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SIMULATION OF PRESSURE RESPONSE DATA FROM GEOTHERMAL RESERVOIRS BY LUMPED PARAMETER MODELS

Gudni Axelsson

National Energy Authority, Grensásvegur 9, 108 Reykjavík, ICELAND

ABSTRACT

Detailed numerical modeling of geothermal reservoirs is time consuming, costly and requires large amounts of field data. Lumped parameter modeling is in some cases a cost effective alternative. A method has been developed that tackles simulation of pressure response data by lumped models as an inverse problem and therefore requires very little time. This method of lumped modeling has been used successfully to simulate data from several low-temperature geothermal reservoirs in Iceland. The lumped simulators have been used to predict future pressure changes and they provide information on the global hydrological characteristics of the geothermal reservoirs.

INTRODUCTION

Modeling of geothermal systems, as a tool for resource assessment, has grown significantly during the last decade. Rapid advances have been made in the development of numerical simulators for detailed and complex modeling of such systems (Bodvarsson et al., 1986). Yet detailed numerical modeling of complex fluid/rock systems, such as geothermal reservoirs, is both time consuming and costly. In addition distributed parameter modeling of geothermal reservoirs requires large amounts of geological, geophysical, geochemical and hydrological data.

Other methods for modeling geothermal systems are available. The most appropriate approach, for a particular modeling study, is determined by the available field data as well as the objectives of the study. In situations where available funds, field data and time are limited, detailed modeling may not be feasible. Lumped parameter modeling is in such cases a viable alternative. Lumped parameter models have been developed for many geothermal reservoirs (Grant et al., 1982; Bodvarsson et al., 1986). Wairakei in New Zealand (Fradkin et al., 1981) and Svartsengi in Iceland (Kjaran et al., 1979; Gudmundsson and Olsen, 1987) can be mentioned as examples. Bodvarsson (1966) discusses the usefulness of lumped methods of interpreting geophysical exploration data.

Energy from several low-temperature (< 150 °C) geothermal reservoirs in Iceland is used for space heating by various district heating services. A limited number of wells have been drilled into many of these reservoirs. But data on the production from the fields as well as data on the pressure in one or two observation wells are often available. Funds for detailed modeling may not be available to the smaller district heating services.

In this paper an effective method of lumped parameter modeling, which has been used successfully for pressure response data from several Icelandic geothermal reservoirs, is discussed. This method tackles the simulation problem as an inverse problem. It automatically fits analytical response functions of lumped models to the observed data by using a non-linear iterative least-squares technique for estimating the model parameters. The theoretical background of this method will briefly be presented, but the details are given by Bodvarsson and Axelsson (1986) and Axelsson (1985).

THEORY AND SOLUTION METHOD

Consider a general lumped network of the type sketched in Figure 1 consisting of a total of N capacitors or boxes with capacitances (storage coefficients) κ. A capacitor has the mass capacitance κ when it responds to a load of liquid mass m with a pressure $p = m/\kappa$. The capacitors are pairwise connected by up to N(N-1)/2 conductors (resistors) of conductances σ_{ik} ($\sigma_{ii} = 0$). The mass conductance of a conductor is σ when it transfers $q = \sigma\Delta p$ units of liquid mass per unit time at the impressed pressure differential Δp. The particular element σ_{ik} connects the i'th and k'th capacitors and because of linearity $\sigma_{ik} = \sigma_{ki}$. The network is open in the sense that the i'th capacitor is connected by a conductor of conductance σ_i to an external capacitor that maintains equilibrium pressure of magnitude zero. The network is closed when $\sigma_i = 0$ for i = 1,2 . . . N.

Let $p_i(t)$ be the pressure in the i'th capacitor and $q_{ik}(t)$ be the mass flow from the k'th to the i'th element. Then the basic equations are the mass flow equation

(1) $$q_{ik} = \sigma_{ik}(p_k - p_i)$$

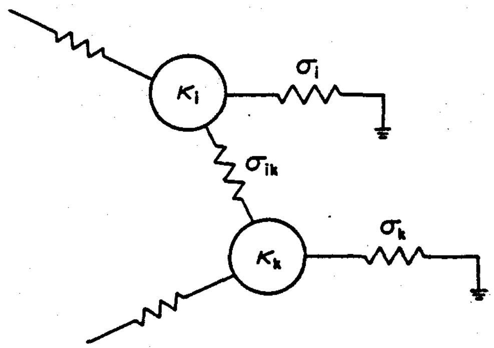

Figure 1. *General lumped capacitor/conductor network.*

and the equation for conservation of mass

$$(2)\qquad \kappa_i\, dp_i/dt = \sum_{k=1}^{N} q_{ik} - \sigma_i p_i + f_i$$

where f_i represents an external source mass flow into the i'th capacitor. Inserting (1) into (2) one obtains the basic system equations in matrix form

$$(3)\qquad \mathbf{K}\, d\vec{p}/dt + \mathbf{A}\vec{p} = \vec{f}$$

where the vectors and matrices are defined as follows

$$\mathbf{K} = [\kappa_i \delta_{ik}]$$

$$(4)\qquad \mathbf{A} = [(\sum_j \sigma_{ij} + \sigma_i)\delta_{ik} - \sigma_{ik}]$$

$$\vec{p} = (p_i), \quad \vec{f} = (f_i).$$

To obtain general solutions of the system of equations (3), one first derives the response of the network to an impulsive drive of the the k'th capacitor, at time t = 0^+, given by

$$(5)\qquad f_i = 0 \text{ for } i \neq k, \quad f_k = \delta_+(t)$$

Here $\delta_+(t)$ is the delta function in time, centered at t = 0^+. The response to this particular drive is $\vec{h}_k(t)$, the k'th impulse response vector of the network that is the solution of (3) with $\vec{f}$ given by (5). If the network is driven by a general causal drive $\vec{f}(t)$, and can be taken to be in equilibrium at t = 0, the response is obtained by the convolution

$$(6)\qquad \vec{p}(t) = \sum_{k=1}^{N}[\int_0^t \vec{h}_k(t-\tau) f_k(\tau)\, d\tau], \quad t>0$$

Equation (3) can be solved by considering the associated eigenvector problem

$$(7)\qquad \mathbf{A} = \lambda \mathbf{K}\vec{\tau}$$

where $\vec{\tau}$ and λ are the eigenvectors and eigenvalues respectively. Equation (7) has up to N non-negative eigenvalues. The matrix **A** can be diagonalized as follows

$$(8)\qquad \mathbf{T}'\mathbf{A}\mathbf{T} = \mathbf{\Lambda} \text{ or } \mathbf{A} = \mathbf{K}\mathbf{T}\mathbf{\Lambda}\mathbf{T}'\mathbf{K}$$

where **Λ** is a diagonal eigenvalue matrix, **T** the eigenvector matrix formed out of the column vectors $\vec{\tau}_j$ and **T′** the transpose of the matrix **T**. The solution of (3) with a drive given by (5) is then given by

$$(9)\qquad \vec{h}_k(t) = \mathbf{T}e^{-t\mathbf{\Lambda}}\mathbf{T}'\vec{\Delta}_k, \quad t>0$$

where $\vec{\Delta}_k$ is a vector having only one non-vanishing component equal to unity at the k'th entry.

The response of the i'th capacitor to an impulsive drive of the k'th capacitor is given by

$$(10)\qquad h_{ik}(t) = \sum_{j=1}^{N} \tau_{ij}\tau_{kj}\, e^{-\lambda_j t}, \quad t>0.$$

In practical situations a step response is often more convenient than the impulse response. The response of the i'th capacitor to a mass flow input q_k, for t>0, into the k'th capacitor is obtained by applying equation (6)

$$(11)\qquad p_{ik}(t) = q_k \sum_{j=1}^{N} \frac{\tau_{ij}\tau_{kj}}{\lambda_j}[1 - e^{-\lambda_j t}], \quad t>0.$$

It should be mentioned that closed networks have a singular matrix **A** such that $\lambda_1 = 0$. The corresponding eigenvector has the components $\tau_{i1} = V^{-1/2}$ where $V = \Sigma\kappa_i$. The solution (10) remains valid, but in the case of the step response (11) the first term of the sum becomes t/V.

To simulate pressure response data from a liquid-dominated geothermal reservoir an appropriate lumped model is chosen. Water is produced from one of the capacitors at a variable rate $q(t)$, the rate of production from the geothermal reservoir. The resulting pressure $p(t)$ is then observed in any given capacitor of the lumped model. One can write

$$(12)\qquad p(t) = \int_0^t h(t-\tau)\, q(\tau)\, d\tau$$

where h is the impulse response of the lumped model for the specific production and observation capacitors. The impulse response is given by equation (10) which can be rewritten

$$(13)\qquad h(t) = \sum_{j=1}^{N} m_j\, e^{-m_{j+N} t}$$

where N is the number of capacitors in the lumped model chosen. An iterative non-linear least-squares technique (Menke, 1984) is used to fit equations (12) and (13) to the observed data $p(t)$ and estimate the parameters m_i, which in turn depend on the properties of the model (Bodvarsson and Axelsson, 1986).

The observed pressure data is written as

$$(14)\qquad p_i = p(t_i); \quad t_i = i\Delta t, \quad i = 1,2 \ldots M,$$

where Δt is a fixed time interval, and the flow rate data is approximated by

$$(15)\qquad q(t) = q_i \text{ for } (i-1)\Delta t \leq t < i\Delta t$$

Equation (12) can be written as

$$(16) \qquad \vec{g}(\vec{m}) = \vec{p}$$

where $\vec{g}$ is a vector-valued function and

$$(17) \qquad \begin{aligned} &\vec{m} = (m_i), \quad i = 1,2\ldots 2N \\ &\vec{p} = (p_i), \quad i = 1,2\ldots M \\ &\vec{g} = (g_i); \quad g_i(\vec{m}) = p(t_i) \end{aligned}$$

Expanding equation (16) into a Taylor series the following iterative scheme can be set up to estimate the best fitting parameters $\vec{m}$ of a given model

$$(18) \qquad \begin{aligned} &\mathbf{G}_n \, \Delta\vec{m}_{n+1} = \vec{p} - \vec{g}(\vec{m}_n^{est}) \\ &\vec{m}_{n+1}^{est} = \vec{m}_n^{est} + \Delta\vec{m}_{n+1} \end{aligned}$$

where $\vec{m}_0^{est}$ is an initial guess for the parameters and the matrix **G** is defined as

$$(19) \qquad (\mathbf{G}_n)_{ij} = \left.\frac{\partial g_i}{\partial m_j}\right]_{\vec{m} = \vec{m}_n^{est}}, \quad j = 1,2..2N$$

The least squares solution of (18) is given by (Menke, 1984)

$$(20) \qquad \Delta\vec{m}_{n+1} = (\mathbf{G}_n^T \mathbf{G}_n)^{-1} \mathbf{G}_n^T \left[\vec{p} - \vec{g}(\vec{m}_n^{est})\right] \qquad n = 0,1,2,\ldots$$

where $\mathbf{G}_n^T$ is the transpose of $\mathbf{G}_n$.

SIMULATION RESULTS FROM ICELAND

Field examples

The procedure outlined above has been used successfully to simulate pressure response data from several low-temperature (<150 °C) geothermal reservoirs in Iceland. Most of these reservoirs provide hot water for local district heating services. The locations of four geothermal fields, that will be presented as examples here, are shown in Figure 2. In each of these four cases long records of pressure response data are available. The pressure changes resulting from variable production from the fields have been monitored as water level changes in one or two observation wells. The water level data and data on the production are presented in Figures 4 through 7. Data on the subsurface geology of three of these reservoirs are, however, limited.

The Hamar-field in N-Iceland is a small geothermal field utilized by a district heating service that serves Dalvík, a small town of 1400 inhabitants. Production from the field started in 1969. Two production wells, with feed zones between depths of 500 and 800 m, are currently in use and the water temperature is 64 °C. Several wells have been drilled into the reservoir, but all within an area of 50x50 m. In view of limited research funds available to this small community and

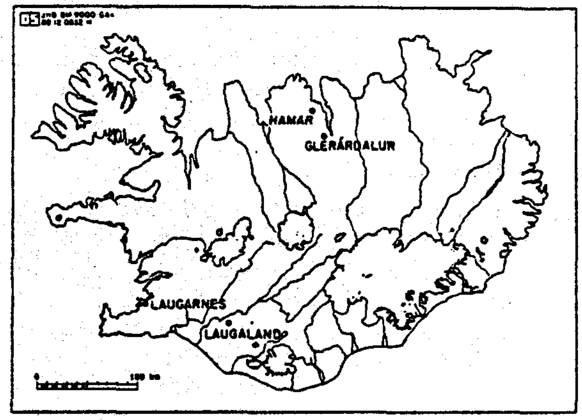

Figure 2. *Location of the low-temperature geothermal fields.*

the limited field data, lumped parameter modeling was used to estimate the production capacity of the Hamar-reservoir (Axelsson, 1988).

The Glerárdalur-field in N-Iceland is one of four small geothermal fields utilized by a district heating service that serves Akureyri, a town of about 13,000 inhabitants. Production from the field started in 1982 and currently one well is used for production. The main feed zone is at 450 m depth and the water temperature is 61 °C. Most of the wells drilled into the reservoir are shallow (100-300m) exploration wells. Due to the limited field data, lumped parameter modeling was determined to be appropriate for the Glerárdalur-reservoir (Axelsson et al., 1988).

The Laugarnes-field in SW-Iceland is considerably larger than the two fields mentioned above. It is one of three fields currently utilized by the Reykjavík Municipal Heating Service that serves about 130,000 inhabitants. Production from the field started in 1930 but increased greatly after 1962. About 44 deep (>500m) wells have been drilled into the field and the deepest well is over 3000 m deep. The major feed zones are between depths of 700 and 1300 m and the water temperature is between 115 and 135 °C. Considerable amounts of data are available on the geological characteristics of the Laugarnes-reservoir. A continuous water level record was, however, only available from one well. In this case lumped modeling and detailed numerical modeling were carried out simultaneously, in order to simulate the pressure response of the field and to estimate its production capacity (Reykjavík Municipal Heating Service, 1986).

The Laugaland-field in S-Iceland is a small geothermal field used by a district heating service that serves two small towns, Hella and Hvolsvöllur, with a total of 1200 inhabitants. Production from the field started in 1982. Three deep wells have been drilled into the field. Two of these wells are productive with the main feed zones between depths of 400 and 900 m. The water temperature is between 85 and 100 °C. Due to limited research funds as well as limited field data, lumped parameter modeling was used to model the Laugaland-reservoir (Georgsson et al., 1987).

Simulations

A closed three capacitor lumped model, as shown in Figure 3, was used to simulate the pressure response data from each of the four reservoirs. These were four different models in the sense that the parameters of the models were different. Water is produced from the first capacitor (κ_1) and the pressure is monitored in the same capacitor. The first capacitor can be considered as representing the innermost part of each geothermal reservoir, the second one as outer and deeper parts of the reservoir and the third one possibly as the surrounding recharge part of each reservoir. These recharge parts may be colder than other parts of the geothermal systems.

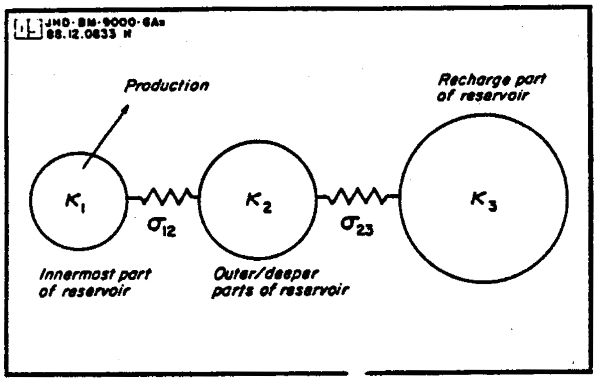

Figure 3. *General three capacitor lumped parameter model used in simulations.*

The simulations were carried out automatically by a computer. A first guess of the lumped model parameters was made and then the parameters were changed by the iterative process described above until a satisfactory fit was obtained. No assumptions were made *a priori* on the properties of the reservoirs. The results of the simulations, that is comparisons between observed and calculated water levels, are presented in Figures 4 through 7 and the parameters of the best fitting lumped models are given in Table 1 below.

Table 1 *Parameters of the best fitting lumped models*

	Hamar	Glerár-dalur	Laugar-nes	Lauga-land
$\kappa_1(ms^2)$	70.0	59.0	773	46.3
$\kappa_2(ms^2)$	6220	666	20900	3400
$\kappa_3(ms^2)$	124000	6080	364000	7100
$\sigma_{12}(10^{-5}ms)$	51.3	3.37	36.8	1.73
$\sigma_{23}(10^{-5}ms)$	18.5	1.89	61.8	9.96

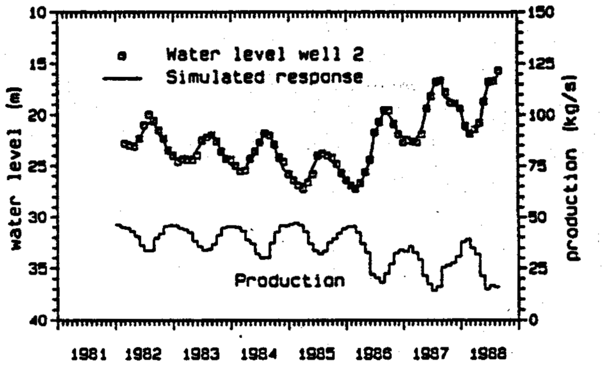

Figure 4. *Comparison of observed and calculated water level changes in the Hamar-reservoir in N-Iceland.*

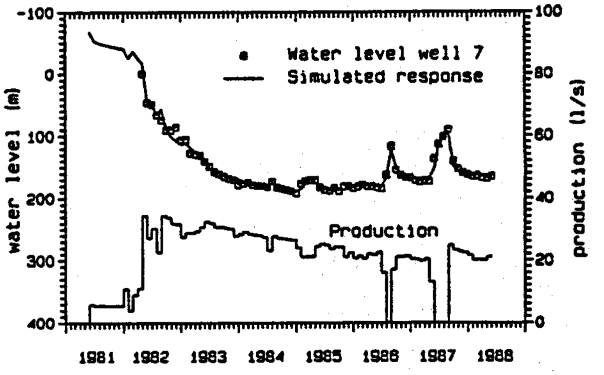

Figure 5. *Comparison of observed and calculated water level changes in the Glerárdalur-reservoir in N-Iceland.*

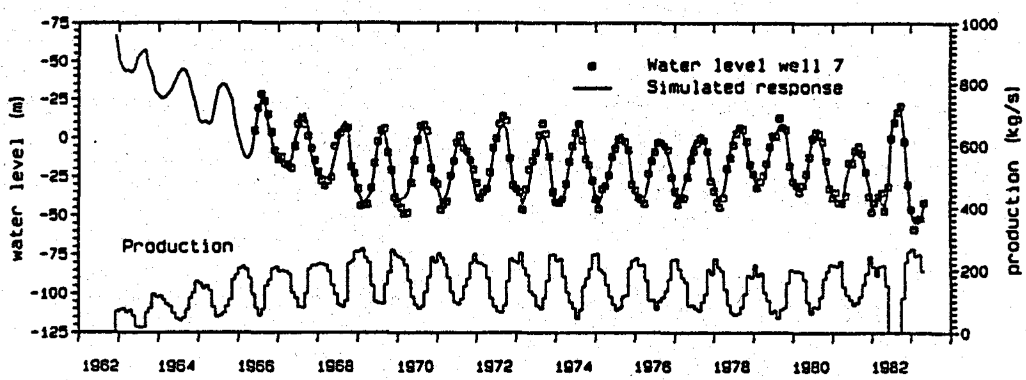

Figure 6. *Comparison of observed and calculated water level changes in the Laugarnes-reservoir in SW-Iceland.*

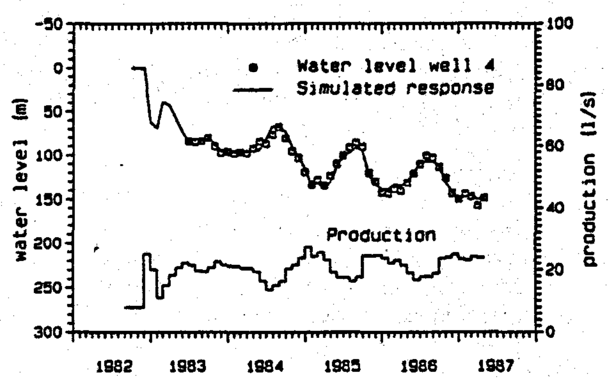

Figure 7. *Comparison of observed and calculated water level changes in the Laugaland-reservoir in S-Iceland.*

Discussion

Considering the results in Figures 4 through 7 one sees that the match between observed and calculated water level changes is quite satisfactory. This is so in spite of the simplicity of the models. The reason for this is the diffusive nature of the pressure response of geothermal systems. In using a closed three capacitor model there are five adjustable parameters which produce a very satisfactory match.

The parameters in Table 1 reflect clearly the highly variable productivity of the four fields. The models for the more productive fields have a higher total capacitance as well as higher conductivity values. The Laugarnes-field, which is the most productive, has the highest capacitance. The Hamar-field is somewhat less productive, which is reflected in a lower capacitance. The conductivity values for the Laugarnes and Hamar fields are, however, similar. The productivities of the Glerárdalur and Laugaland fields are quite poor. The total capacitance, as well as the conductivity values, are an order of magnitude greater for the Hamar and Laugarnes fields than for the Glerárdalur and Laugaland fields.

Capacitance, or storage, in a liquid-dominated geothermal system can result from two types of capacitive effects (storage mechanisms). The capacitance may on one hand be controlled by liquid/formation compressibilty. In that case the capacitance of a capacitor in a lumped model is given by

(21) $\kappa = V \rho c_t$

where V is the volume of that part of the reservoir in question the capacitor simulates, ρ the liquid density and c_t the compressibility of the liquid saturated formation. The compressibility is given by

(22) $c_t = \phi c_w + (1-\phi) c_r$

where c_w is the compressibility of the water and c_r the compressibility of the rock matrix. The capacitance may on the other hand be controlled by the mobility of a free surface. Then

(23) $\kappa = A \phi / g$

where A is the surface area of that part of the reservoir in question a capacitor simulates, ϕ its porosity and g the acceleration of gravity.

If the total capacitances of the Hamar and Laugarnes fields were solely due to compressibilty, based on equation (21) and porosities between 5 and 10 %, they would have to cover areas of the order of 1000 - 5000 km^2. Such large areas are unacceptable. Their capacitance must partially be due to free surface mobility. In that case the Hamar and Laugarnes systems would only cover areas of 10 - 70 km^2 (equation (23)). Therefore the third capacitor in the models for the Hamar and Laugarnes reservoirs appears to represent some unconfined part of the hydrological systems, perhaps the groundwater system in each area. It is also likely that some parts of the Glerárdalur and Laugaland systems are unconfined as well.

The interpretation of the conductivity values is not straight forward. The conductivity values reflect the permeability in the systems, but they also depend on their internal geometry. Because of the limited knowledge on the subsurface geological characteristics of the systems the conductivity values will not be interpreted further.

The main objective of modeling a geothermal system is to assess its production potential. In the cases discussed here the lumped models were used to predict the pressure changes in the reservoirs in question for different cases of future production. The maximum allowable drawdown in the fields determines the maximum potential of the systems. Two examples of such predictions are presented in Figures 8 and 9 below.

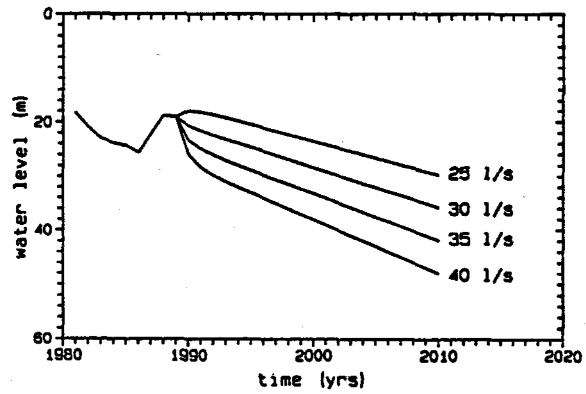

Figure 8. *Predicted water level changes in the Hamar-reservoir in N-Iceland.*

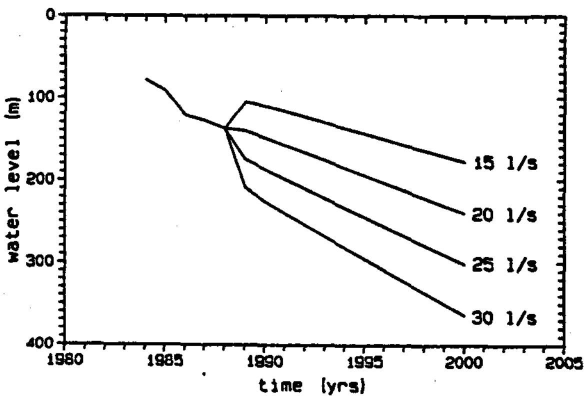

Figure 9. *Predicted water level changes in the Laugaland-reservoir in S-Iceland.*

CONCLUDING REMARKS

A method of simulating pressure response data from liquid-dominated geothermal reservoirs by simple lumped parameter models has been developed. The method uses an automatic non-linear least-squares iterative technique which requires very little time compared to more detailed/complex numerical modeling techniques. The use of this method is appropriate in cases where data on subsurface conditions are scarce but where the pressure response of a reservoir has been monitored carefully for some time. In such cases highly detailed/complex modeling, being much more costly, can hardly be justified. This method can also be used as a first stage in a modeling study of a reservoir as well as to provide independent checks on results of more complex modeling techniques.

Lumped parameter models can simply be considered as distributed parameter models with a very coarse spatial discretization (Bodvarsson et al., 1986). But the method presented here tackles the modeling as an inverse problem which requires far less time than direct, or forward, modeling. This makes lumped parameter simulations highly cost effective.

Examples of simulations of pressure response data from four low-temperature geothermal reservoirs in Iceland show that quite a satisfactory match between observed and calculated data can be obtained. Because of the satisfactory degree of approximation achived by the lumped models they have a strong power of predicting the future evolution from the observed past.

Detailed numerical modeling has also been performed for the Glerárdalur and Laugarnes fields (Axelsson and Tulinius, 1988; Reykjavík Municipal Heating Service, 1986). A comparison of the pressure data match by the two methods shows that in both cases the lumped models were able to match the pressure data with the same accuracy as the detailed numerical models. The time required for the lumped modeling, however, was only a fraction of the time required for the more detailed modeling.

At this point, it is appropriate to emphasize that a clear distinction has to be made between liquid reservoirs and reservoirs of thermal energy. In the individual areas, the extent of each type of reservoir depends on local geological and physical conditions. This paper deals with modeling of the liquid reservoirs only. Variations in temperature within the systems are not taken into account. This is justified by the fact that significant changes in the temperature of the water produced have not been observed in any of the cases presented here. It appears evident, however, that some parts of the liquid reservoirs of the Hamar and Laugarnes systems are unconfined. These two geothermal reservoirs are possibly connected to local groundwater systems and the recharge into the systems may be cold groundwater. Thus the temperature of the water produced from the two fields may eventually decrease. In cases where changes in temperature and/or chemical content have been observed, lumped models can also be developed to simulate such data and to predict the future evolution.

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