobile region P_{e1}. As expected, if P_{e1} increases we get longer tracer breakthrough times. This can be explained by saying that as P_{e1} increases, the coefficient D_1 will be smaller and this will result in longer breakthrough times. However, after breakthrough, the increase of tracer concentration is faster for the higher P_{e1} values; this means that the slope of the tracer concentration responses increases and as shown in Fig. 8, for values of P_{e1} greater than 5 the response will be similar to that of piston like displacement predicted by the Buckley-Leverett frontal theory of linear waterflooding displacement in oil reservoirs (Craig, 1971).

In summary, from the foregoing discussion it has been concluded that for a fixed value of α, the Peclet number P_{e1} affects the tracer concentrations responses (different shapes and also changing breakthrough times), and that for a constant P_{e1} the responses for various α values show curves of similar shape, and that the response for α equal to 0.01 is an upper limit for all solutions. These conclusions also hold for the finite-step and spike injection cases to be discussed next.

With regard to the finite-step case, Fig. 9 shows results for an injection period expressed in dimensionless time equal to 0.3, $P_{e1} = 1$ and various α values. It can be observed that the time to reach maximum concentration conditions is independent of α ; this means that the diffusion and adsorption of the tracer in the immobile region has no influence on the travel of the maximum concentration towawrd the producing well.

Fig. 10 shows results of tracer concentration for the finite-step case for $\alpha = 0.01$ and various values of the P_{e1} parameter for an injection period of 0.3; it can be noticed that the time to reach maximum concentration conditions will increase as P_{e1} increases.

Fig. 11 shows results of tracer concentration for the finite-step case for the same α and P_{e1}'s parameters already stated for Fig. 10, but for an injection period of 0.1; comparing the results of this Fig. 11 with those of Fig. 5 for the spike injection case, it can be concluded that the tracer responses for both cases are similar and that the finite-step solution will only converge to the spike solution for large dimensionless times. Another important point is that the time to reach maximum concentration and the breakthrough time are essentially the same for both solutions, for the particular case of an injection period equal to 0.1.

Next, Fig. 12 presents the results of tracer concentration for the finite-step case, for a value of $\alpha = 0.01$, $P_{e1} = 2$ and values of the injection period of 0.1, 0.3 and 0.5. As expected, it is observed from results of this figure that as the injection time increases, so does the maximum tracer concentration and that the time at which this maximum is attained is larger than the injection period, due to the dispersion effects, taken into account in the Peclet number P_{e1}. It can also be noticed that the tracer concentration response for different injection periods is the same for times smaller than the injection time. This implies that for times smaller than the injection time, the continuous solution is valid to analyze tracer concentration response. This can be further verified if the results of this figure for an injection time of 0.5 are compared to those of Fig. 3 for α = 0.01, finding that both responses are the same for times smaller than the injection time.

CONCLUSIONS

The main pourpose of this work was to present a semianalytical solution of the integral form for the flow of a tracer in naturally fractured reservoirs.

Based on the material presented in this study, the following conclusions are pertinent.

1. A model is presented for the flow of a tracer in naturally fractured reservoir that considers the following mechanisms: diffusion, convection, adsorption and radioactive decay.

2. The integral semianalytical solutions presented for the continuous, the finite-step and the spike injection cases do not show numerical dispersion as shown by previous investigators when using the Stehfest inverter Laplace transform algorithm.

3. For practical pourposes the thickness (size) of the matrix blocks does not affect the tracer concentration results. Under this condition, the tracer concentration response is expressed in terms of only two dimensionless parameters: α and P_{e1}.

4. The parameter α determines the influence of the immobile region upon the tracer concentration response. An analytic solution was derived for the case of no diffusion to the matrix (α = 0). It was shown that this solution can be applied for α< 0.01.

5. The immobile region influences the tracer concentration response for values of α> 0.01. It was found that the breakthrough time of the tracer is essentially constant for all α values and that the response for α =0.01 is an upper limit for all solutions.

6. The Peclet number of the mobile (fractured) region P_{e1} has an important effect upon the tracer concentration response. It was observed that if P_{e1} increases the breakthrough time also increase and also that the shape (slope) of the tracer concentration response is affected by this parameter.

7. The continuous injection solution is valid to analyze tracer concentration response for times smaller that the injection time.

NOMENCLATURE

- C = tracer concentration, M/L^3
- C_D = dimensionless tracer concentration
- D = diffusion coefficient, L^2/T
- E = fracture spacing, L
- H = step function, Eqs. 12, 13 and 14
- k = adsorption constant, L^3/M
- L = distance from the injector to producing well, L
- L^{-1} = inverse Laplace's
- m_1 = parameter, Eq. 3
- P_e = Peclet number, dimensionless
- R^1 = dimensionless group, Eq. A-10
- s = Laplace variable
- t = time, T
- t_D = dimensionless time, Eq. A-7
- v = velocity, L/T
- w = fracture half-width, L
- x = distance in the x direction
- x_D = dimensionless distance in the x direction Eq. A-6
- y = distance in the y direction
- y_D = dimensionless distance in the y direction, Eq. A-2

Greek simbols.

- α = dimensionless group defined by Eq. A-11.
- γ = dimensionless group, defined by Eq. A-4.
- δ = Stagnant fluid film thickness, L.
- ϕ = porosity, referred to bulk volume
- τ = dummy variable of integration
- ρ = density, M/L^3
- λ = radioactive decay constant, T^{-1}

Subscripts

- D = dimensionless
- i = initial
- sp = spike
- 1 = mobile or fractured region
- 2 = immobile region

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APPENDIX A

DIMENSIONLESS GROUPS USED TO DESCRIBE THE FLOW OF TRACERS

Dimensionless distances

$$x_D = \frac{x}{L} \tag{A-1}$$

$$y_D = \frac{y}{L} \tag{A-2}$$

Dimensionless parameter ξ

$$\xi = \frac{\phi_2 D_2}{V_1 (w-\delta)} \tag{A-3}$$

Dimensionless parameter γ

$$\gamma = \frac{L\lambda}{V_1} \tag{A-4}$$

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APPENDIX A

DIMENSIONLESS GROUPS USED TO DESCRIBE THE FLOW OF TRACERS

Peclet number.

Mobile (fractured) region (1)

$$P_{e1} = \frac{V_1 L}{D_1} \tag{A-5}$$

Immobile region (2)

$$P_{e2} = \frac{V_1 L}{D_2} \tag{A-6}$$

Dimensionless time (t_D)

$$t_D = \frac{V_1 t}{L} \tag{A-7}$$

Dimensionless concentration

Mobile (Fractured) region (1)

$$C_{1D} = \frac{C_1 - C_i}{C_o - C_i} \tag{A-8}$$

Immobile region

$$C_{2D} = \frac{C_2 - C_i}{C_o - C_i} \tag{A-9}$$

Dimensionless parameter R

$$R = \frac{\phi_2}{\phi_2 + \rho k (1-\phi_2)} \tag{A-10}$$

Dimensionless parameter α

$$\alpha = \xi \sqrt{P_{e2}} \tag{A-11}$$

Table 1. Comparison of the limit analytic solution for homogeneous systems (EQ. 8), the limit solution (EQ. 14), and the numerical integration solution of the general EQ. 12, for $\alpha = 0.01$.

TD	CDla. (Eq. 8)	CDlint. (Eq. 14)	CDint. (Eq. 12)
0.10	0.004077	0.004076	0.004047
0.20	0.063753	0.063753	0.063309
0.30	0.165726	0.165726	0.164584
0.40	0.270613	0.270613	0.268769
0.50	0.364975	0.364975	0.362520
0.60	0.446383	0.446383	0.443422
0.70	0.515738	0.515738	0.512365
0.80	0.574723	0.574723	0.571021
0.90	0.625022	0.625023	0.621058
1.00	0.668101	0.668102	0.663929

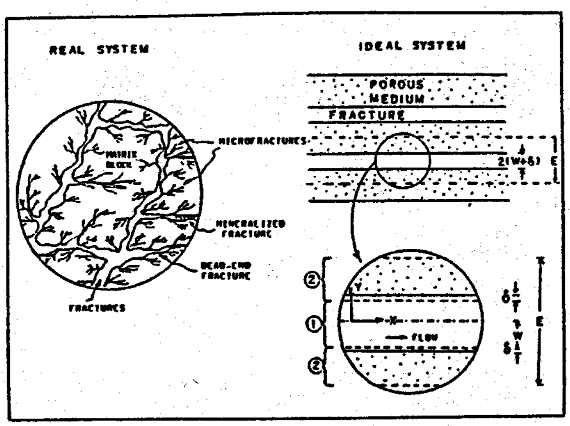

Fig. 1 Idealized proposed model for representation of the naturally fractured medium.

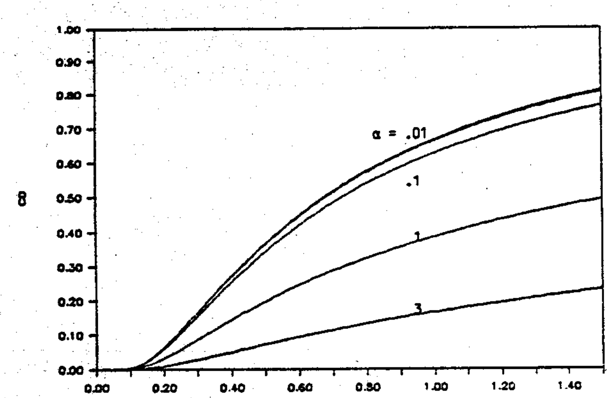

Fig. 3. Continuous injection tracer concentration semianalytical solutions.

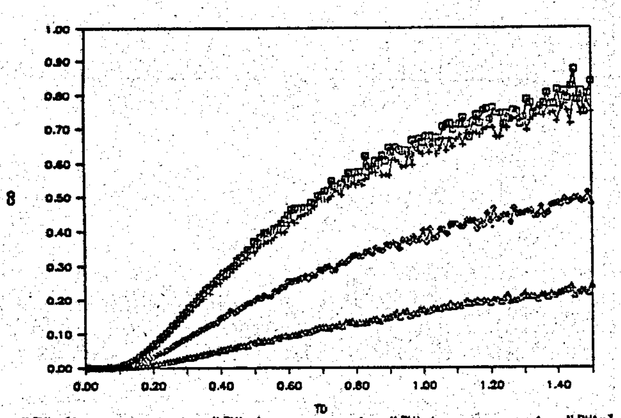

Fig. 2. Continuous injection tracer concentration solutions obtained through numerical inversion using the Stehfest algorithm

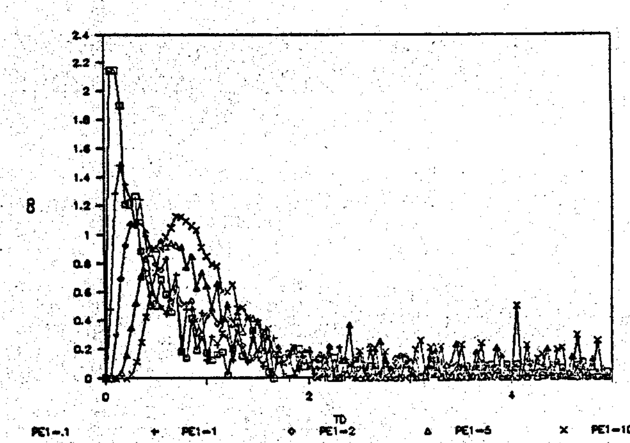

Fig. 4. Spike injection tracer concentration solutions obtained through numerical inversion using the Stehfest algorithm

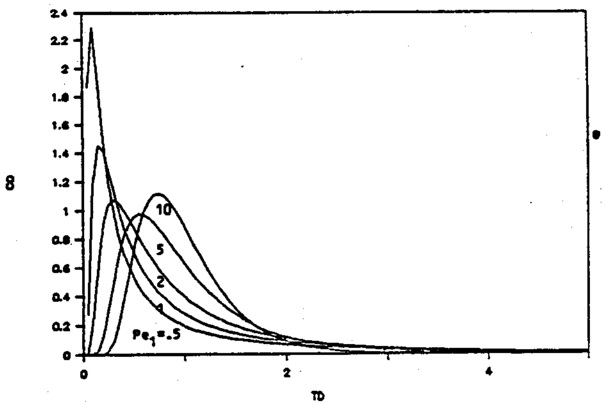

Fig. 5. Spike injection tracer concentration semianalytical solutions

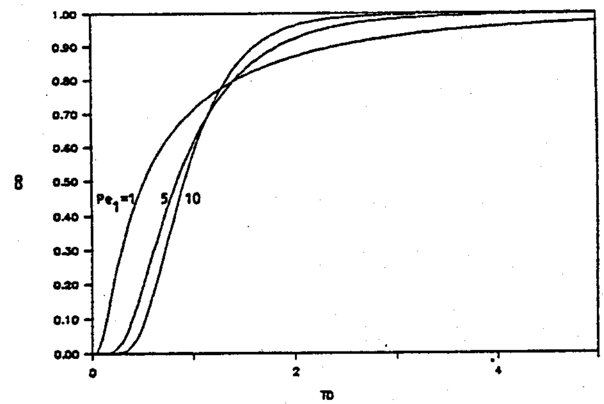

Fig. 8. Influence of the peclet number on the continuous injection tracer concentration for long times.

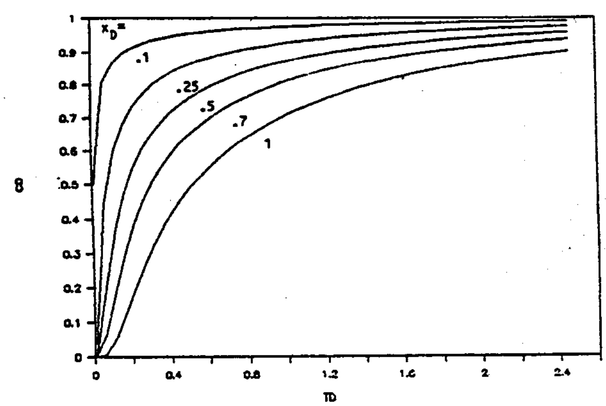

Fig. 6. Continuous injection tracer concentration for various x_D values

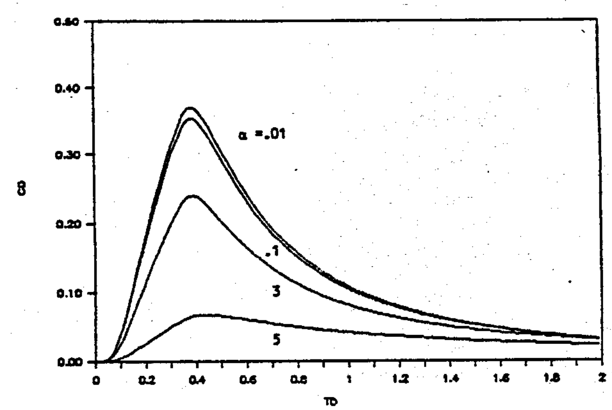

Fig 9. Influence of the a parameter on the finite-step injection tracer concentration for $t_D = 0.3$, $Pe_1 = 1$.

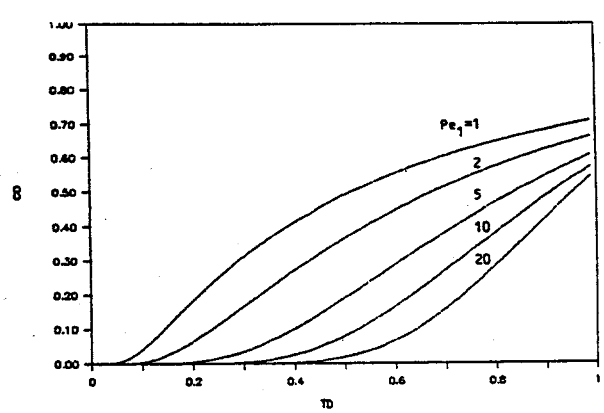

Fig. 7. Influence of the peclet number on the continuous injection tracer concentration for short times.

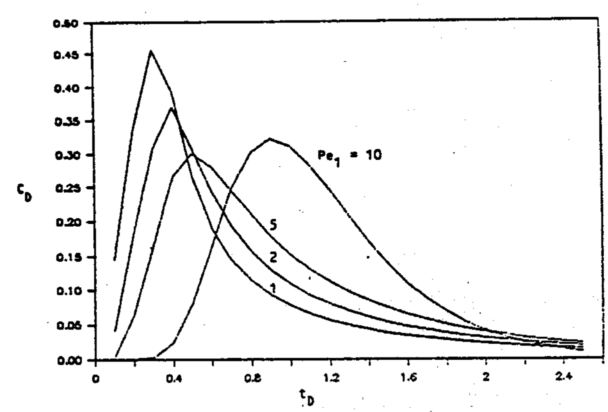

Fig. 10. Influence of the peclet number on the finite-step injection tracer concentration for $t_D = 0.3$, $\alpha = 0.01$.

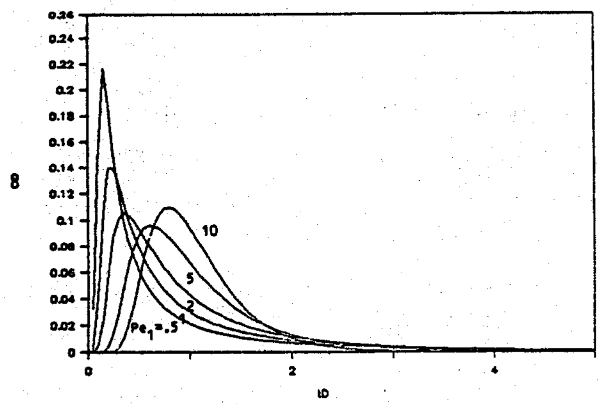

Fig. 11. Influence of the peclet number on the finite-step injection tracer concentration for $t_D = 0.1$, $\alpha = 0.01$.

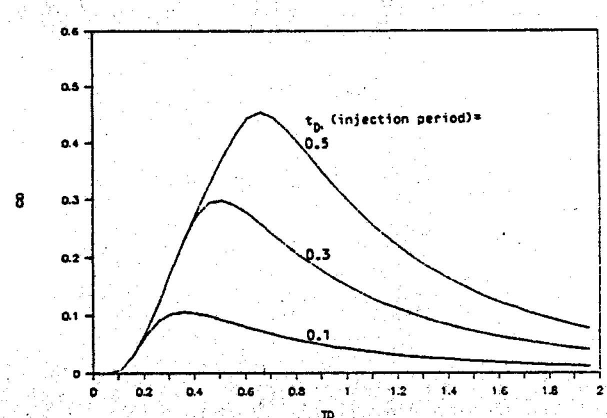

Fig. 12. Influence of the injection period on the finite-step injection tracer concentration for $Pe_1 = 2$, $\alpha = 0.01$.