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TRACER FLOW MODEL FOR NATURALLY FRACTURED GEOTHERMAL RESERVOIRS

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ABSTRACT

The model proposed has been developed to study the flow of tracers through naturally fractured geothermal reservoirs. The reservoir is treated as being composed of two regions: a mobil region where diffusion and convection take place and a stagnant or immobile region where only diffusion and adsorption are allowed.

Solutions to the basic equations in the Laplace space were derived for tracer injection and were numerically inverted using the Stehfest algorithm. Eventhough numerical dispersion is present in these solutions, starting at moderate dimensionless time values, a definite trend was found as to infer the behavior of the system under different flow conditions. For practical purpouses, it was found that the size of the matrix blocks does not seem to affect the tracer concentration response and the solution became equivalent to that previously presented by Tang et al. Under these conditions, the behavior of the system can be described by two dimensionless parameters: the Peclet number for the fractures, P_{e1}, and a parameter $\alpha (\alpha = \xi \sqrt{P_{e2}})$, where ξ is $\xi = \phi_e D_e / v(w-\delta)$ and P_{e2} is the Peclet number for the matrix. Tracer response for spike injection was also derived in this work. A limiting analytical solution was found for the case of α approaching zero and a given P_{e1}, which corresponds to the case of a homogeneous system. It is shown that this limiting solution is valid for $\alpha < 10^{-2}$. For the case of continuous injection this solution reduces to that previously presented by Coats and Smith. For the spike solution it was found that the breakthrough time for maximum tracer concentration is directly related to the dimensionless group $\frac{\sqrt{9+X_D^2 P_{e1}^2}-3}{P_{e1}}$. Therefore it is possible to obtain the value of P_{e1} or X_D. A set of graphs of dimensionless concentration in the fracture vs. dimensionless time for tracer response were developed. It was found that if P_{e1} is held constant while α is changing, the limiting solution becomes a limiting curve for a family of curves in a plot of C_D vs t_D. In this graph P_{e1} fixes the range in which the family of curves evolves. It was also found that the breakthrough time for a given concentration is a strong function of α.

INTRODUCTION

Reinjection of separated hot brine back into the geothermal reservoir can be considered as a promising reservoir pressure maintenance technique. In addition to this, secondary heat mining from the reservoir seems to be desirable. However, several field experiences in the past years indicate that unless careful and detailled studies on the selection of the reinjection-production pattern are made, adverse effects such as early thermal breakthrough can result (Horne, 1982). Therefore, before a reinjection project could be implemented, a proper reservoir characterization should be given a major role in designing the project.

Geothermal reservoirs are highly-fractured, complex systems, which require the use of sophisticated techniques in order to be properly characterized. In addition to well testing, tracer flow interpretation techniques provide the means to obtain basic reservoir parameters, as well as the connectivity between several regions of the reservoir and a good estimation of transit times of injected fluids. Most interpretation models published to describe tracer flow through both aquifers and oil reservoirs cannot be directly applied to geothermal reservoirs, because they treat the flow system as a homogeneous porous medium, which does not correspond to the case of most geothermal fields. Most of the models for geothermal reservoirs published to date provide only qualitative estimation of reservoir parameters (Fossum, 1983, Tester et al, 1982, Jensen, 1983) and only a few of them allow quantitative determination of these parameters (Walkup and Horne, 1985, Tang et al., 1981).

In this study a fractured, two flow region model was developed to quantitatively determine basic *geothermal* reservoir parameters from tracer return curves. This model takes into account the main mass transport mechanisms that take place in the reservoir when a tracer is flowing under actual reservoir conditions. In spite of this, the model can still be considered as simpler than other published models (Walkup and Horne, 1985), since it requires only one numerical inversion in order to bring to real space the analytical solution for the concentration profile, which was written in Laplace's Space. It is also shown that under certain conditions, the fractured system can be properly characterized by a minimum of two fitting parameters, which compare favorable with four or five fitting parameters required by other published models.

MODEL DESCRIPTION

The model proposed in this work is actually an extension of that previously presented (Rivera et al., 1987) at the Eleventh Workshop on Geothermal Reservoir Engineering. Dimensionless parameters were redefined in order to simplify the solution to the model and also to eliminate some numerical dispersion that was present in the solution to the former model. In this work the characteristc length was taken as the distance from the injection point to that in which the concentration profile has to be calculated.

The proposed model is shown in Fig. 1. The fractured heterogeneous medium is represented by means of a system of equally spaced parallel fractures alternated with porous blocks. As shown in Fig. 1, this system is made of two connected regions; a mobile region constituted by the fracture itself, where diffusion and convection processes are taking place and an *immobile region where only diffusion and adsorption are allowed.* Connecting both regions there is a very thin, stagnant *fluid* layer of thickness δ, which acts as a resistance to mass transfer from the mobile region to the immobile one. For a more detailed description of the model the reader should refer elsewhere (Ramírez, 1987, Rivera et al., 1987). The idea of representing the flow system by means of two interconnected regions has been used in the past by several anthors (Deans, 1963, Walkup and Horne, 1985, Maloszewski and Zuber, 1985, among others).

The governing equations of the model are as follows:

a) For the mobile region:

$$D_m \frac{\partial^2 C_m}{\partial x^2} - V_m \frac{\partial C_m}{\partial x} - \lambda C_m - \frac{\phi_e}{w-\delta} D \frac{\partial C_e}{\partial y}\bigg|_{(w-\delta)} - \frac{\partial C_m}{\partial t} = 0 \qquad \ldots(1)$$

b) For the immobile region:

$$\frac{D_e}{1 + \frac{\rho k(1-\phi_e)}{\phi_e}} \frac{\partial^2 C_e}{\partial y^2} - \lambda C_e - \frac{\partial C_e}{\partial t} = 0 \qquad \ldots(2)$$

The main assumptions for development of the model are the following: a) no production by reaction of the *chemical* species *within the* control volume; b) continuous injection of tracer into the fracture system; c) tracer transport in the fractures is due to diffusion and convection; d) tracer distribution across the fracture width can be assumed constant due to efficient transverse diffusion and dispersion; e) constant density; f) in the immobile region only diffusion in the y-direction is important; g) reversible adsorption with a linear adsorption isotherm is taking place.

In dimensionless form, eqs. (1) and (2) can be expressed as follows:

$$\frac{1}{P_{e1}} \frac{\partial^2 C_{D1}}{\partial x_D^2} - \frac{\partial C_{D1}}{\partial x_D} - \gamma C_{D1} + \xi \frac{\partial C_{D2}}{\partial y_D}\bigg|_{\frac{w-\delta}{L}} - \frac{\partial C_{D1}}{\partial t_D} = 0 \qquad \ldots(3)$$

$$\frac{R}{P_{e2}} \frac{\partial^2 C_{D2}}{\partial y_D^2} - \gamma C_{D2} - \frac{\partial C_{D2}}{\partial t_D} = 0 \qquad \ldots(4)$$

where:

$$P_{e1} = \frac{V_m L}{D_m} \qquad \ldots(5)$$

$$P_{e2} = \frac{V_m L}{D_e} \qquad \ldots(6)$$

$$\xi = \frac{\phi_e D_e}{V_m (w-\delta)} \qquad \ldots(7)$$

$$\gamma = \frac{L\lambda}{V_m} \qquad \ldots(8)$$

$$R = \frac{\phi_e}{\phi_e + \rho k(1-\phi_e)} \qquad \ldots(9)$$

The initial and boundary conditions are as follows:

$$C_{D1}(x_D, 0) = 0 \qquad \ldots(10)$$

$$C_{D2}(x_D, y_D, 0) = 0 \qquad \ldots(11)$$

$$C_{D1}(0, t_D) = 1 \qquad \ldots(12)$$

$$C_{D1}(\infty, t_D) = 0 \qquad \ldots(13)$$

$$C_{D2}(x_D, \frac{w-\delta}{L}, t_D) = C_{D1}(x_D, t_D) \qquad \ldots(14)$$

$$\left.\frac{\partial C_{D2}}{\partial y_D}\right|_{(x_D, \frac{E}{2L}, t_D)} = 0 \qquad \ldots(15)$$

where:

$$C_{D1} = \frac{C_m - C_i}{C_o - C_i} \qquad \ldots(16)$$

$$C_{D2} = \frac{C_e - C_i}{C_o - C_i} \qquad \ldots(17)$$

$$x_D = \frac{x}{L} \qquad \ldots(18)$$

$$y_D = \frac{y}{L} \qquad \ldots(19)$$

$$t_D = \frac{V_m t}{L} \qquad \ldots(20)$$

Equations (3) and (4) can be solved by means of the Laplace's transform. Solution for concentration distribution in the mobile and immobile regions in Laplace's space are as follows:

$$\overline{C}_{D1} = \frac{C_o}{s} \exp\left\{\frac{P_{e1} x_D}{2}\left[1 - \sqrt{1 + \frac{4}{P_{e1}}\left(s + \gamma + \alpha\sqrt{\frac{s+\gamma}{R}}\tanh(a_1)\right)}\right]\right\} \qquad \ldots(21)$$

$$\overline{C}_{D2} = \left\{\frac{\exp[-m_1(\frac{E}{L} - y_D)] + \exp(-m_1 y_D)}{\exp[-m_1(\frac{E-w+\delta}{L})] + \exp[-m_1(\frac{w-\delta}{L})]}\right\}\overline{C}_{D1} \qquad \ldots(22)$$

where:

$$m_1 = \sqrt{\frac{P_{e2}}{R}(s+\gamma)} \qquad \ldots(23)$$

$$a_1 = \frac{m_1(E - 2w + 2\delta)}{2L} \qquad \ldots(24)$$

$$\alpha = \xi\sqrt{P_{e2}} \qquad \ldots(25)$$

A particular case of eq.(21) results when α as defined by eq.(25) above becomes very small. In a given situation, this can be produced by either one of the following factors or any combination of them: the porosity or the diffusion coefficient for the immobile region are very small, or the velocity in the mobile region is very large. In this case, the immobile region will behave as if it were impermeable, so that only one flow region will contribute to any change in the tracer concentration. Under these conditions, eq.(21) becomes:

$$\overline{C}_{D1} = \frac{1}{s}\exp\left[\frac{x_D P_{e1}}{2}\right]\exp\left[-x_D\sqrt{\frac{P_{e1}^2}{4} + P_{e1}(s+\gamma)}\right] \qquad \ldots(26)$$

In real space, the inverse transform of eq. (26) can be written as follows:

$$C_{D1}(x_D, t_D) = \frac{1}{2}\exp\left[-x_D\left(\sqrt{a_2} - \frac{P_{e1}}{2}\right)\right]\operatorname{erfc}\left(\frac{x_D}{2}\sqrt{\frac{P_{e1}}{t_D}} - \sqrt{\frac{a_2 t_D}{P_{e1}}}\right)$$
$$+ \frac{1}{2}\exp\left[x_D\left(\sqrt{a_2} + \frac{P_{e1}}{2}\right)\right]\operatorname{erfc}\left(\frac{x_D}{2}\sqrt{\frac{P_{e1}}{t_D}} + \sqrt{\frac{a_2 t_D}{P_{e1}}}\right) \qquad \ldots(27)$$

where:

$$a_2 = \frac{P_{e1}^2}{4} + \gamma P_{e1} \qquad ...(28)$$

Eq.(27) corresponds to that case in which the influence of the immobile region is negligible. This equation will reduce to that previously presented by Coats and Smith (1963) for an infinite homogeneous system when $\gamma=0$.

On the other hand, if for practical purpouses the size of the immobile region in the vertical direction, E, does not show any influence on the behavior of the system, eqs. (21) and (22) can be expressed as follows:

$$\bar{C}_{D1} = \frac{C_o}{s} \exp\left\{ \frac{P_{e1} x_D}{2} \left[1 - \sqrt{1 + \frac{4}{P_{e1}}(s+\gamma+\alpha\sqrt{s})} \right] \right\} \qquad ...(29)$$

$$\bar{C}_{D2} = \exp\left[- m_1 (y_D - \frac{w}{L}) \right] \qquad ...(30)$$

The system described by eqs. (29) and (30) was previously studied by Tang et al. (1981) and their solutions in Laplace's space agree very well with the solution presented here. Numerical inversion was used for the evaluation of eqs. (29) and (30) in real space. The algorithm presented by Stehfest (1970) was used for this purpouse. Data reported by Tang et al were used in the evaluation of eqs. (29) and (30) and then compared with those obtained from the integral solution previously presented by Tang et al. Results obtained from both evaluations agreed very well with each other.

To determine the influence that the size of the repetitive element, E, could have under several combinations of practical values for the parameters involved in the solution to the model, extensive evaluations of the general solution, eq. (21), and the particular solution when the effect of E is negligible, eq. (29), were carried out. It was observed that for all conditions considered, the results obtained that can be expected in practical applications, the size of the repetitive element, E, does not have influence in the magnitude of the tracer concentration obtained at the producing end, or at any other location within the flow path. Under these conditions the system can be properly described by just two dimensionless parameters: the Peclet number for the mobile region, P_{e1}, and α. If the tortuosity of the fractured medium is considered, then x_D should be added to these parameters.

Thus far, all discussion has been centered in the continuous injection case. However, in field applications tracers are injected as finite slugs in a short period of time , which has been called the "spike injection case" in the literature. As it was pointed out by Walkup and Horne, the solution for an spike-input is the time derivative of a step input, that is:

$$(C_{D1})_{spike} = \frac{\partial C_{D1}}{\partial t_D} = L^{-1}\left\{ s\bar{C}_{D1} \right\} \qquad ...(31)$$

Therefore to obtain the solution for a spike-input all that is needed is to multiply the expression for the step input in the (x,y,s) space by s and then invert the resulting equation to the (x,y,t) space. Two cases can be considered; the first one results by considering the solution when the effect of E is negligible. From eq. (29):

$$(\bar{C}_{D1})_{spike} = s\,\bar{C}_{D1} = C_o \exp\left\{ \frac{P_{e1} x_D}{2} \left[1 - \sqrt{1 + \frac{4}{P_{e1}}(s+\gamma+\alpha\sqrt{s})} \right] \right\} \qquad ...(32)$$

By analyzing eq. (32) it can be seen that a maximum (or limiting) solution can be obtained when $\alpha=0$. Applying the inverse transformation to eq. (32) when $\alpha=0$ the following expression is obtained:

$$(C_{D1})_{\substack{spike\\limit}} = \frac{x_D}{2t_D}\sqrt{\frac{P_{e1}}{\pi t_D}} \exp\left\{ \frac{P_{e1}}{4}\left[x_D(2 - \frac{x_D}{t_D}) - t_D(1 + \frac{4\gamma}{P_{e1}}) \right] \right\} \qquad ...(33)$$

An estimation of the time of arrival of the maximum tracer concentration can be obtained by equating to zero the time derivative of eq. (33). Thus, the following expression is obtained:

$$t_{D_{cmax}} = \frac{\sqrt{9 + x_D^2 P_{e1}^2} - 3}{P_{e1}} \qquad ...(34)$$

This corresponds to that dimensionless time when the maximum dimensionless tracer concentration breaks through at the producing point location. Since it has been calculated from eq. (33), it will hold for $\alpha=0$. However, from the cases studied, it has been determined that although the maximum tracer concentration at a given location will change with α departing from zero, the time of arrival of the maximum concentration will have a small variation compared with that value predicted by eq. (34). Therefore, this expression should be considered as a good estimation for the time of arrival of the maximum tracer concentration.

DISCUSSION OF RESULTS

Analysis of results generated from Eqns. (21) and (26) show that for practical purposes, $w/L \geq 0.005$, $1 \leq P_{e1} \leq 10^2$ and $10^7 \leq P_{e_2} \leq 10^{12}$, tracer response seems not to be affected by the size of the matrix blocks, this is, diffusion into the matrix blocks does not reach far enough as for matrix boundary effects to be felt in the solution. Under these conditions the system can be described by two dimensionless parameters, α and P_{e1}, as seen from equation (29)

Diffusion of tracer into the matrix is governed by α. When $\alpha \leq 0.01$, tracer response is the same as would be for $\alpha=0$, which corresponds to the zero diffusion into the matrix case.

An analytical solution, equation (26), was found for this limiting case; this is shown in Fig.2 for $P_{e1}=2$ and $x_D=1$ along with solutions for $\alpha\neq0$, generated from the numerical inversion of Eq. (26). As α becomes larger, for a given t_D, concentration becomes smaller, thus indicating that more tracer has been transferred into the matrix along the injection path. Also, notice in Fig.2 that instabilities on the solutions, due to the numerical Laplace space invertor used, become worst as $\alpha \longrightarrow 0$ and t_D becomes larger.

The effect of P_{e1} on the tracer response at $X_D=1$, for $\alpha=0.01$, is presented in Fig. 3. These results were obtained from the numerical inversion of Eq. (29). It can be seen that as P_{e1} becomes larger, the time for tracer breaktrough become also larger. This seems to be contradictory, note however that both α and P_{e1} depend on velocity and system length and that setting α to a constant implies that changes in P_{e1} migth only be produced by changing the diffusivity of the tracer on the fractures, Dm. Then, as Dm becomes smaller, P_{e1} becomes larger and so will the breakthrough time. Fig. 4 shows the same results as Fig. 3 but obtained analytically from Eq. (27).

Figure 5 shows tracer concentration responses at $X_D = 1$ for the case of a spike injection, $P_{e_1} = 2$ and several values of α. These results were obtained by numerical inversion of Eq.(29) and as it can be seen, instabilities due to the inversion algorithm become worse as time increases and for samll values of α. Dimensionless concentration values greater than unity were obtanied for relatively small and large values of P_{e_1}, ≤ 2 and ≥ 20, when $\alpha \leq 0.01$. This indicates that the mathematical treatment of the spike injection, $C_{D_1}(0, t_D) = \delta(t_D)$, must only be valid under certain conditions. These conditions are currently being investigated.

NOMENCLATURE

a_1= Dimensionless constant defined by eq.(24)

a_2= Dimensionless constant defined by eq.(28)

C= Concentration, (M/L^3)

D= Diffusion coefficient, (L^2/t)

E= Fracture spacing, (L)

k= Adsorption constant, (L^3/M)

L^{-1}= Inverse Laplace's operator

m_1 = Dimensionless group defined by eq. (23)

P_e = Peclet number, (dimensionless)

R= Dimensionless group defined by eq. (9)

s= Laplace parameter

t= Time, (t)

V= Velocity, (L/t)

w= Fracture half-width, (L)

x= Distance in x-direction, (L)

y= Distance in y-direction, (L)

Greek Symbols

α = Dimensionless group defined by eq. (25)

γ = Dimensionless group defined by eq. (8)

δ = Stangnant fluid film thickness, (L)

ξ = Dimensionless group defined by eq. (7)

ϕ = Porosity, referred to total-bulk volume, (dimensionless)

λ = Radioactive decay constant, (t^{-1})

ρ = Density, (M/L^3)

Subscripts

D = Dimensionless variable (distance, time or concentration)

e = Refers to the immobile (stagnant) region

i = Refers to initial conditions

m = Refers to the mobile (fractured) region

0 = Refers to inlet conditions

1 = Refers to mobile region

2 = Refers to immobile region

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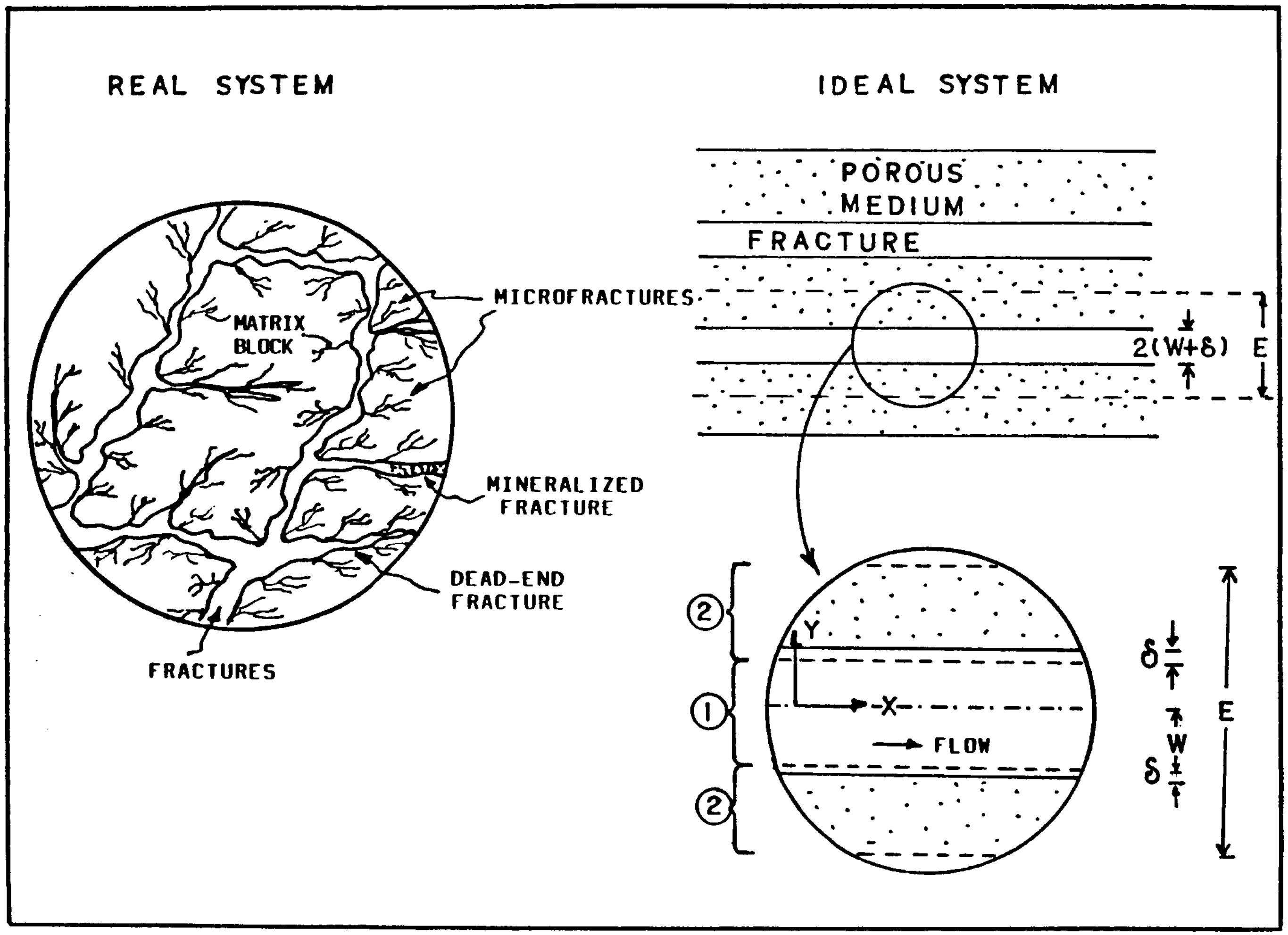

Fig. 1. Idealized proposed model for representation of the naturally fractured medium.

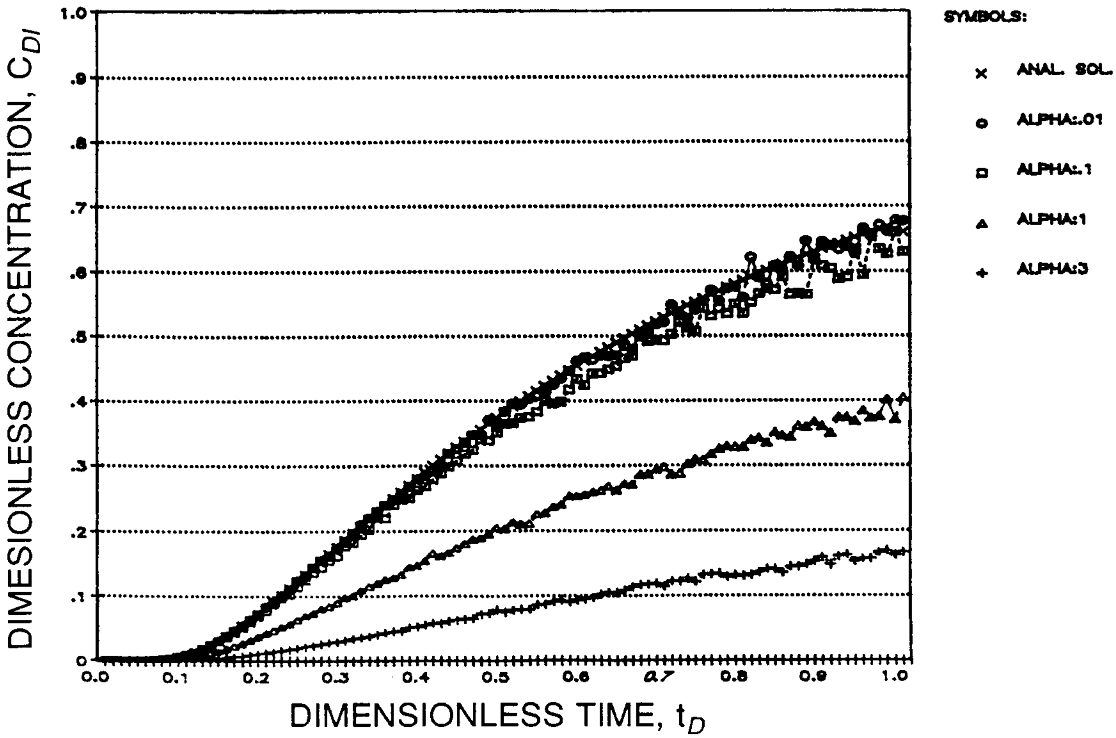

Fig. 2. Effect of α on the concentration profile at X_D=1.0. Step up injection case.

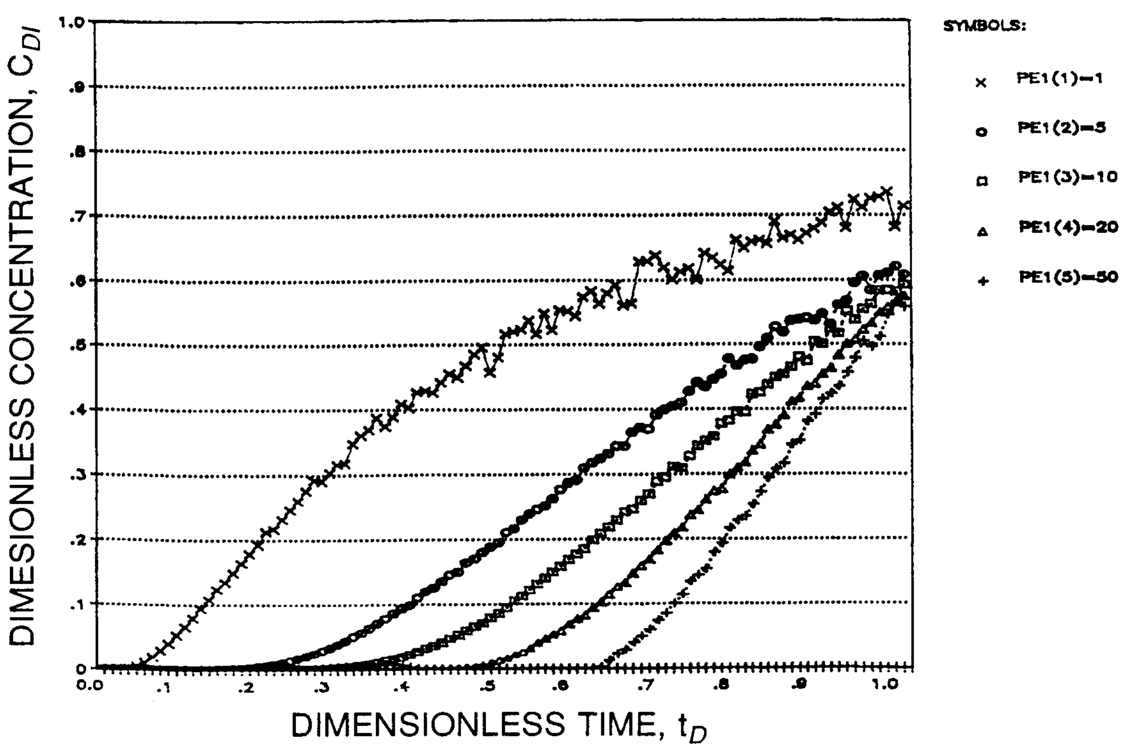

Fig. 3. Effect of the Peclet number in the mobile region, P_{e1}, on the concentration profile X_D=1.0. Step injection, α=0.01

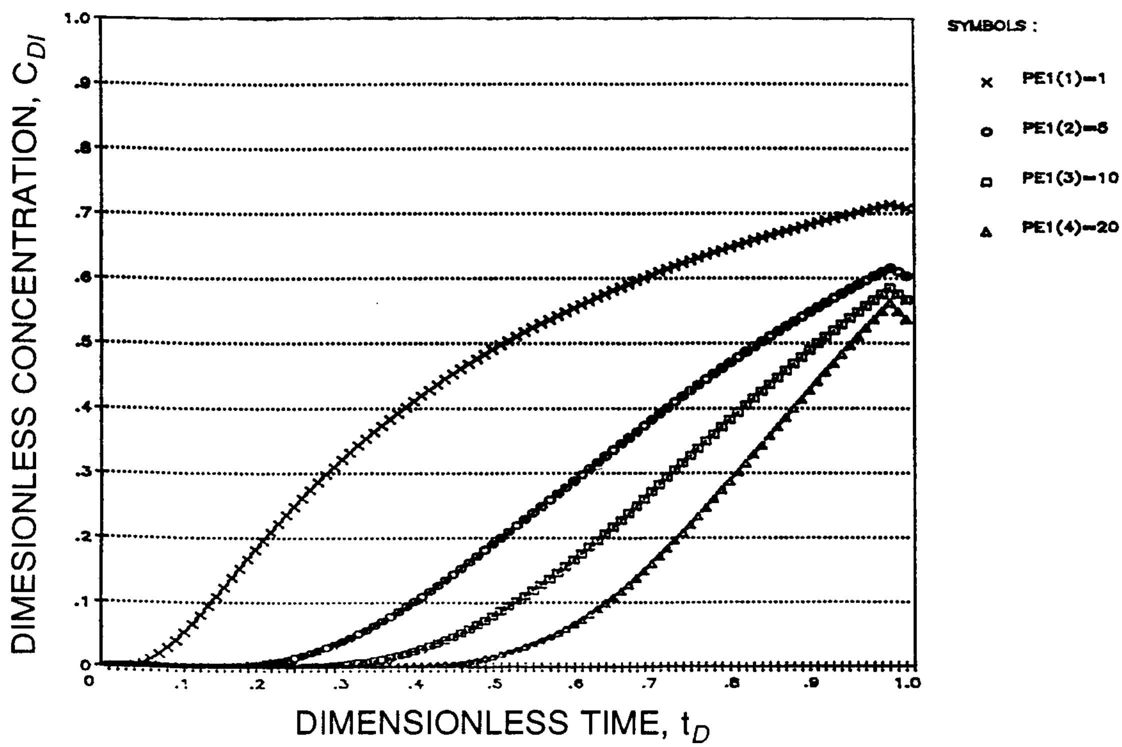

Fig. 4. Limiting analitical solution given by Eq. (27), as a function of P_{e1}. Step injection case.

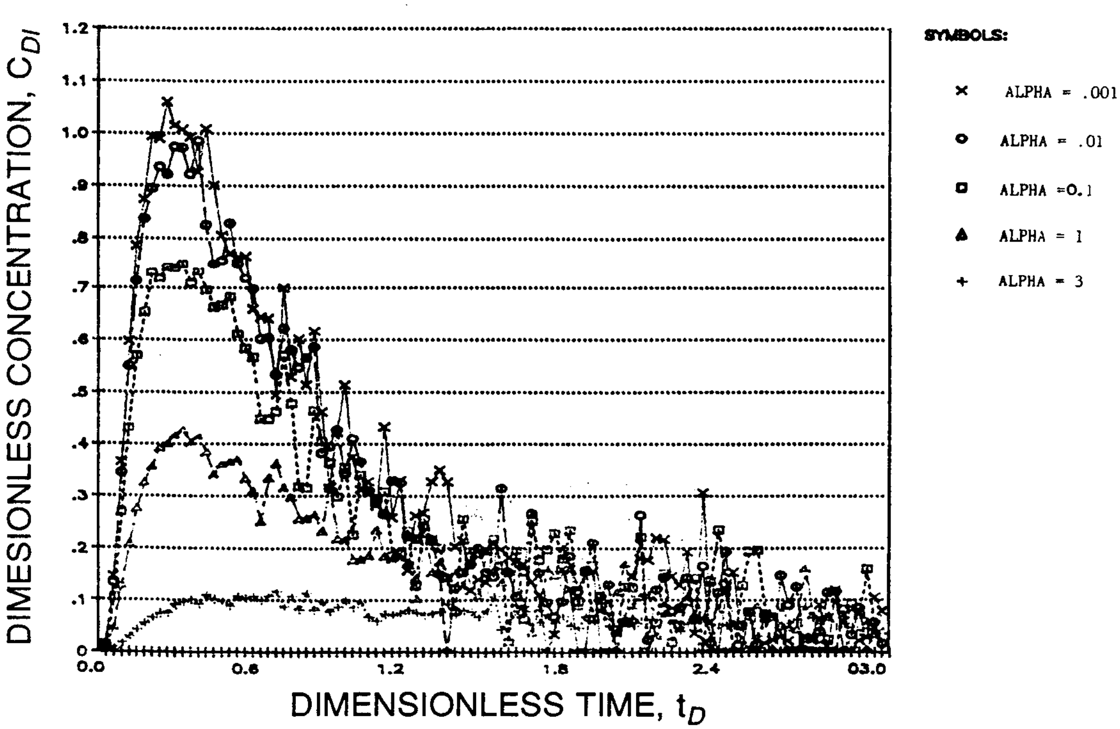

Fig. 5. Tracer concentration responses at X_D=1, P_{e1}=2 and several values of α. Spike injection case.