

## LONGEVITY CALCULATIONS FOR HDR GEOTHERMAL RESERVOIRS

Derek Elsworth

Waterloo Centre for Groundwater Research  
University of Waterloo, Waterloo  
Ontario, Canada N2L 3G1

### ABSTRACT

A conceptual model is presented to describe thermal recovery from a semi-infinite hot dry rock (HDR) geothermal reservoir containing an equidimensional permeable zone. Transient behaviour may be represented uniquely by five dimensionless parameters. Variation in production temperature ( $T_D$ ) with time ( $t_D$ ) is influenced by reservoir throughput ( $Q_D$ ), thermal porosity ( $\Phi_D$ ) and depth ratio ( $a/z$ ). Of these, only throughput ( $Q_D$ ) exercises significant control on transient performance, the parameter being directly proportional to reservoir circulation rate and inversely proportional to the effective radius of the stimulated zone. Steady production temperature ( $T_D$ ) is indexed to throughput ( $Q_D$ ) and depth ratio ( $a/z$ ), only. Steady production temperatures are always highest for a host medium bounded by a proximal constant temperature surface and lowest for an insulated boundary. Boundary effects are insignificant for reservoir burial depths up to an order of magnitude greater than the reservoir radius. A threshold behaviour in time ( $t_D Q_D$ ) is evident for very large reservoir throughput ( $Q_D$ ). This bounding behaviour describes, in dimensionless time ( $t_D Q_D$ ), the maximum rate at which thermal depletion may occur. This state is evident for large dimensionless throughput magnitudes ( $Q_D$ ) corresponding directly with high circulation rates within the reservoir. Predictions compare favourably with results from a 300-day circulation test at the Fenton Hill Geothermal Energy Site, New Mexico.

### INTRODUCTION

The original concept for HDR geothermal energy production involved propagation of a massive hydraulic fracture to facilitate full hydraulic connection with a secondarily introduced borehole. This concept spawned a number of conceptual models to allow evaluation of thermal drawdown within the hydraulically closed system for single (Gringarten and Sauty, 1975) and multiple fractures (Gringarten and Witherspoon, 1973; Gringarten et al., 1975). Recent attempts to stimulate HDR reservoirs have resulted in

inflation of a voluminous and roughly equidimensional region through which complex flow paths penetrate (Baria et al., 1987; Fehler, 1987). The equiaxed form of this zone of permeability enhancement may be discerned from both the passive seismic record and the results of tracer testing (Robinson and Tester, 1984).

The following suggests a conceptual model to represent the thermal drawdown behaviour of a spherical stimulated zone embedded within impermeable host rock.

### CONCEPTUAL MODEL

A schematic of the reservoir model is illustrated for free surface and insulated caprock conditions in Figures 1 and 2, respectively. Thermal energy is withdrawn from the system through circulation of fluid within a spherical and hydraulically closed zone of radius,  $a$ . The secondary porosity of this zone is assumed constant and of magnitude,  $\phi$ , with the external medium returning zero secondary porosity. Fluid is circulated at constant volumetric flow rate,  $q_F$ , with a prescribed injection temperature  $T_F$  and unknown outlet temperature  $T_F(t)$ . The following assumptions are germane to the model:

1. All spatial, temporal, and material parameters are assumed constant with time excepting the outlet fluid temperature,  $T_F(t)$ .
2. Fluid temperature within the stimulated zone is prescribed as uniform and of magnitude  $T_F$ , neglecting any spatial dependence. This condition requires that the blocks comprising the stimulated zone remain in thermal equilibrium with the circulating fluid; a condition that is justified as block size, and therefore thermal diffusion length, is reduced.
3. Thermal transport within the spherical stimulated zone is by forced convection and in the surrounding medium by pure conduction.

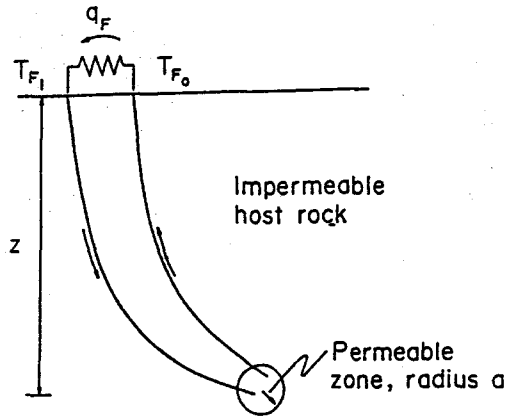


Figure 1. Circulation geometry for constant surface temperature HDR model

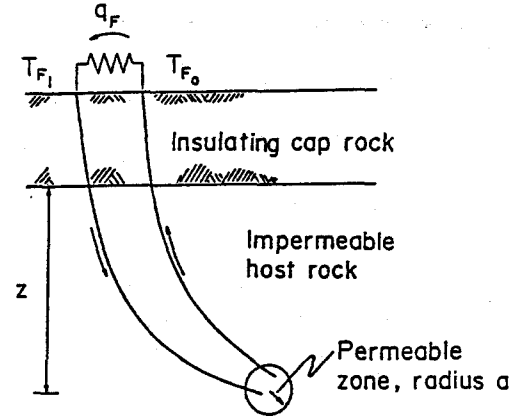


Figure 2. Circulation geometry for insulated caprock HDR geothermal reservoir model

### ENERGY BALANCE

An energy balance relationship may be defined for the production zone. Thermal energy is withdrawn from the system by uniform drawdown within the spherical reservoir. This drawdown in turn stimulates a conductive heat flux,  $q_T(a,t)$ , from the surrounding rock where equivalence is maintained on the periphery of the spherical zone between the temperatures of the circulating fluid ( $T_F$ ) and the external rock medium. In this manner, the energy balance equation may be stated as

$$q_T(a,t) = q_F \rho_F c_F (T_F(t) - T_{F_i}) + \frac{4}{3} \pi a^3 \rho_S c_S \frac{\partial T_F(t)}{\partial t} \quad (1)$$

where, the three component terms, from left to right, represent: (i) the conductive heat flux supplied from the external host rock; (ii) the thermal flux removed from the system through forced convection; and (iii) the thermal inertia of the system. The heat capacity of the fluid is incorporated as  $\rho_F c_F$  and the aggregated heat capacity of the fluid saturated reservoir zone is given as  $\rho_S c_S = (1-\phi)\rho_R c_R + \phi\rho_F c_F$  where  $\rho_R c_R$  is the heat capacity of the rock.

Initial conditions are applied to equation (1) where the system is initially at rest and the temperature distribution is prescribed by the geothermal gradient,  $T_R$ . At the initiation of thermal drawdown, heat flux to the reservoir volume is stimulated through  $q_T(a,t)$ . The magnitude of this flux is determined directly from solution of the spherically symmetric initial value problem for conductive heat flow to a

spherical source (eg. Carslaw and Jaeger, 1959). With this final component, equation (1) may be solved to represent thermal recovery from a spherical reservoir within an infinite medium. Image theory is used to approximately represent the behaviour of a semi-infinite host medium for the constant surface temperature and zero flux conditions represented by Figures 1 and 2, respectively. Linear superposition in time, through Duhamel's theorem, is used to formally evaluate equation (1) for the infinite and semi-infinite cases. The interested reader is referred to Elsworth, (1989), for further information regarding transient solution.

### COMPUTATIONAL RESULTS

The transient thermal production behaviour is uniquely represented by five dimensionless parameters, namely:

$$T_D = \frac{(T_{F_i} - T_{F_o})}{(T_{F_i} - T_R)} \quad (2a)$$

$$Q_D = \frac{q_F \rho_F c_F}{K_R a} \quad (2b)$$

$$\Phi_D = \frac{\rho_S c_S}{\rho_R c_R} \quad (2c)$$

$$t_D = \frac{K_R t}{\rho_R c_R a^2} \quad (2d)$$

and

$$a/z \quad (2e)$$

where  $K_R$  is the thermal conductivity of the intact rock surrounding the reservoir and  $z$  is the depth of the reservoir below the surface or insulating caprock as illustrated in Figures 1 and 2. The controlling parameters are therefore dimensionless output temperature ( $T_D$ ), dimensionless circulation rate ( $Q_D$ ), dimensionless thermal porosity ( $\Phi_D$ ), dimensionless time ( $t_D$ ), and dimensionless depth ratio ( $a/z$ ).

As a specialization of the general transient result, the steady long term behaviour is given by

$$T_D = \left[ 1 + \frac{1}{4\pi} Q_D \left( 1 \pm \frac{a}{2z} \right) \right]^{-1} \quad (2)$$

where the positive and negative signatures applied against the dimensionless depth ratio refer to cases of zero flux and constant (with time) temperature constraints on the surface of the half-space, respectively. This suggests that thermal supply from the exterior, by conduction, is most significant for small circulation rates,  $Q_D$ . This intuitive result is reinforced by the significant magnitude of the dimensionless temperatures that may be recovered. As  $Q_D \rightarrow 0$ , no thermal drawdown is experienced, and hence thermal recovery remains finite even as  $t_D \rightarrow \infty$ . This result is important for geothermal energy production schemes operating under low circulation rates since thermal recovery magnitudes are guaranteed. For production at larger circulation rates, knowledge of the transient thermal drawdown is important in predicting useful lifetimes.

### TRANSIENT BEHAVIOUR

Of prime interest in determining the projected longevity of energy production is knowledge of the reduction in reservoir production temperature,  $T_D$ , as a function of dimensionless time,  $t_D$ , for different production rates,  $Q_D$ . The results are represented most compactly if thermal drawdown ( $T_D$ ) is represented as a function of the product  $t_D Q_D$ . This method of compressing the time dimension is used throughout the following.

Thermal histories are most markedly sensitive to variations in the circulation rate,  $Q_D$  and less sensitive to the thermal porosity,  $\Phi_D$ , which is sensibly restricted to a relatively narrow band of variation. For igneous rocks, the thermal porosity term  $\Phi_D$  will always be greater than unity. Physical constants representative of granite as the host and water as the percolating fluid impart  $\Phi_D \approx 1.1$  for an unrealistically high secondary porosity of 10%. As secondary porosity is reduced ( $\phi \rightarrow 0$ ), the dimensionless reservoir porosity term approaches

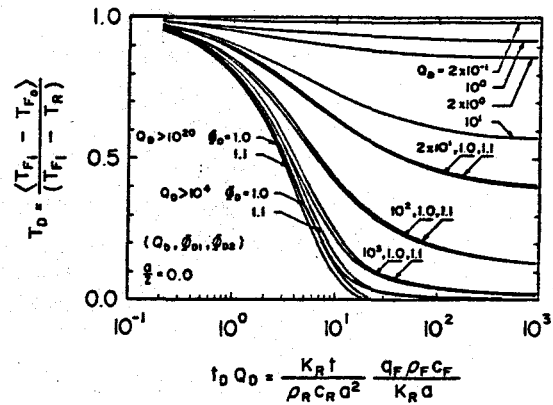


Figure 3. Thermal drawdown histories for a reservoir within an infinite host medium. Thermal porosities ( $\Phi_D$ ) of 1.0 and 1.1.

unity. A suitable range for the reservoir porosity parameter would therefore appear to be  $1.0 \leq Q_D \leq 1.1$ .

Results for temperature histories in an infinite medium ( $a/z=0$ ) are reported in Figure 3. As a natural consequence, the solution exhibits uniform rock temperature at the periphery of the permeable zone. It is apparent that thermal history is sensitive to  $\Phi_D$  only for large dimensionless throughputs  $Q_D$ . For  $Q_D < 10^1$  the results are indistinguishable for the two thermal porosities  $\Phi_D$  of 1.0 and 1.1. Even for large  $Q_D$  values the discrepancy appears insignificant. The sense of the modification to the thermal histories suggests that increased porosities slightly forestall thermal depletion. As evidenced in equation (3) and also in Figure 3, long-term (steady) withdrawal temperatures remain finite and, more importantly, significant for throughput values,  $Q_D$  less than  $10^3$ . Depending on the threshold temperatures required to ensure efficient energy conversion, the productive life of the geothermal reservoir may remain unbounded.

For very large magnitudes of  $Q_D$ , a threshold behaviour is apparent. Figure 3 illustrates the predicted temperature as  $Q_D \rightarrow \infty$ . This behaviour corresponds to high circulation rates where the thermal supply from the surrounding rock becomes negligible. In this instance, solution of equation (1) with  $q_R(a,t)=0$  becomes  $T_D = \exp[-3t_D Q_D / 4\pi]$  resulting in the limiting curvature of Figure 3 for  $Q_D = 10^{20}$ . Physically, the limiting behaviour corresponds to thermal drawdown in a sphere of radius  $a$ , embedded within an insulating medium. The total thermal energy extracted at full depletion is given by the product of sphere volume, specific heat capacity of that volume and

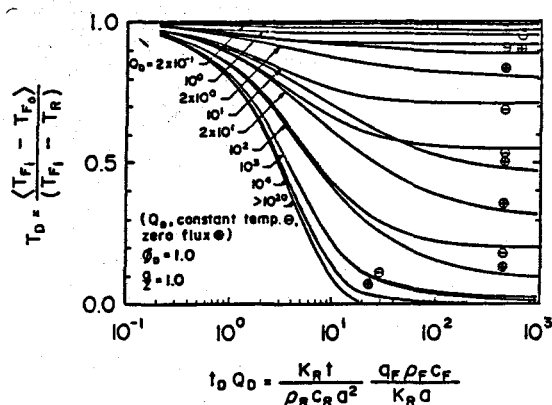


Figure 4. Thermal drawdown histories for a reservoir within a semi-infinite host medium. Depth ratio  $a/z=1.0$ . Positive and negative demarcations represent insulating caprock and constant surface temperature constraints, respectively.

temperature change. For fixed thermal diffusivities of the host medium, together with a constant reservoir dimension,  $a$ , the thermal histories are moved laterally (earlier) in real time by one order of magnitude for every increase in real circulation rate ( $q_F$ ) of one order of magnitude. Thus, as anticipated, the energy production rate is directly conditioned by the circulation rate through the system. Intuitively, minimal thermal depletion is evident in Figure 3 for small flow rates  $q_F$  or large reservoir diameters as evidenced by small values of dimensionless throughput  $Q_D$ .

The thermal response of semi-infinite systems are bounded, in the most extreme case, by a depth ratio,  $a/z$ , of unity. Although results are physically meaningless for the instance of a constant temperature surface, comparisons are useful since thermal histories bound candidate responses for all depth ratios between unity and zero. Semi-infinite responses are illustrated in Figure 4 for a depth ratio of unity. The influence of an insulated surface is to reduce the ultimate steady outlet temperature  $T_D$  over that of the infinite case. This result appears reasonable since a reduced volume, and hence thermal reservoir, is available for depletion in the semi-infinite case. Conversely, the presence of a surface retained at constant (ambient) temperature elicits a more favourable (hotter) thermal response to that of the infinite case. The influence of boundaries is only significant for intermediate values of dimensionless throughput  $Q_D$  and even in these instances is only apparent at large dimensionless times  $t_D Q_D$ . The range  $10^{-1} < Q_D < 10^3$  brackets this boundary sensitive region. The branching of thermal histories occurs earlier in dimensionless time with decreasing throughput  $Q_D$ .

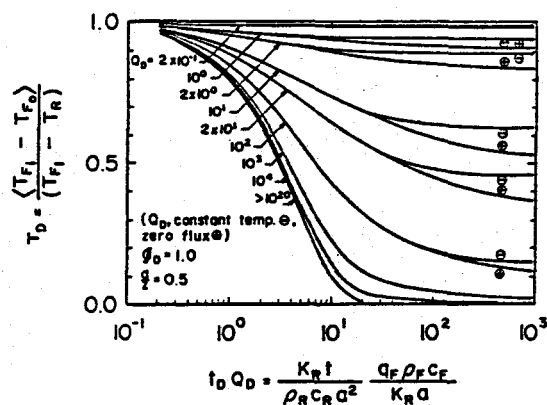


Figure 5. Thermal drawdown histories for a reservoir within a semi-infinite host medium. Depth ratio  $(a/z)=0.5$ . Positive and negative demarcations represent insulating caprock and constant surface temperature constraints, respectively.

Thermal responses for depth ratios of one half are illustrated in Figure 5. Similar to the results for  $a/z=1$ , the influence of boundaries are only significant in the range  $10^{-1} < Q_D < 10^3$ . As the reservoir depth  $a/z$  decreases to  $10^{-1}$ , the results for bounded reservoirs are indistinguishable from the infinite case and may be correctly interpolated from Figure 3.

From knowledge of thermal response, depletion times may be directly determined to aid in defining useful lifetimes of individual reservoir configurations. Ultimate withdrawal temperatures may be determined from equation (3). The time required for the outlet temperature to reach fifty per cent and ninety-five per cent of the steady temperature values are defined as  $t_D^{50} Q_D$  and  $t_D^{95} Q_D$ , respectively. These results are illustrated in Figure 6 for the limiting cases of bounded and infinite reservoir geometries for all significant values of  $Q_D$ . Reservoirs bounded by a constant temperature surface deplete more rapidly than either insulated or infinite geometries. This behaviour is most marked in the very long-term performance as evidenced by the  $t_D^{95} Q_D$  parameter.

#### FENTON HILL HDR RESERVOIR

Comparison is possible between predictions elicited from the thermal recovery model and results from a 300-day circulation test conducted at the Fenton Hill experimental reservoir in New Mexico. Results are reported from Run Segment 5 of Experiment 217 in Zyvoloski et al. (1981), where long-term circulation was induced between the injection level at 2903 m and withdrawal level at 2708 m below surface. The known separation of the



indexed directly to dimensionless throughput  $Q_D$  and is apparent for  $Q_D > 10^3$ .

Large radii or low circulation rate systems exhibit the most desirable thermal behaviour in the long-term. This response is further conditioned, however, by the flow impedance characteristics of the stimulated zone and cannot, therefore, be viewed in isolation.

#### ACKNOWLEDGEMENTS

The foregoing represents partial results of work supported by the National Science Foundation under Grant No. MSM-8708976. The assistance of Steven Birdsell, Michael Fehler, and Bruce Robinson, of Los Alamos National Laboratory, in providing data from the Fenton Hill reservoir is gratefully acknowledged.

#### REFERENCES

- Baria, R., A.S.P. Green, and R.H. Jones, (1987) "Anomalous Seismic Events Observed at the CSM HDR Project", Proceedings International Workshop on Forced Fluid Flow through Strong Fractured Rock Masses, Commission of European Communities, EUR 11164/1, April, pp. 321-336.
- Carslaw, H.S. and J.C. Jaeger, (1959) "Conduction of Heat in Solids," 2nd edition. Clarendon, Oxford.
- Elsworth, D. (1989) "Theory of Thermal Recovery from a Spherically Stimulated HDR Reservoir," J. Geophys. Res., in press.
- Gringarten, A.C. and P.A. Witherspoon (1973) "Extraction of Heat from Multiple Fractured Hot Dry Rock," Geothermics, 2(3/4), pp. 119-122.
- Gringarten, A.C., P.A. Witherspoon, and Y. Ohnishi (1975) "Theory of Heat Extraction from Hot Dry Rock," J. Geophys. Res, 80(8), pp. 1120-1124.
- Gringarten, A.C. and J.P. Sauty (1975) "A Theoretical Study of Heat Extraction from Aquifers with Uniform Regional Flow," J. Geophys. Res., 80(35), pp. 6956-6962.
- Robinson, B.A. and J.W. Tester (1984) "Dispersed Fluid Flow in Fractured Reservoirs: An Analysis of Tracer-Determined Residence Time Distributions," J. Geophys. Res., 89(B12), 10, pp. 374-384.
- Zyvoloski, G.A. et al. (1981) "Evaluation of the Second Hot Dry Rock Geothermal Reservoir: Results of Phase I, Run Segment 3," Los Alamos Scientific Laboratory, LA-8940-HDR, 94 pages.