PROCEEDINGS, Fifteenth Workshop on Geothermal Reservoir Engineering
Stanford University, Stanford, California, January 23-25, 1990
SGP-TR-130

# A New Method of Forecasting the Thermal Breakthrough Time During Reinjection in Geothermal Reservoirs

I. Kocabas R. N. Horne

Department of Petroleum Engineering
Stanford University
Stanford, CA 94305

## Abstract

Success of a reinjection process depends on the method used for estimating the thermal breakthrough time. Estimating the thermal breakthrough time may be based solely on tracer data. Alternatively, it may be estimated solely by using thermal interference tests. While the tracer method usually yields ambiguous estimates, thermal interference tests are infeasible because durations of these tests are similar to thermal breakthrough times. Estimation of the thermal breakthrough time must be based on both tracer and thermal data. A new method of forecasting the thermal breakthrough time during reinjection was developed by using results of both interwell tracer tests and thermal injection-backflow tests. If Lauwerier model which represents the heat transport in a system consisting of a fracture located in a porous matrix is used, the thermal breakthrough time is determined by two parameters, the water transit time $t_w$ and the parameter $\lambda$ which is a measure of thermal interaction between the fracture and the adjacent matrix. While the water transit time is obtained from analysis of interwell tracer tests, $\lambda$ may be obtained from thermal injection-backflow tests. Analysis of temperature return profiles of thermal injection-backflow tests are discussed. Also, for the analysis of temperature return profiles of thermal injection-backflow tests, a new solution of Lauwerier model is presented.

## 1 Introduction

This work involves development of a new method to forecast the thermal breakthrough time during reinjection in geothermal reservoirs.

Reinjection of the waste water[2,10,17] is commonly practiced in many liquid-dominated geothermal fields worldwide. Most of the time the objective of the reinjection is to dispose of the waste water[9,10], since it usually contains silica and toxic minerals such as arsenic, boron and mercury[3]. Reinjection of the waste water is also used to maintain the reservoir pressure[9,10] and to enhance the energy recovery[15]. Regardless of the objective, however, the low temperature of the waste water is a serious constraint upon the reinjection. Many field experiences have shown that the reinjected water may move through the fractures to the production zones in a very short time. The rapid migration of the reinjected water is undesirable, because it can produce thermal drawdown at the production wells. This thermal drawdown has two detrimental effects. First, it reduces the discharge enthalpy causing the steam discharge rates to decline. Second, it decreases the total production because of the increasing hydrostatic pressure of the fluid in the well[10]. We can avoid a rapid propagation of the thermal front if we are able to identify these fast flow channels and their thermal characteristics prior to the start of reinjection.

Analysis of the return profiles of tracer tests is the tool most commonly used to identify these fast flow channels and to estimate the fracture aperture, which is the most important parameter controlling the propagation of the thermal front[15]. However, Pruess and Bodvarsson[15] argued that while tracer breakthrough time is determined by the volume of the flow path, thermal breakthrough is determined by the available surface area for heat transfer from the matrix to the fracture. As a result, the speed of the thermal front is partially determined by the speed of the tracer. They proposed thermal interference tests to make reliable estimates of the thermal characteristics of fast flow paths. They reported that these tests had been carried out in several small experimental hot dry rock reservoirs[7,20]. However, a thermal interference test on a large scale requires a test period of duration similar to the thermal breakthrough time. A small scale test is also unattractive, since it requires drilling a new observation well into the zone connecting the injector and the producer, which is a costly operation. It is also possible that the observation well would not intersect the fast flow path.

A thermal injection-backflow test may be the best way to estimate the thermal characteristics of a system and to avoid high cost or long periods of thermal interference tests.

## 2 Analysis of Thermal Injection-Backflow Tests

In a thermal injection-backflow test, the cold fluid is injected into the system at a well for a period of time, after which the same well is produced until the temperature of the produced fluid reaches to the original reservoir temperature. Flow conditions are assumed to be steady state and the flow due to injection to be dominant compared to the natural formation flow. Temperature return profiles obtained during the backflow period may be analyzed to determine thermal characteristics of the system. The Lauwerier[14] model is assumed to represent the heat transport in the system adequately.

### 2.1 Injection Period Solutions

Injection period solutions of transport equations can be found in the paper by Lauwerier[14].

### 2.2 Backflow Period Solutions

Since the flow directions in injection and backflow periods are opposite, the sign of the convective transport term in the transport equation of the backflow period is of opposite sign from the convective transport term in the equation of the injection period. The temperature distribution at the end of the injection period is the initial temperature distribution for the backflow period. Therefore, the injection period solution provides the initial condition of the backflow period transport equation.

The solutions of transport equations were obtained by using a double Laplace transformation method(see Kocabas[13] for details). Transport equations were transformed first with respect to the injection period time variable $t_j$, and then with respect to the backflow period time variable, t. Using the Laplace transformation, permits avoiding the difficulties caused by the effect of the step function, which is in the solutions for the injection period.

The Laplace space solution for equal injection and backflow rates were presented in previous works[11,12]. The dependent variable of the system is temperature, and the parameter of the model for flow in a porous stream tube is:

$$\lambda = \frac{\sqrt{k_m \rho_m c_m}}{\phi_f b \rho_w c_w} \frac{1}{\sqrt{hrs}} \quad (1)$$

Defining a new variable, $\lambda_D$:

$$\lambda_D = \lambda\sqrt{t_j} \quad (2)$$

and normalizing the time variables by $t_j$:

$$\frac{t_j}{t_j} = 1 \quad (3)$$

$$t_{DP} = \frac{t}{t_j} \quad (4)$$

the Laplace space solution can be expressed as:

$$\overline{\overline{C}} = \overline{\overline{C}}_1 \left[\frac{1}{s} + \frac{2\lambda_D}{s\sqrt{p}} + \frac{2\lambda_D}{s-p}\left(\frac{1}{\sqrt{s}} - \frac{1}{\sqrt{p}}\right)\right] \quad (5)$$

where

$$\overline{\overline{C}}_1 = \frac{1}{s + p + 2\lambda_D(\sqrt{s} + \sqrt{p})} \quad (6)$$

In Eq. 5, $s$ corresponds to $t_j/t_j = 1$ and $p$ corresponds to $t_{Dp}$ respectively.

To obtain temperature return profiles, Eq. 5 was inverted by a double numerical inversion technique based on the Stehfest algorithm[19]. However, the effect of numerical dispersion on the Stehfest algorithm was not determined for the case of temperature return profiles with steep slopes, which are likely to be observed when the parameter, $\lambda_D$, in Eq. 5 is small. To compute solutions for small $\lambda_D$, either a more accurate numerical inversion algorithm must be used or the real space solution must be found and evaluated.

Using the Laplace transform inversion method, *functions of functions*, discussed by Ditkin and Prudnikov[4], and the inversion formula given by Voelker and Doetsch[21], the real space function of Eq.5 is obtained[13]:

$$\mathrm{C} = \int_0^{\theta_1} \left\{ \mathrm{erfc}\left(\frac{\lambda_D\theta}{\sqrt{1-\theta}}\right) \frac{\lambda_D\theta}{\sqrt{\pi(t_{Dp}-\theta)^3}} \exp\left(-\frac{\lambda_D^2\theta^2}{t_{Dp}-\theta}\right) + 2\lambda_D\,\mathrm{erfc}\left(\frac{\lambda_D\theta}{\sqrt{1-\theta}}\right)\frac{1}{\sqrt{\pi(t_{Dp}-\theta)}} \exp\left(-\frac{\lambda_D^2\theta^2}{t_{Dp}-\theta}\right)\right\} d\theta + 2\lambda_D \int_0^1 d\eta \int_0^{\theta_2} \mathrm{C}_2\, d\theta \quad (7)$$

where

$$\theta_1 = min(1, t_{Dp}) \quad (8)$$

$$\theta_2 = min(1-\eta, t_{Dp}+\eta) \quad (9)$$

$$C_2 = \frac{1}{\sqrt{\pi(1-\eta-\theta)}} \exp\left(-\frac{\lambda_D^2\theta^2}{1-\eta-\theta}\right)$$
$$\frac{\lambda_D\theta}{\sqrt{\pi(t_{Dp}+\eta-\theta)^3}} \exp\left(-\frac{\lambda_D^2\theta^2}{t_{Dp}+\eta-\theta}\right)$$
$$-\frac{\lambda_D\theta}{\sqrt{\pi(1-\eta-\theta)^3}} \exp\left(-\frac{\lambda_D^2\theta^2}{1-\eta-\theta}\right)$$
$$\frac{1}{\sqrt{\pi(t_{Dp}+\eta-\theta)}} \exp\left(-\frac{\lambda_D^2\theta^2}{t_{Dp}+\eta-\theta}\right) \quad (10)$$

For several values of $\lambda_D$, Eq. 7 was evaluated numerically and Eq. 5 was inverted numerically by using a double numerical inversion technique based on the Stehfest[19] algorithm. Figs. 1 to 3 show the results

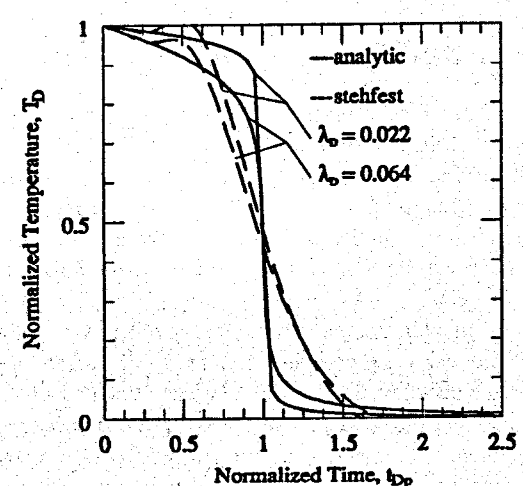


Figure 1: Solutions to MD Model for Small $\lambda_D$

of numerical integration of the solution as well as the numerically inverted Laplace space solution. From these figures, temperature return profiles computed by both methods preserve an energy balance. The energy balance can be checked by drawing a vertical line at $t_{Dp} = 1$, and determining whether the areas above the curve before the line and under the curve after the line are equal or not. For small values of $\lambda_D$, the curves for the two methods differ considerably. For large values of $\lambda_D$, the curves become smooth, and both methods produce the same result. All of the temperature return profiles have a common feature. It is that because the matrix provides a time-dependent storage, for the temperature of backflowing fluid to reach the initial reservoir temperature, an infinitely long backflow period is required.

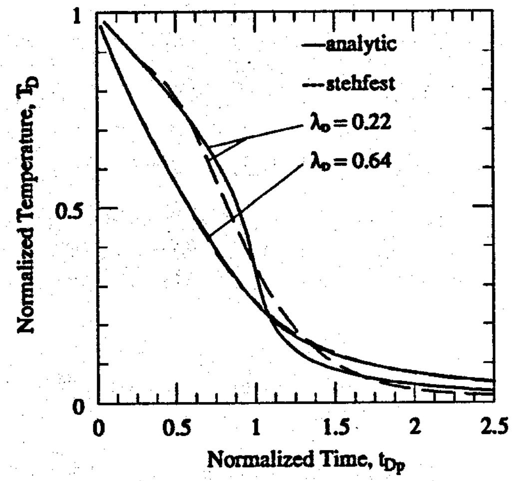


Figure 2: Solutions to MD Model for Medium $\lambda_D$

Fig. 1 shows the temperature return profiles for small values of $\lambda_D$. A small $\lambda_D$ may occur because of either $\lambda$ or $t_j$ being small. While a small $\lambda$ means that the rate of heat transfer into the matrix is small, a small value for $t_j$ means that the time is not enough for considerable heat transfer into the matrix. In Fig. 1, the total amount of heat transfer is small, and the temperature discontinuity at the convective front is reduced slightly. As a result, when $t_{Dp} = 1$ a large sudden temperature drop is observed in the temperature return profile.

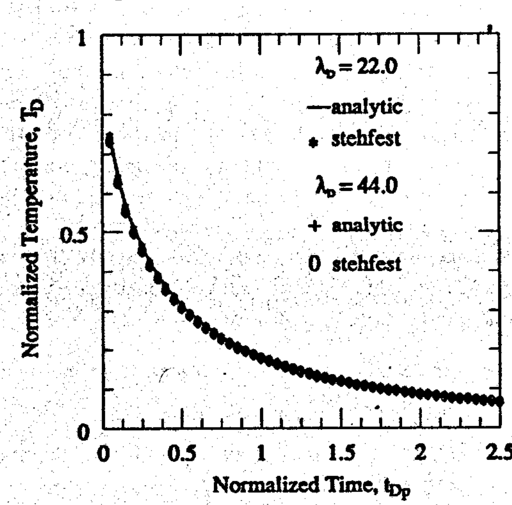


Figure 3: Solutions to MD Model for Large $\lambda_D$

In Fig. 2, values of $\lambda_D$ are moderate, and there is a moderate amount of heat transfer, but the rate of transfer is not enough to smooth the temperature discontinuity at the convective front. In Fig. 3, $\lambda_D$ values are high, which means either a high $\lambda$ or $t_j$ value or both being moderate such that the product $\lambda_D$

is high. If $\lambda$ is high, then the injected fluid is heated up to the initial temperature before it is transported far in the fracture. Thus, the temperature discontinuity is virtually eliminated. If, on the other hand, $t_j$ is high, the continuous sorption of heat from the matrix into the fracture eliminates the temperature discontinuity at the convective front.

The value of parameter $\lambda_D$ is obtained by matching temperature return profiles of thermal injection-backflow tests with theoretical temperature return profiles.

## 2.3 Estimation of the Thermal Breakthrough Time

A new method of estimating the thermal breakthrough time during reinjection in geothermal reservoirs is presented in the following. In heat transport problems, a dimensionless temperature variable is:

$$T_D = \frac{T - T_o}{T_{in} - T_o} \tag{11}$$

Based on the Lauwerier model, the thermal breakthrough time, if equal the breakthrough time of $T_D = 0.75$, is[15]:

$$t_t = \frac{\rho_1 c_1}{\rho_w c_w} \frac{t_w}{\phi_f} + \frac{\lambda_D^2}{t_j} \frac{t_w^2}{0.8134^2} \tag{12}$$

The contribution of the first group of terms in Eq.12 is small compared to the contribution of the second group of terms, because the lateral heat conduction is the main mechanism retarding propagation of the thermal front. Therefore, only two parameters $t_w$ and $\lambda_D$ are needed to estimate the thermal breakthrough time. The water transit time $t_w$ is a measure of flow speed and can be obtained from interwell tracer tests. The parameter $\lambda_D$ is a measure of thermal interaction between the fracture and the matrix, and may be determined by thermal injection-backflow tests.

In conclusion, the new method requires the following steps to estimate the thermal breakthrough time during reinjection:

1. estimate the water transit time $t_w$ from interwell tracer tests,
2. estimate the parameter $\lambda_D$ from thermal injection-backflow tests,
3. substitute values of $t_w$ and $\lambda_D$ into Eq. 12 to evaluate the thermal breakthrough time.

This new technique does not have the disadvantages of previously suggested methods namely, ambiguity of estimates from non-thermal methods and high cost and long periods of thermal interference tests.

| $\phi$ | $k_w$ $\frac{W}{m^oC}$ | $\rho_w$ $\frac{kgm}{s}$ | $c_w$ $\frac{kJ}{kg^oC}$ | $k_r$ $\frac{W}{m^oC}$ | $\rho_r$ $\frac{kg}{m^3}$ | $c_r$ $\frac{kJ}{kg^oC}$ |
|---|---|---|---|---|---|---|
| 0.05 | 0.677 | 890 | 4.371 | 2.855 | 2640 | 0.82 |

Table 1: Fluid and Rock Properties in the System

| $\phi_f b\,(mm)$ | 0.5 | 1 | 2 | 5 | 10 | 50 |
|---|---|---|---|---|---|---|
| $\lambda_D/\sqrt{t_j}$ | 76.6 | 38.3 | 19.2 | 7.6 | 3.8 | 0.76 |
| $t_t\,(years)$ | 2531 | 633 | 159 | 25 | 6 | 0.25 |

Table 2: Estimated Thermal Breakthrough Times

# 3 Application of the New Method

Three aspects namely, design considerations of thermal injection-backflow tests, use of these tests for identifying dominant flow geometry and temperature dependence of physical and thermal properties must be considered in the applications of the new method.

In design considerations, injection periods of thermal injection-backflow tests must be as small as possible. In addition, temperature measurements must be as frequent and precise as possible at early times during the backflow period. Figs. 1 to 3 show that as $\lambda_D$ values increase, temperature return profiles converge to single curve. Since the parameter $\lambda_D$ is directly proportional to $\sqrt{t_j}$, the injection period must be as small as possible so that distinct temperature return profiles are obtained.

Using the values given in Table 1 for the rock and fluid properties[16] and for $t_j = 1\ hr$ and $t_w = 50\ hrs$, corresponding breakthrough times were calculated and are given in Table 2. Temperature return profiles for $\lambda$ values corresponding to a range of effective fracture apertures given in Table 2 are shown in Figs. 4 to 6.

These figures show that in order to distinguish between the temperature profiles from flow paths with aperture sizes less than 2 $mm$, injection periods must be in the order of one hour. Conducting such a test, however, has major technical difficulties and reliability problems due to a small radius of investigation.

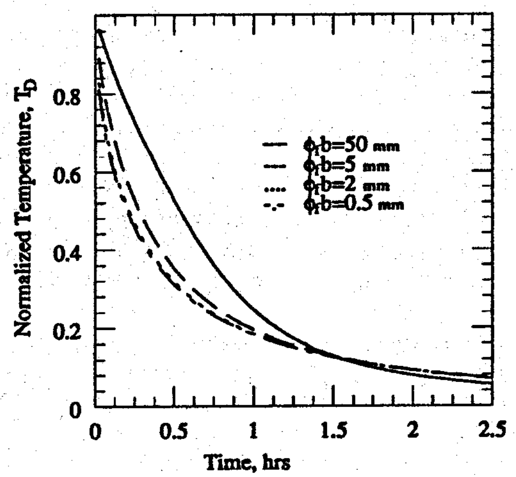


Figure 4: One-Hour Injection Period Solutions to MD Model

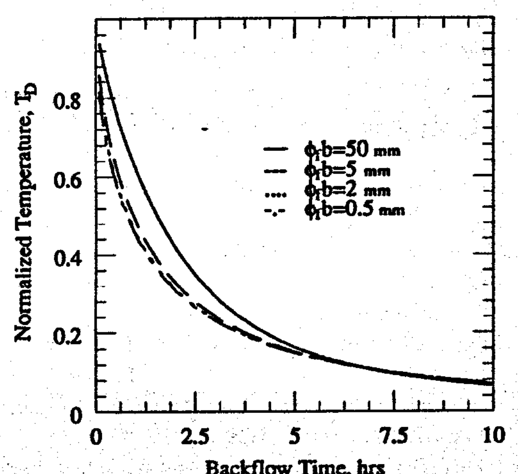


Figure 5: Four-Hour Injection Period Solutions to MD Model

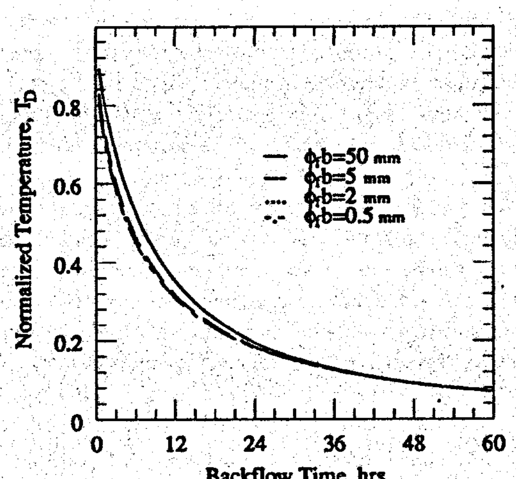


Figure 6: Twenty-Four-Hour Injection Period Solutions to MD Model

Fortunately, a premature thermal breakthrough is not a concern for flow paths with aperture sizes 2 *mm* or less. These figures also indicate that temperature return profiles reveal the most information at early times. Nevertheless, small temperature differences between temperature return profiles requires a precise measurement of temperatures.

Figs. 7 to 9 show temperature return profiles for $\lambda$ values given in Table. 2 at early backflow times.

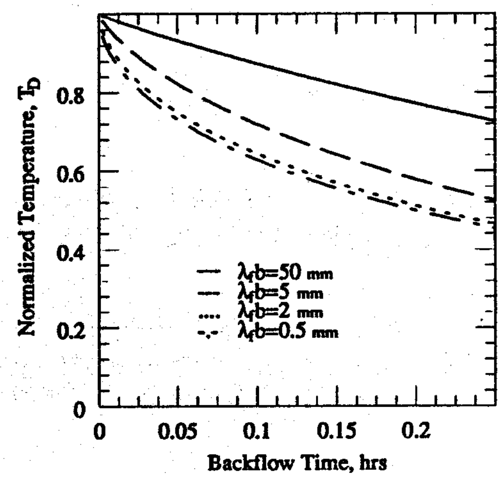


Figure 7: Early Time Temperature Return Profiles for One-Hour Injection Period

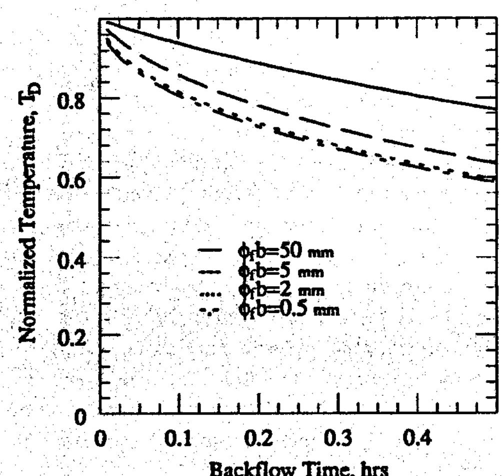


Figure 8: Early Time Temperature Return Profiles for Four-Hour Injection Period

In these figures, the difference between temperature return profiles for effective fracture aperture sizes of 5 and 0.5 *mm* varies from five to ten percent only. For the two paths of aperture sizes of 5 and 0.5 *mm*, a premature breakthrough is likely only for the former.

These results lead to an important conclusion on the nature of fast flow paths: Since, a fracture with

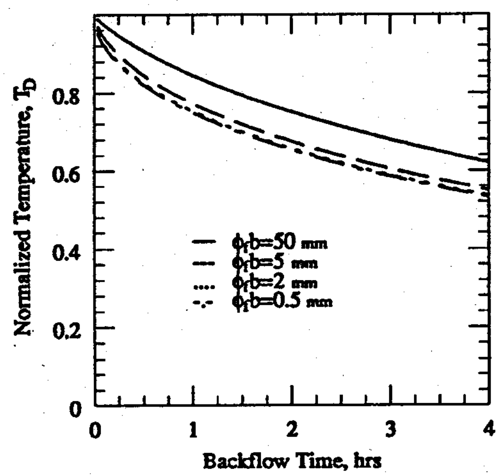


Figure 9: Early Time Temperature Return Profiles for Twenty-Four-Hour Injection Period

an aperture size in the order of several milimeters or larger is unrealistic, a premature thermal breakthrough is unlikely due to a single fracture as long as a thermal equilibrium exists at the fracture matrix interface. However, a fast flow path consisting of several fractures along a fault zone can have an effective fracture aperture size in the order of several milimeters or larger, and cause a premature thermal breakthrough during reinjection.

The second aspect is the application of the new method of estimating the thermal breakthrough time to other flow geometries. The assumptions of the method are that flow occurs in a single vertical fracture and lateral heat conduction is the main mechanism retarding the propagation of the thermal front. In fact, the assumption that the flow path is vertical can be removed since the solution given by Eq. 7 is valid for both linear and radial flow geometries, and for other general flow paths. The expressions for the breakthrough time, however, must be modified according to the assumed geometry. Eq. 12 can be used to estimate the breakthrough time for any flow path with a constant surface to volume ratio independent of the position. As for radial flow, a similar expression can be derived by using the solution for unidirectional flow.

If a radial geometry exists, the effective flow path aperture could be the effective aperture of a horizontal fracture as well as the effective thickness of the reservoir. However, horizontal fractures are unlikely to occur in natural flow systems. If a radial flow exists and assumption of a uniform temperature distribution over the reservoir thickness holds, corresponding $\lambda_D$ values would be much smaller than unity due to high $\phi b$ values representing effective reservoir thickness. As a result, temperature profiles would be similar to the ones in Figs. 1 and 2. Consequently, since nature of temperature return profiles for radial flow in a homogeneous reservoir is different than of linear flow in fracture, thermal injection-backflow tests may be used for identifying the flow geometry of the system.

Finally, the effect of temperature on fluid and rock properties are of importance. While the effect of temperature on some of the rock and the fluid properties such as $\rho_w c_w$ and $\rho_r c_r$ are not important others such as $k_w$ and $k_r$ may be affected by temperature. In Table 1 the properties of water were evaluated at $176°C$, and the reservoir rock was assumed to be granitic. It is reported[16] that $k_r$ of granite may have values ranging from 1.73 to 3.98. If $k_r$ is taken to be 1.73, then for $\phi_f b = 1mm$, the corresponding breakthrough time in Table 2 would have decreased from 633 to 398 years. This demands an accurate determination of the thermal conductivity of the rock. An in-situ determination of thermal conductivity and other parameters as well may be achieved by using thermal injection-backflow tests.

# 4 Conclusions

A new method of forecasting the thermal breakthrough time during reinjection is presented. Based on the Lauwerier model, the thermal breakthrough time depends on water transit time $t_w$ and $\lambda_D$ which is a measure of thermal interaction between the fracture and the adjacent matrix. Interwell tracer tests permit estimating the water transit time $t_w$. The parameter $\lambda_D$, on the other hand, may be obtained from thermal injection-backflow tests. This new method seems to avoid the disadvantages of previously suggested methods based on either tracer tests or thermal interference tests.

For the application of the method, thermal injection-backflow tests with small injection periods are essential to identify flow paths likely to cause a premature thermal breakthrough. Since early parts of temperature return profiles yield the most information, frequent and precise temperature measurements are necessary at early backflow times.

The solution to the Lauwerier model for thermal injection-backflow tests is valid for both radial and linear geometries. Since, temperature return profiles of linear and radial flow geometries may be significantly different, thermal injection backflow tests may permit identifying flow geometry of the system.

# References

[1] Avdonin, N. A.: "Some Formulas for Calculating the Temperature Field of a Stratum Subject

to Thermal Injection," *Nefti Gaz*, (1964), *3*, 37-41.

[2] Budd C. F. Jr.: " Geothermal Energy for Electrical Generation," *J. Pet. Tech.*, (Feb. 1984), 189-195.

[3] Bullivant, D. P.: "Tracer Testing of Geothermal Reservoirs," Ph.D. Thesis, Department of Theoretical and Applied Mechanics, School of Engineering, University of Auckland, Auckland, New Zealand, (1988).

[4] Ditkin, V.A. and Prudnikov, A. P.: *Operational Calculus in Two variables*, Int. Ser. Pure Appl. Math., Pergamon Press, London, (1962), 46-47.

[5] Dubner, H. and Abate, J.: "Numerical Inversion of Laplace Transforms by Relating Them to the Finite Fourier Cosine Transform," *J. ACM.*, (Jan. 1968) *15*, 1, 115-123.

[6] Fossum, M. P.: "Tracer Analysis in a Fractured Geothermal Reservoir: Field Results From Wairakei, New Zealand," Stanford Geothermal Program, SGP-TR-56, Stanford, CA, (1982).

[7] Gringarten, A. C., Witherspoon, P. A. and Ohnishi, Y.: "Theory of Heat Extraction From Fractured Hot Dry Rock," *J. Goephys. Res.*, (1975), *80*, 8, 1120-1124.

[8] Horne, R. N. and Rodriguez, J.: "Dispersion in Tracer Flow in Fractured Geothermal Systems," *Proceedings, 7th Annual Stanford University Geothermal Workshop*, (1981).

[9] Horne, R. N.: "Geothermal Reinjection Experience in Japan," *J. Pet. Tech.*, (March 1982a), 495-503.

[10] Horne, R. N.: "Effects of Water Injection into Fractured Geothermal Reservoirs : A Summary of Experience Worldwide," *Geothermal Resources Council*, Davis, CA (1982), Special Report 12, 47-63.

[11] Kocabas, I. and Horne, R. N.: "Analysis of Injection-Backflow Tracer Tests in Fractured Geothermal Reservoirs," *Proceedings*, Thirteenth Workshop on Geothermal Engineering, Stanford U., Stanford, CA (Jan. 1987).

[12] Kocabas, I. : "Analysis of Injection-backflow Tracer Tests" Stanford Geothermal Program, SGP-TR-96, Stanford, CA, (1986).

[13] Kocabas, I. : "Analysis of Tracer and Thermal Transients During Reinjection" Ph.D. Thesis, Stanford University, (1989).

[14] Lauwerier, H. A.: "The Transport of Heat in an Oil Layer Caused by the Injection of Hot Fluid," *Appl. Sci. Res.*, (1955), *5*, 2-3, 145-150.

[15] Pruess, K. and Bodvarsson, G. S.: "Thermal Effects of Reinjection in Geothermal Reservoirs With Major Vertical Fractures," *J. Pet. Tech.*, (September 1984), 1567-1578.

[16] Reynolds, W. C. and Perkins, H. C.: *Engineering Thermodynamics*, 2nd ed., McGraw-Hill, New York, (1977).

[17] Satman, A.: " Reinjection, " *Reservoir Engineering Assessment of Geothermal Systems*, H. J. Ramey, Jr., (Editor), Department of Petroleum Engineering, Stanford University, (1981), 10.3-10.18.

[18] Sauty, J. P., Gringarten, A. C. and Landel, P. A.: "The Effect of Dispersion on Injection of Hot Water in Aquifers," *Proceedings of Second Invitational Well Testing Symposium*, Lawrence Berkeley Lab., Berkeley, Calif., (1979), 122-131.

[19] Stehfest, H.: "Numerical Inversion of Laplace Transforms," *Communications, ACM 13*, (1970), 144-149.

[20] Tester, J. N., Bivins, R. L. and Potter, R. M.: " Interwell tracer Analysis of a Hydraulically Fractured Granitic Geothermal Reservoir," *SPEJ*, (Aug. 1982), 537-554.

[21] Voelker, D. and Doetsch, G.: *Die Zwiedimensionale Laplace-Transformation* Birkhauser, Basel Switzerland, (1950), 186-187.