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RESERVOIR COMPRESSIBILITY FROM WATER-INFLUX MODELING OF LIQUID-DOMINATED SYSTEMS

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ABSTRACT

A water influx model was used to history match the pressure drawdown behavior of three liquid-dominated geothermal reservoirs. The compressibilities of confined and unconfined liquid-only reservoir systems were shown to differ by one to two orders of magnitude in the temperature range 300°C to 100°C. History matching of production data from three high temperature fields (Ahuachapan, Svartsengi and Wairakei) yielded reservoir compressibility values similar to what would be expected for unconfined (free liquid level) systems.

INTRODUCTION

The success of reservoir modeling depends largely on what information is available concerning production history, reservoir shape and size, and the properties of rock and fluid. Many different methods have been used to model the behavior of geothermal reservoirs (Grant, 1983; Bodvarsson et al. 1986). The Hurst simplified model was formulated for use in lumped parameter modeling of petroleum reservoirs with edge-water drive (Hurst, 1958). It is easily modified to describe a liquid-dominated geothermal reservoir with aquifer support.

Examples of water influx modeling of geothermal reservoirs are those of Whiting and Ramey (1969), Olsen (1984), Marcou (1985) and Brock (1986); also Gudmundsson and Olsen (1987). Because liquid-dominated geothermal reservoirs have compressibilities much higher and more highly variable than petroleum reservoirs, the model was formulated to output the compressibility by history matching in the present work.

PHYSICAL PROPERTIES

The compressibility of geothermal reservoirs depends on many factors, including the fluid state and production mechanism. Considering liquid-dominated reservoirs only, two extreme, idealized conditions can be identified: confined and unconfined (free liquid surface). The storage coefficients (m/kPa) for the two types of reservoirs are

$$S_c = \phi c h \qquad (1)$$

and

$$S_u = \frac{\phi}{\rho g} \qquad (2)$$

where the subscripts c and u refer to confined and unconfined conditions (Grant et al., 1982). Other symbols are formation porosity ϕ, fluid compressibility c, formation thickness h, fluid density ρ and the gravity constant g.

Sample values of the storage coefficients with temperature are plotted in Fig. 1, assuming 10 percent porosity ($\phi = 0.1$) and a reservoir thickness h = 1000 m. The confined storage coefficient S_c ranges from about 5×10^{-5} to 3×10^{-4} (m/kPa) from 100°C to 300°C. In this temperature range water density decreases from 958 to 712 (kg/m^3), its viscosity decreases from 283×10^{-6} to 91×10^{-6} (Pa.s) and the compressibility (liquid water only) increases from about 0.50 to 3.22 (1/GPa). In other words, the density changes 1.3 times, viscosity 3.1 times and compressibility 6.4 times for liquid water. For the same temperature range the unconfined storage coefficient S_u increases from 1×10^{-2} to 1.4×10^{-2} (m/kPa). Sveinsson (1987) has presented values for the compressibility of liquid water.

For ideal conditions, therefore, an unconfined reservoir is 200 times more compressible than a confined reservoir at 100°C and 40 times more compressible at a temperature of 300°C. These findings agree with Zais and Bodvarsson (1980) who state that the compressibility of unconfined (free liquid surface) systems is 100 to 1000 times greater than that of confined systems.

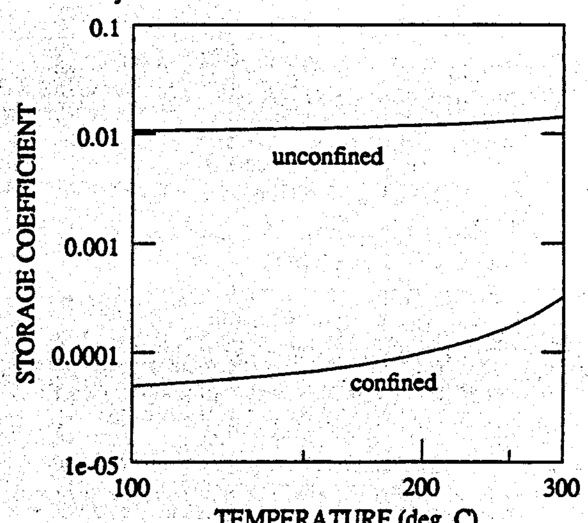

Fig. 1. Sample values of storage coefficients.

HURST'S SIMPLIFIED METHOD

It was reported by Gudmundsson and Olsen (1987) that the Hurst (1958) simplified water influx method gives a satisfactory match to production data from the Svartsengi high temperature, liquid-dominated geothermal field. Likewise, Marcou and Gudmundsson (1986) found the Hurst method satisfactory in the modeling of the Ahuachapan and Wairakei liquid-dominated reservoirs.

In the Hurst (1958) water influx method the reservoir and aquifer compressibility are assumed different and constant. See Olsen (1984) and Brock (1986) for complete derivation of the method for liquid-dominated reservoirs. For a radial reservoir/aquifer system, Hurst (1958) introduced the ratio

$$\sigma = 2\frac{c_a \rho_a}{c\rho} \tag{3}$$

where the numerator refers to aquifer properties and the denominator to that of the reservoir. Hurst (1958) gave the following drawdown solution for an infinite radial aquifer

$$\Delta p = \frac{\mu_a \sigma}{2\pi (kh)_a \rho_a} \sum_{j=0}^{n} \Delta w_{pj} N(\sigma, t_D - t_{Dj}) \tag{4}$$

written in superposition form. The Hurst function N is in Laplace space and is expressed as

$$N(\sigma, t_D) = L^{-1}\left\{ \frac{K_0(\sqrt{s})}{s^{3/2}[\sigma K_1(\sqrt{s}) + \sqrt{s}K_0(\sqrt{s})]} \right\} \tag{5}$$

The function is not analytically invertible to real space. Therefore, a numerical inversion method must be used; The Stehfest (1970) algorithm was used in the present work.

Consider two limiting solutions of the pressure drawdown in Equation 4: σ small and σ large. When $c \gg c_a$ the reservoir will dominate the overall system pressure behaviour; surrounding aquifers will not affect drawdown in reservoir pressure with time. In this case, the reservoir responds to fluid production as a confined system (Gudmundsson and Olsen, 1987).

When $c \ll c_a$ the aquifer will dominate the overall system pressure response. In effect, the aquifer is the reservoir. Hurst (1958) showed that in this case the pressure drawdown solution is the same as the general radial system solution; the line source solution applies.

HISTORY MATCHING

In history matching the production data of a reservoir is fitted to a model. The fitting consists of adjusting one or more parameters of the model to find the best match; permeability and permeability-thickness product are commonly used. Compressibility is the parameter that exhibits the greatest range in values; it is also likely to be the least known parameter in reservoir modeling. It follows, that compressibility should be a good parameter to use in history matching for liquid-dominated reservoirs. In the Hurst water influx model the compressibility ratio σ in Equation 3 was used.

The history matching method used in the present work has been detailed by Brock (1986), Marcou (1985) and Olsen (1984). The matching procedure consisted of plotting the drawdown in terms of water head

$$y_n = \frac{\Delta p_n}{\rho g} = \Delta h_n \tag{6}$$

against the Hurst function term

$$x_n = \sum_{j=0}^{n} \Delta w_{pj} \sigma N(\sigma, t_D - t_{Dj}) \tag{7}$$

A liquid-dominated reservoir system conforming to the Hurst water influx model assumptions will exhibit a straight line having the slope

$$m = \frac{\mu_a}{2\pi (kh)_a \rho_a \rho g} \tag{8}$$

The fitting procedure was the following:

(1) Select a value for σ

(2) Calculate x_n and y_n

(3) Find slope m using least squares fit on $y_n = mx_n$

(4) Calculate standard deviation of fit

(5) Select a new σ value and repeat above steps

(6) Plot standard deviation versus σ values

(7) Select σ value giving minimum standard deviation

(8) Select corresponding slope m

Fortran 77 computer programs were written for history matching and forecasting (Brock, 1986). Because the Hurst function in Equation 5 cannot be expressed analytically in real space, numerical inversion had to be used; the Stehfest (1970) algorithm was used. Although this algorithm is well behaved in the Hurst function application, it is slow in execution. In the history matching method x_n and y_n are calculated several times for each data point (often in the hundreds); the Hurst function is inside a doubly nested loop. For a data history of 200 points, say, the Hurst function is evaluated over twenty thousand times. Therefore, a table lookup method was devised to speed up the execution time. For a given σ value a table of $N(t_D)$ was calculated. A table lookup subroutine was then used to obtain by interpolation the appropriate Hurst function value, rather than repeatedly performing the Stehfest algorithm inversion. On a data set of 66 points (time, flowrate, drawdown) the execution time on a VAX 11/750 was more than 1100 seconds of CPU-time while the table lookup method took only 45 seconds. By using the table lookup method it seems model calculations can even be carried out on a typical microcomputer.

FIELD DATA

The production histories of two liquid-dominated fields have been compiled by Marcou (1985): Ahuachapan in El Salvador and Wairakei in New Zealand. The production history of the Svartsengi liquid-dominated field in Iceland has been reported by Olsen (1984). These data were readily available for the purpose of the present study (Brock, 1986; Gudmundsson et al., 1985) report the data also. The

Ahuachapan, Svartsengi and Wairakei fields are all high-temperature: 240°C, 240°C and 260°C, respectively. Grant et al. (1982) provide general information about these fields.

In addition to temperature (both in reservoir and aquifer) and production history (flow rate and drawdown with time), three reservoir parameters are required for the Hurst (1958) water influx model: reservoir radius, porosity and permeability. These parameters were guesstimated for the three liquid-dominated reservoirs and used in the present study. The parameters are shown in Table 1, where reservoir radius is expressed in terms of surface area.

RESULTS AND DISCUSSION

The first step in the history matching procedure was the selection of the σ value, which gave the minimum standard deviation (optimum match) between field data and water influx model. The standard deviations vs. σ values for the three fields are shown in Fig. 2. A minimum was observed for the three fields (Ahuachapan, Svartsengi, Wairakei). The optimum σ value and the corresponding slope m are shown in Table 2 for the three fields.

History matches for the three liquid-dominated fields are shown in Figs. 3, 4 and 5.

Table 1. Reservoir parameters input to history matching.

Field Name	Temperature T (°C)	Area (km^2)	Porosity ϕ	Permeability k (mD)
Ahuachapan	240	15	0.20	50
Svartsengi	240	4	0.05	500
Wairakei	260	15	0.20	30

Table 2. Hurst-model parameters output from history matching.

Field Name	Ratio $\sigma \times 10^{-3}$	Slope m	Standard Deviation
Ahuachapan	37.3	0.133	5.69
Svartsengi	1.6	0.087	1.77
Wairakei	72	0.072	6.56

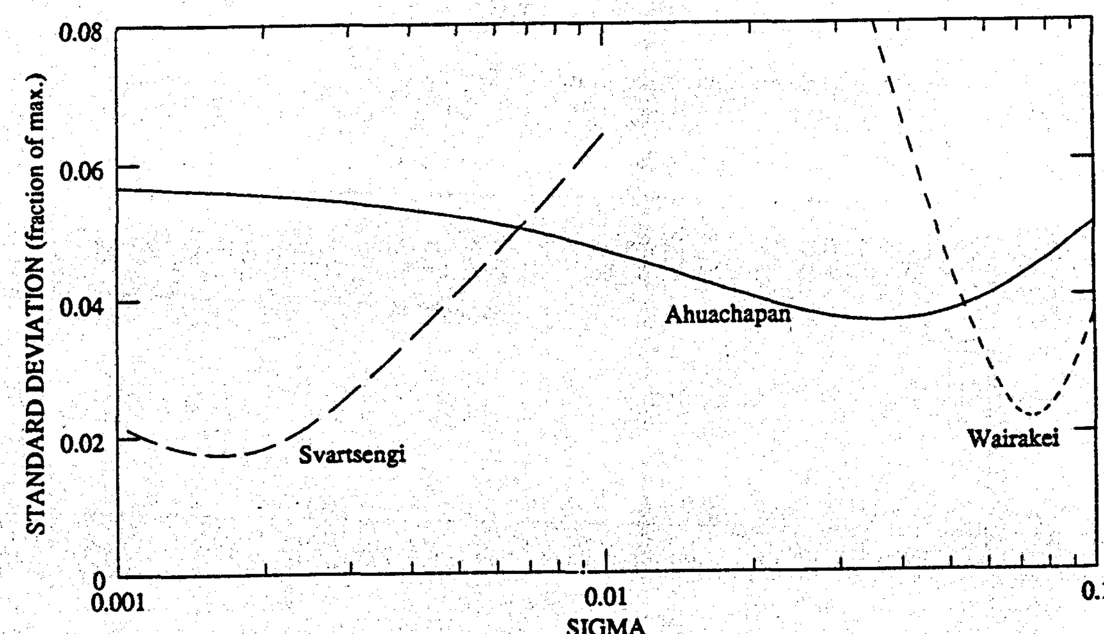

Fig. 2. Choosing the best fit: Standard Deviation vs. σ for the three fields.

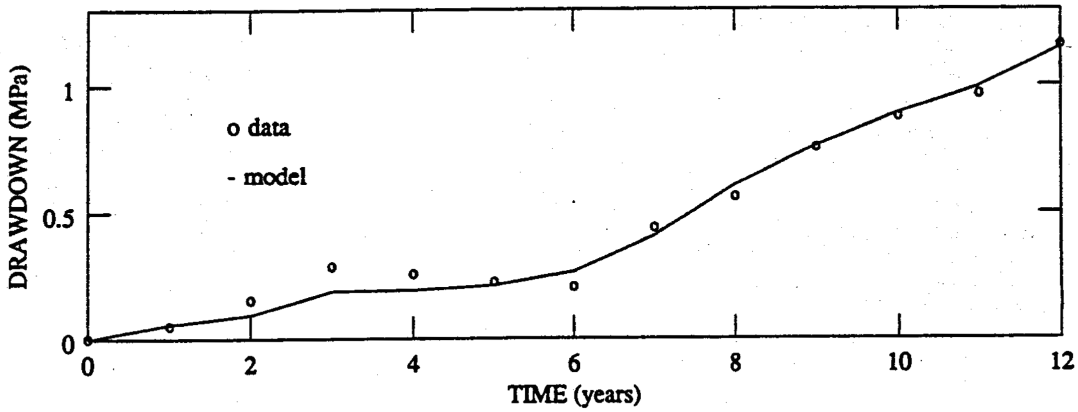

Fig. 3. Drawdown match to the Ahuachapan field data.

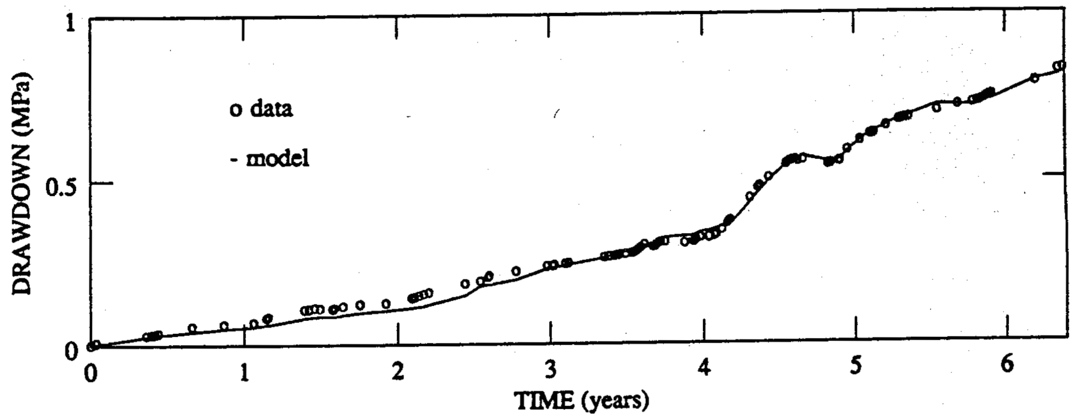

Fig. 4. Drawdown match to the Svartsengi field data.

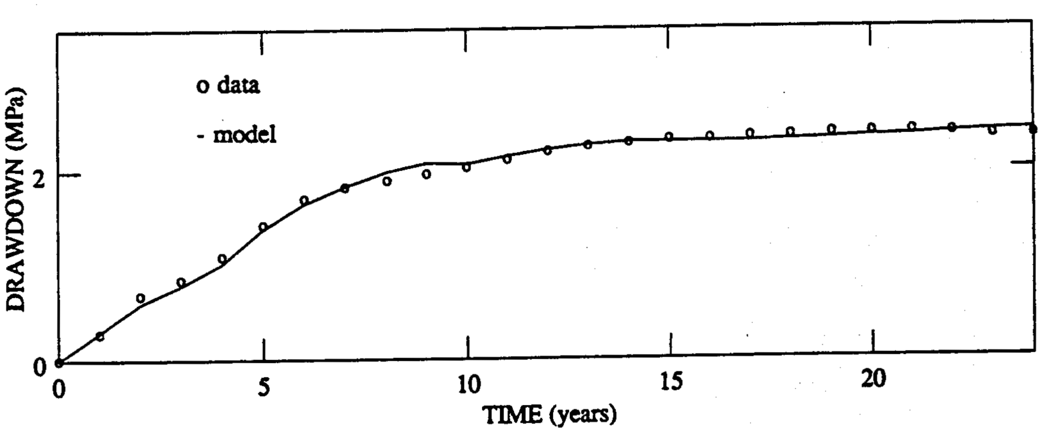

Fig. 5. Drawdown match to the Wairakei field data.

To estimate the reservoir compressibility, c, of the high- temperature fields from their optimum σ value, the aquifer compressibility must first be estimated. However, should the aquifer be assumed confined or unconfined? The density of the reservoir and aquifer fluids must also be estimated - the aquifer temperature was assumed 100°C in the present work. In Table 3 are shown the calculated compressibility values for the Ahuachapan, Svartsengi and Wairakei high-temperature, liquid-dominated reservoirs, for both confined and unconfined aquifer conditions. Also shown are the aquifer permeability-thickness product, $(kh)_a$, derived from the slope, m, obtained by history matching.

The reservoir compressibility values in Table 3 are rather high, particularly when unconfined aquifer conditions are assumed. The storage coefficients for each of the three reservoirs are given in Table 4 and plotted in Fig. 6. They were calculated using the porosity values in Table 1, the compressibility values in Table 3, and for an assumed reservoir thickness of 1000 m. Two values are shown for each reservoir; confined (lower) and unconfined (higher).

Grant et al. (1982) presented a numerical approximation for the compressibility of two-phase reservoir zones. At 240°C this approximation gives the porosity-compressibility product, ϕc, a value of 1400 (1/GPa). For a reservoir thickness of 1000 m, therefore, it corresponds to a storage coefficient of 1.4 (m/kPa). This value exceeds that of an unconfined reservoir by two to three orders of magnitude when the aquifer is confined, and by one to two orders when the aquifer is unconfined.

The results show that the Svartsengi reservoir is more compressible, by one order of magnitude, than both Ahuachapan and Wairakei. The compressibility of the

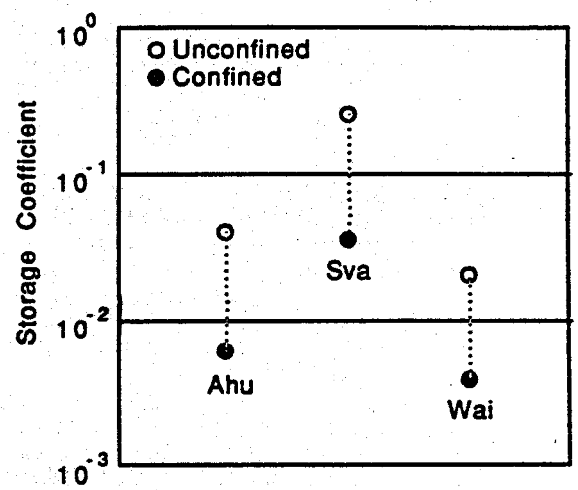

Fig. 6. Storage coefficients for the three fields.

Table 3. Reservoir compressibility and aquifer permeability-thickness product calculated from history matching.

Field Name	Confined Compressibility $(GPa)^{-1}$	Unconfined Compressibility $(GPa)^{-1}$	Permeability-Thickness $(kh)_a$ (Dm)
Ahuachapan	32	206	44
Svartsengi	736	4740	68
Wairakei	17	110	85

Table 4. Reservoir storage coefficients.

Field Name	Confined Aquifer (m/kPa)	Unconfined Aquifer (m/kPa)
Ahuchapan	6.4×10^{-3}	4.1×10^{-2}
Svartsengi	36.8×10^{-3}	23.7×10^{-2}
Wairakei	3.4×10^{-3}	2.2×10^{-2}

Svartsengi reservoir is well above that of an idealized, unconfined system (free liquid surface), no matter what the confinement of the aquifer. This suggests that the two-phase zone, known to exist near the top of the Svartsengi reservoir (Gudmundsson and Thorhallsson, 1986), may to some extent affect the overall pressure response of the system.

CONCLUSIONS

(1) For idealized conditions, an unconfined reservoir is 200 times more compressible at 100°C than a confined reservoir and 40 times more compressible at 300°C.

(2) The Hurst (1958) simplified water influx method gave a satisfactory match to production data from the high-temperature Ahuachapan, Svartsengi and Wairakei fields.

(3) Compressibility is a highly variable parameter in liquid-dominated reservoirs, much more so than porosity and permeability. In water influx and other lumped-parameter modeling, therefore, compressibility should be an output rather than an input parameter.

(4) The effective compressibility of high-temperature liquid-dominated reservoirs is similar to that shown by unconfined (free liquid surface) systems.

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