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COMPRESSIBILITY AND SONIC VELOCITY IN STEAM/WATER MIXTURES

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ABSTRACT

A simplified expression is derived for the compressibility of steam/water mixtures at ideal thermodynamic conditions. It is shown how the same expression can be used to estimate the sonic velocity also. Empirical correlations are presented for both the compressibility and sonic velocity.

INTRODUCTION

The maximum fluid velocity in two-phase geothermal wells and surface equipment, is the sonic velocity of the mixture. In locations where this velocity is attained, the flow conditions are said to be choked or critical. The James-method is based on such conditions in a discharge tube (James, 1962). Also, when calculating the output of two-phase geothermal wells using a wellbore simulator, the velocity in the steam/water mixture at outlet conditions needs to be known, to check that the calculated mixture velocity does not exceed the sonic velocity (Bilicki et al., 1982).

Experimental data on sonic velocity in steam/water mixtures at geothermal wellbore conditions are not readily available. To solve this problem, theoretical calculations need to be carried out to find the sonic velocity for ideal thermodynamic conditions. Such calculations have been carried out by Kieffer (1977) and Maeder et al. (1981), for example. In both studies the results were presented in tabular and graphic form, not convenient for engineering calculations. Karamarakar and Chen (1980) reviewed several choked flow correlations for two-phase flow metering of geothermal wells. They concluded that the nuclear-industry-type correlations agreed reasonably well with the James-method correlation.

We have derived a simplified expression for the compressibility of steam/water mixtures at conditions typical in geothermal wells. The expression can be used to estimate the sonic velocity in such mixtures. We started this work to better understand the nature of two-phase flow in high-enthalpy geothermal wells, producing from boiling-dominated reservoirs (Gudmundsson, 1986a, 1986b). Many such wells flow steam vapor with a few percent liquid water. Perhaps, if the compressibility of steam/water mixtures can be found from a simple expression, it would be possible to calculate directly the pressure drop in high-enthalpy two-phase wells, using a similar method as used for steam wells (Morales G. et al., 1979). The details of our work are given by Sveinsson (1987).

DERIVATION OF COMPRESSIBILITY

Our derivation follows similar reasoning as that of Grant and Sorey (1979), who investigated the compressibility of a steam/water mixture in contact with reservoir rock. Consider a two-phase geothermal well where thermodynamic equilibrium exists between the phases and where the flow is homogeneous, steady-state and one-dimensional. Furthermore, assume the wellbore flow to be adiabatic; no heat loss or gain.

A balance equation can be written for the steam/water mixture flowing from one infinitesimal cross-section to another

$$x_1h_{g1}+(1-x_1)h_{f1}=x_2h_{g2}+(1-x_2)h_{f2} \quad ...(1)$$

where x represents the mass fraction of steam and h_g and h_f the enthalpy of steam vapor and liquid water, respecively. The equation can also be written as

$$h_{f1}+x_1h_{fg1}=h_{f2}+x_2h_{fg2} \quad ...(2)$$

We assume that the latent heat of evaporization, h_{fg}, changes negligibly between adjacent cross-sections; from one infinitesimal cross-section to another. We can hence rewrite Equation 2 in the form

$$h_{f1}-h_{f2}=(x_2-x_1)h_{fg} \quad ...(3)$$

An examination of Steam Table values shows that the heat of evaporization of water remains reasonably constant over a wide range. This does not mean that the heat of evaporization is constant throughout the system considered; the appropriate value is used for each saturation condition.

Next we assume that constant pressure heat capacity, C_p, can be used. For the liquid water phase we can write

$$C_{pf}\Delta T=h_{f1}-h_{f2} \quad ...(4)$$

and we note that the right-hand side of this equation is the same as the left-hand side of Equation 3. When liquid water evaporates due to pressure lowering, the steam/water mixture volume changes. The dominant volume change is that caused by phase change; liquid water turning into steam vapor. We ignore the slight volume change in the water and steam already present. Therefore, the change in mixture volume becomes

$$\Delta V=\Delta m(v_g-v_f) \quad ...(5)$$

where Δm is the mass of water changing phase and v_g and v_f the specific volume of steam and liquid, respectively. And since this mass of water is simply

$$\Delta m=(x_2-x_1)m \quad ...(6)$$

where m is the mass of steam/water mixture present, the change in mixture volume can be written

$$\Delta V=m(x_2-x_1)(v_g-v_f) \quad ...(7)$$

It is possible to define fluid compressibility in several ways, depending on what physical property is assumed constant: temperature, enthalpy, entropy. Usually the temperature is assumed constant, giving an isothermal compressibility. For two-phase wellbore flow without heat loss or gain, an adiabatic definition of compressibility seems most appropriate. Thus

$$K_h=-(1/V)(\Delta V/\Delta p) \quad ...(8)$$

Substituting for ΔV using Equation 7 and then Equations 3 and 4 to eliminate (x_2-x_1), the adiabatic compressibility becomes

$$K_h=-(mC_{pf}\Delta T(v_g-v_f))/(V\Delta p h_{fg}) \quad ...(9)$$

The total mass of the steam/water mixture, m, is given by the relationship

$$m=V(\alpha\rho_g+(1-\alpha)\rho_f) \quad ...(10)$$

At all conditions the steam/water mixture follows the saturation curve. Therefore, for a small pressure drop, Δp, the following will apply

$$\Delta p/\Delta T=(dp/dT)_{sat} \quad ...(11)$$

Combining Equations 9, 10 and 11, gives our simplified derivation for the adiabatic compressibility of steam/water mixtures

$$K_h = \frac{C_{pf}(\alpha\rho_g+(1-\alpha)\rho_f)(1/\rho_g-1/\rho_f)}{h_{fg}(dp/dT)_{sat}} \quad (12)$$

COMPRESSIBILITY AND SONIC VELOCITY

We now turn to the relationship between compressibility and sonic velocity. In our derivation above we assume an adiabatic process. Sonic velocity, c, is always defined in thermodynamic texts at isentropic conditions, s, namely

$$c^2=(dp/d\rho)_s \quad ...(13)$$

where ρ is the fluid density. By using the definition of compressibility at isentropic conditions

$$K_s=1/\rho(d\rho/dp)_s \quad ...(14)$$

the following relationship results

$$K_s=vc^{-2} \quad ...(15)$$

In other words, by knowing the isentropic compressibility, we also know the sonic velocity. This was the method used in the present work.

From thermodynamics we know that an isentropic process is both adiabatic and reversible. The question arises whether we can equate compressibility and sonic velocity for adiabatic and isentropic conditions. For fluid flow in wellbores and pipelines, frictional losses make the process non-reversible. This aspect of fluid flow is particularily important in situations where rapid pressure drops occur, for example in nozzles. It is less important in situations where the pressure changes gradually with distance, for example in wellbores and long pipelines. Therefore, the validity of sonic velocity values derived from adiabatic compressibility values, will depend on the flow situation. The sonic velocity obtained in our work represents an approximation.

RESULTS

Equation 12 gives the compressibility of steam/water mixtures we want to find. The properties needed are the liquid water heat capacity, the density of liquid water and steam vapor and the latent heat of evaporization. These were calculated from correlations reported by Michaelides (1981). [A few errors (in Equations 2, 3 and 21) were found in the paper - they were corrected by contacting a co-worker of the author. The corrections are given by Sveinsson (1987) and also by Gudmundsson and Thráinsson (to be published).] The liquid water heat capacity was calculated from a correlation giving enthalpy with temperature; the other properties were calculated directly. The pressure-temperature gradient on the saturation line, however, was calculated from the Clausius-Clapeyron equation.

$$(dp/dT)_{sat}=(v_g-v_f)^{-1}[h_{fg}/(T+273.15)] \qquad ...(16)$$

The remaining property in Equation 12, the steam/water mixture void fraction, α, can be expressed as

$$\alpha=[1+((1-x)/x)(\rho_g/\rho_f)]^{-1} \qquad ...(17)$$

where x has the usual meaning of steam mass fraction

$$x=(h_o-h_f)/h_{fg} \qquad ...(18)$$

A computer program was written where the steam/water mixture enthalpy was specified and calculations performed for saturation temperature in the range 100°C to 300°C. The Michaelides (1981) physical property correlations are reasonably accurate over this temperature interval. All calculations were performed assuming saturation conditions. Examples of calculated compressibility values are shown with pressure on a semilog plot in Figure 1, for steam/water mixture enthalpy [$(1-x)h_f+xh_g$] from 1000 kJ/kg to 2400 kJ/kg, in intervals of 200 kJ/kg.

We were interested in correlating the results, to obtain an expression convenient for engineering calculations. Figure 1 shows the compressibility with saturation pressure for several constant mixture enthalpy values. We found that the results could be collapsed into a single line by plotting the compressibility, divided by the steam/water mixture density

$$\rho=(\alpha\rho_g+(1-\alpha)\rho_f) \qquad ...(19)$$

with saturation pressure. The correlated results are plotted on a semilog plot in Figure 2, showing the calculated values (solid line) and fitted values (broken line). An examination of Equation 12 shows, that moving the term expressing the mixture density (Equation 18) from the right-hand-side to the left-hand-side of the equation, suggests a way to correlate the results. We used a standard power-fit program, which resulted in the following expression

$$K_h/\rho=(789.4)p^{-1.616}\ (1/\mathrm{GPa})(\mathrm{m}^3/\mathrm{kg}) \qquad ...(20)$$

In the range 10-20 bara pressure, the fitted line is just above the calculated

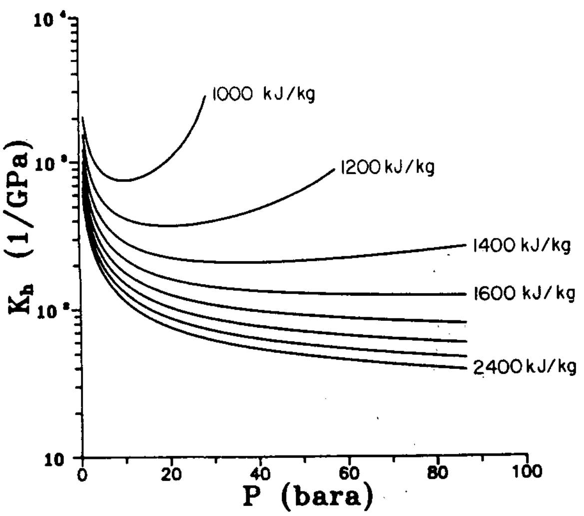

Figure 1 - Calculated compressibility with saturation pressure and steam/water mixture enthalpy.

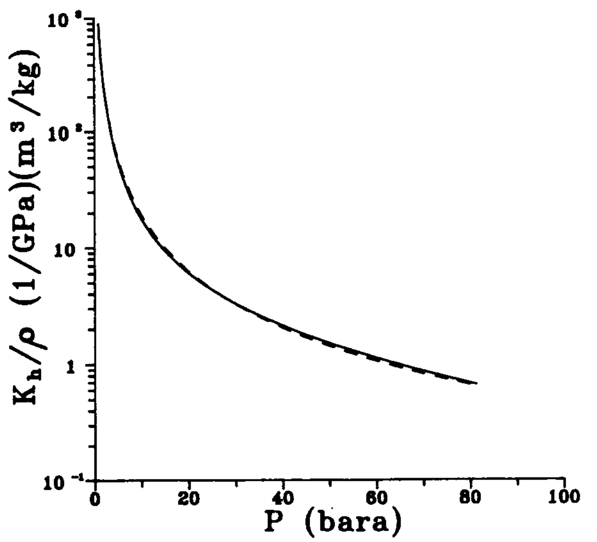

Figure 2 - Calculated (solid line) and correlated (broken line) compressibility with saturation pressure.

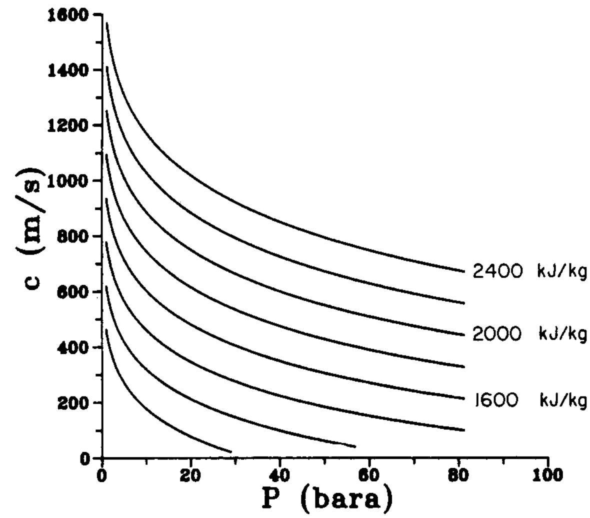

Figure 3 - Calculated sonic velocity with saturation pressure and steam/water mixture enthalpy.

values. In the range 40-80 bara the fitted line falls just below the calculated values. We did not evaluate the goodness of fit, but estimate that the calculated and fitted values differ by only a few percent at the most.

Using Equation 15, we calculated the sonic velocity, c, from the steam/water mixture compressibility values already obtained. Our results are shown on a linear plot in Figure 3. It can be shown that the compressibility correlation (Equation 20) gives rise to the following relationship for sonic

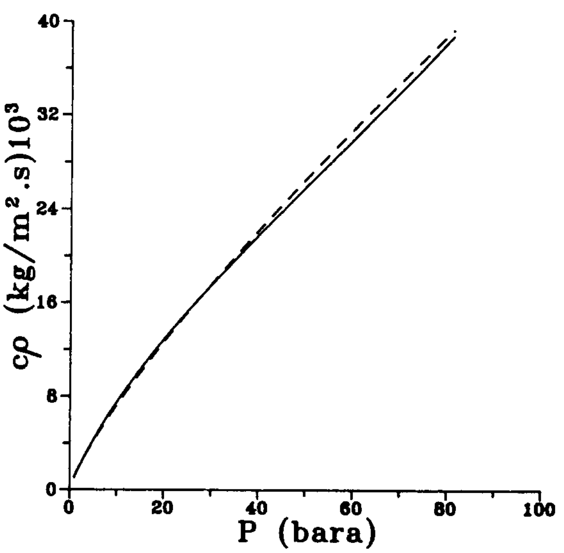

Figure 4 - Calculated (solid line) and correlated (broken line) sonic velocity with saturation pressure.

velocity in terms of mass flux at choked or critical flow conditions

$$c\rho=(1125)p^{0.808}\ (kg/m^2.s) \qquad ...(21)$$

We obtained this correlation assuming thermal equilibrium between the phases, a homogeneous steam/water mixture and isentropic (adiabatic and reversible) conditions. Equation 21 is shown on a linear plot in Figure 4 (broken line), along with the calculated sonic velocity values (solid line).

DISCUSSION

Well 11 in the Námafjall field in Iceland, was tested in 1987 to flow 20 kg/s of a steam/water mixture having an enthalpy of 2072 kJ/kg at a wellhead pressure of 27 bar-a. The steam mass fraction at these conditions should be 0.6 or 60 percent. The James-method was used to arrive at the mixture flowrate and enthalpy (James, 1966).

From Equation (21) we calculate the sonic velocity in the steam/water mixture at the wellhead, to be about 720 m/s. For comparison, from the results of Kieffer (1977), we estimate the sonic velocity to be about 400 m/s. Similarily from Meader et al. (1981) we estimate the sonic velocity to be about 350 m/s. We observe that our results give a much higher sonic velocity than estimated from the other studies.

The critical lip pressure in the Námafjall well 11 output test was 1.7-1.8 bar-g, say 1.85 bar-a. The James (1962, 1966) equation can be written

$$(Gh^{1.102}/p_c^{0.96})=16.8x10^6 \qquad ...(22)$$

where G ($kg/m^2.s$) is the mass flux, p_c (MPa) the critical lip-pressure and h (kJ) the mixture enthalpy. The mass flux at critical/choked flow conditions can be approximated by

$$G=c\rho \qquad ...(23)$$

where c is the sonic/critical/choked flow velocity and ρ the mixture density. We estimate the mixture density (from mass and energy balance and Equation 19) of Námafjall well 11 at 1.85 bar-a pressure to be 1.47 kg/m^3 and calculated the mass flux from Equation (22) to be 736.4 $kg/m^2.s$. Using these values we calculated from Equation 23 the sonic velocity to be 500 m/s. Equation (22) shows the critical mass flux in the James-tube for some mixture enthalpy and critical lip-pressure. The lip-pressure is not the pressure in the steam/water mixture immediately upsteam at the wellhead.

Table 1 - Sonic velocity (m/s) in steam/water mixtures with enthalpy corresponding to 250°C liquid water (1086 kJ/kg), flashed to lower temperatures.

Temperature (°C)	Pressure (bar)	Steam Mass Fraction	Present Work	Kieffer (1977)	Maeder (1981)
225	25.50	0.065	95	68	82
200	15.55	0.120	172	160	126
175	8.92	0.170	250	175	160
150	4.76	0.215	330	200	181

We note that our compressibility equation is a simplification of the equations used by both Kieffer (1977) and Maeder et al. (1981). [There are typographical errors in the Kieffer (1977) paper. In Equations 17 and 18 the wrong symbol is used for steam mass fraction. In Equations 17, 28 and 29, there are missing overall brackets and the exponent -1]. In our study and the two other studies, the flow is assumed frictionless. Therefore, the sonic velocity estimated from all three studies, should be greater than the empirical value obtained from the James (1962, 1966) equation. We note that the sonic velocity obtained in the present study is much greater, while the Kieffer (1977) and Maeder et al. (1981) values are smaller than the empirical James-equation value. This calls for further investigation.

To check our results in another way, we calculated the sonic velocity at several conditions typical for a two-phase geothermal well producing from liquid-dominated conditions at 250°C, see Table 1. For comparison we show the sonic velocity estimated from the results of Kieffer (1977) and Maeder et al. (1981). We observe that our results are above the others, but not by as much as for the wellhead conditions of well 11 in the Námafjall field. We note that the lower the steam mass fraction (0.07 to 0.22 in Table 1), the closer our results agree with the two other studies.

CONCLUSIONS

1. A simplified equation was derived for the compressibility of steam/water mixtures at ideal thermodynamic conditions. This equation can also give the sonic velocity in the same mixtures.
2. Calculated values of compressibility and sonic velocity were easily correlated by including the steam/water mixture density. The empirical power-fit correlations are convenient for engineering calculation.
3. The calculated sonic velocity was compared to two previous studies. The present results were 80-100 percent higher when the steam fraction was large (mainly steam vapor) and 40-80 percent higher when the steam fraction was small (mainly liquid water).

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