PROCEEDINGS, Thirteenth Workshop on Geothermal Reservoir Engineering
Stanford University, Stanford, California, January 19-21, 1988
SGP-TR-113

A SEMI-ANALYTICAL METHOD FOR HEAT SWEEP CALCULATIONS IN FRACTURED RESERVOIRS

K. Pruess and Y. S. Wu

Earth Sciences Division, Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

ABSTRACT

An analytical approximation is developed for purely conductive heat transfer from impermeable blocks of rock to fluids sweeping past the rocks in fractures. The method was incorporated into a multi-phase fluid and heat flow simulator. Comparison with exact analytical solutions and with simulations using a multiple interacting continua approach shows very good accuracy, with no increase in computing time compared to porous medium simulations.

INTRODUCTION

Despite major advances in recent years, the mathematical modeling of fluid and heat flow in fractured geothermal reservoirs remains a difficult problem. Porous medium approximations have been shown to be inadequate for many flow processes in fractured systems (Pruess, 1983), while double- or multiple-porosity techniques may involve excessive amount of numerical work or large discretization errors.

Combining finite-difference and analytical techniques, we have developed a method which can provide an accurate and efficient representation of fluid and heat flow in fractured reservoirs under conditions of strong heat transfer and insignificant fluid exchange between rock matrix and fractures. The method is an adaptation of a technique developed by Vinsome and Westerveld (1980) to describe heat exchange with impermeable confining beds in thermally enhanced oil recovery.

THE METHOD OF VINSOME AND WESTERVELD

An important aspect of thermal oil recovery schemes is the transfer of heat by conduction from the reservoir to adjacent strata of low permeability. In steam and hot waterfloods this represents a heat loss which may have significant effects on process economics. In steam soak operations (huff-and-puff) heat lost to cap and base rock during injection can be partially conducted back to the reservoir during the production cycle, providing beneficial effects. The heat exchange with impermeable strata can be large and must be included in numerical simulations of thermal recovery. At early times the conductive temperature profile has rather steep gradients near the surface of the conductive zone, while at late time it extends to large distance from the boundary. A reasonably accurate representation of heat conduction by numerical methods (e.g. finite differences) therefore requires many grid blocks and can greatly increase the computational work.

A number of semi-analytical and variational approaches have been developed which permit modeling of conductive heat exchange with impermeable strata without requiring these strata to be explicitly included in the domain of a finite difference model (Weinstein, 1972, 1974; Chase and O'Dell, 1973; Vinsome and Westerveld, 1980). Of these the method of Vinsome and Westerveld is the most attractive due to its elegance and simplicity. Observing that the process of heat conduction tends to dampen out temperature variations, Vinsome and Westerveld suggested that cap- and base-rock temperatures would vary smoothly even for strong and rapid temperature changes at the boundary of the conductive zone. Arguing that heat conduction perpendicular to the conductive boundary is more important than parallel to it, they proposed to represent the temperature profile in a conductive layer by means of a simple trial function, as follows:

$$T(x,t) - T_i = (T_f - T_i + px + qx^2)e^{-x/d} \qquad (1)$$

Here x is the distance from the boundary, T_i is initial temperature in cap- or base-rock (assumed uniform), T_f is the time-varying temperature at cap- or base-rock boundary, p and q are time-varying fit-parameters, and d is the penetration depth for heat conduction, defined by

$$d = \frac{\sqrt{\kappa t}}{2} \qquad (2)$$

where $\kappa = K/\rho c$ is the thermal diffusivity, with K the heat conductivity, ρ the density of the medium, and c the specific heat. In connection with a finite-difference simulation of non-isothermal flow, each grid block in the top and bottom layers of the computational grid will have an associated temperature profile in the adjacent impermeable rock as given by Eq. (1). The coefficients p and q will be different for each grid block; they are determined concurrently with the flow simulation from simple physical principles, namely: (1) temperature at the conductive boundary obeys the heat conduction equation for the impermeable stratum, and (2) the rate of change in total cap- or base-rock heat content is equal to the heat flux at the boundary. Vinsome and Wester-

veld presented test calculations which showed that their method was able to accurately represent monotonic as well as non-monotonic temperature profiles. We incorporated their technique into our MULKOM simulator (Pruess, 1983b, 1988) and verified that it gave accurate results (Pruess and Bodvarsson, 1984).

HEAT EXCHANGE WITH BLOCKS OF IMPERMEABLE ROCK

The method of Vinsome and Westerveld treats heat exchange between a surface with time varying temperature and a semi-infinite conductive half-space. It can be easily adapted to the problem of heat exchange between impermeable rock matrix blocks and fluids flowing in fractures or porous materials around these blocks. The required modifications involve the equation of heat conduction and the calculation of total heat content in the blocks, both of which differ from those for a semi-infinite medium.

Following concepts developed in the method of "multiple interacting continua" or "MINC" (see Fig. 1) we approximate heat flow in impermeable blocks of rock as being one-dimensional, with temperatures in the blocks depending only on the distance x from the nearest surface (i.e. from the nearest fracture; Pruess and Narasimhan, 1982, 1985). We use the concept of "proximity function" (Pruess and Karasaki, 1982) to describe one-dimensional flow in blocks of arbitrary shape, as well as flow in stochastic assemblages of matrix blocks which are encountered in fractured reservoirs. For matrix blocks of volume V_m having a volume V(x) within a distance x from the fractures (i.e., from the block surfaces), the proximity function is defined as:

$$\mathrm{PROX}(x) = \frac{V(x)}{V_m} \tag{3}$$

The interface area for flow in the matrix blocks at distance x from the surface is

$$A(x) = \frac{dV}{dx} = V_m \, d\frac{\mathrm{PROX}}{dx} \tag{4}$$

Considering a heat balance for a volume element dV = A(x) dx, we obtain the following equation for one-dimensional heat conduction in the blocks:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} + \kappa \frac{\partial T}{\partial x} \frac{\partial \ln A}{\partial x} \tag{5}$$

Choosing the same form Eq. (1) for the temperature profile in the blocks as was used by Vinsome and Westerveld for the semi-infinite solid, the condition that Eq. (5) must be satisfied at the surface of the blocks gives

$$\frac{T_f - T_f^\circ}{\kappa \Delta t} = \frac{\theta}{d^2} - \frac{2p}{d} + 2q + \left.\frac{\partial \ln A}{\partial x}\right|_o \left(p - \frac{\theta}{d}\right) \tag{6}$$

Here we have replaced the time derivative by a first-order forward finite difference, as required for incorporating the method into our numerical simulator MULKOM. T_f° and T_f are temperatures in the fracture at the beginning and end of the time step Δt, respectively. θ is an abbreviation for T_f - T_i. For a semi-infinite solid the derivative term involving A(x) vanishes, so that Eq. (6) then reduces to the form given by Vinsome and Westerveld. Energy conservation in the blocks is expressed as follows.

$$\frac{d}{dt} \int_{V_m} \rho c T dv = -K \left.\frac{\partial T}{\partial x}\right|_o A(x=0) \tag{7}$$

With a slight rearrangement of terms the integral on the left hand side becomes

$$I(t) = \int_0^{D/2} [T(x,t) - T_i] \frac{A(x)}{A(0)} dx \tag{8a}$$

The integration extends to D/2, which for fracture spacing D is the largest distance from the block surfaces. Inserting from Eq. (1) this integral can be written as

$$I(t) = \beta q + \gamma p + \delta \theta \tag{8b}$$

The coefficients β, γ and δ represent a weighting of the x-dependent terms $x^n \exp(-x/d)$ (n = 0, 1, 2) in Eq. (1) with the function A(x)/A(0) characterizing the matrix block shapes. Even for irregular blocks and stochastic assemblages the proximity function and its derivative A(x) can be written as polynomials in x (Pruess and Karasaki, 1982), so that the integral in Eq. (8) can be evaluated by elementary means. Evaluating the spatial derivative from Eq. (1) the finite difference version of Eq. (7) becomes

$$I(t+\Delta t) - I(t) = \kappa \Delta t \left(\frac{\theta}{d} - p\right) \tag{9}$$

Eqs. (6) and (9) (with the definition Eq. 8) represent two linear equations for the two unknown time-dependent parameters p and q. Solution of these is trivial once the coefficients β, γ, and δ in Eq. (8b) have been obtained. The heat flux from the blocks to the fractures is calculated as in Vinsome/Westerveld by

$$Q = K \left.\frac{\partial T}{\partial x}\right|_0 = K \left(\frac{\theta}{d} - p\right) \tag{10}$$

Calculation of the time-dependent coefficients p and q from Eqs. (6) and (9), and of heat exchange between the permeable and the conductive domains from Eq. (10), has to be done at each time step separately for all grid blocks which contain purely conductive material. It is possible to apply the conductive exchange calculation only for certain grid blocks, while others may be treated as homogeneous porous media, or as fractured media with permeable matrix using the MINC method. The temperature T (x=0,t) at the surface of the conductive domain is identified with the temperature in the permeable portion (fractures) of the grid block. In a fully implicit scheme this temperature is evaluated at the new time level t + Δt, and the heat exchange calculation is done in a fully coupled manner as part of the iterative

process to solve the fluid and heat flow equations in the permeable domain.

To incorporate the above scheme into a numerical simulator we partition grid block volumes into a permeable and a purely conductive part.

$$V_n = V_{n,per} + V_{n,cond} \qquad (11)$$

Fluid and heat flow in the permeable portions $V_{n,per}$ of the grid blocks is handled by numerical simulation. Heat transferred by conduction from the impermeable portion $V_{n,cond}$ is represented by including Eq. (10), properly scaled for the total conductive interface area in V_n, as a source term into the heat balance equation for $V_{n,per}$.

EVALUATION

We have implemented the method described in the foregoing section into our general purpose simulator MULKOM, and have performed several tests and comparisons. For simplicity this was done for matrix blocks of cubic shape. For cubes the proximity function can be directly obtained from the definition Eq. (3); it is given by

$$\mathrm{PROX}(x) = \frac{6x}{D} - \frac{12x^2}{D^2} + \frac{8x^3}{D^3} \qquad (12)$$

This leads to a particularly simple form for the expression A(x)/A(0) appearing in the integral Eq. (8), namely,

$$\frac{A(x)}{A(0)} = \left(1 - \frac{2x}{D}\right)^2 \qquad (13)$$

In order to evaluate the accuracy of the semi-analytical approximation we have studied a problem for which exact analytical solutions are available, namely, heat exchange with a cube of initially uniform temperature, which at time $t = 0$ is subjected to a step change in temperature at the surface. The parameters of the problem are given in Table 1.

The heat flow rate at the surface of the cube was computed as function of time using the following four approaches: (1) numerical evaluation of the exact three-dimensional Fourier series solution (Carslaw and Jaeger, 1959); (2) a one-dimensional approximation to heat flow in a cube, for which the exact solution is identical to heat flow in a sphere (Carslaw and Jaeger, 1959); (3) the semi-analytical solution as developed above, incorporated into the MULKOM simulator; and (4) method of multiple interacting continua ("MINC"; Pruess and Narasimhan, 1985). Results are given in Tables 2 and 3.

Table 1. Parameters for test problem (heat exchange with unit cube).

side length of cube	1 m
rock density	2650 kg/m^3
specific heat	1000 J/kg°C
heat conductivity	2.1 W/m°C
initial temperature	300 °C
surface temperature for $t > 0$	100 °C

The "exact 3-D" and the "exact 1-D" results are virtually identical, with the exception of very early and very late times, which have little significance for overall heat transfer. Heat flow rates calculated in the semi-analytical approximation agree very well with the exact results, being typically 1-2 % larger. Cumulative heat transfer in the semi-analytical approximation is underpredicted by typically 10% at most times, but it approaches the correct asymptotic value of 5.3×10^8J at late times. It may appear inconsistent that heat flow rates in the semi-analytical approximation are slightly on the high side at all times while cumulative heat transfer is somewhat low. This effect is caused by the time discretization: In the semi-analytical approach the heat flow rate is constant during each time step; moreover, in our fully implicit scheme it is equal to the heat flow rate at the end of the time step. Because heat flow rates are monotonically declining this leads to some underprediction of cumulative heat transfer. The accuracy of the semi-analytical calculation could be improved by taking smaller time steps (we used 4 time steps per log-cycle), or by using a mid-point weighting in time (Crank-Nicolson equation; Peaceman, 1977) rather than a fully implicit treatment. However, in practical problems one is seldom interested in accurate answers

Table 2. Heat flow rates from unit cube.

Time	Heat Flow Rate (W)			
(s)	exact 3-D	exact 1-D	semi-analytical	MINC
1	1.410E6	1.592E6	1.821E6	1.315E6
10	4.987E5	5.000E5	5.038E5	6.122E5
10^2	1.534E5	1.547E5	1.546E5	1.683E5
10^3	4.429E4	4.547E4	4.514E4	4.913E4
10^4	1.020E4	1.093E4	1.063E4	1.207E4
10^5	6.354E2	4.414E2	9.478E2	8.372E2
10^6	4.275E-7	2.610E-10	1.021E1	5.676E-2

Table 3. Cumulative heat flows from unit cube.

Time	Cumulative Conductive Heat Transfer (MJ)			
(s)	exact 3-D	exact 1-D	semi-analytical	MINC
1	3.285	3.286	2.645	1.22
10	10.04	10.05	9.079	8.52
10^2	31.31	31.44	28.40	30.28
10^3	94.73	95.97	86.09	93.52
10^4	259.6	269.0	235.7	261.8
10^5	503.0	515.9	453.9	503.3
10^6	530.0	530.0	524.9	529.9

over many orders of magnitude in time, so that time steps do not need to grow as fast as in our test case, and better time truncation accuracy will be attainable.

Heat flow rates calculated in the MINC approach, using 50 subcontinua of equal volume, differ by as much as 10 - 20% from the exact values. For rates that change with time by many orders of magnitude this is not a bad approximation. In terms of cumulative heat transfer the MINC approximation does extremely well (Table 3). After a brief period with significant space discretization effects at very early times the MINC results agree to better than 1% with the exact solution.

FIVE-SPOT

We have applied the semi-analytical heat exchange approach to a two-dimensional five-spot production/injection problem similar to that previously studied by Pruess (1983a). Problem parameters are given in Table 4.

The grid used in the numerical simulations represents 1/8 of a five-spot; it has six rows and eleven columns for a total of thirty-six volume elements (see Fig. 2). We assume three sets of equidistant, plane, parallel fractures at right angles, so that the impermeable matrix blocks are cubes. Calculations were done for two different fracture spacings. In addition to using the semi-analytical approach we also performed simulations with the MINC method, and with a uniform porous medium model (with same total void space, i.e., porosity of 1%). The MINC approach uses five subcontinua, with volume fractions of .02, .08, .20, .35, and .35. Results are given in Figure 3 and in Table 5.

Table 4. Specifications of five-spot problem.

Formation	
rock grain density	2650 kg/m^3
specific heat	1000 J/kg°C
heat conductivity	2.1 W/m°C
permeable volume fraction	2%
porosity in permeable domain	50%
impermeable blocks: cubes with side length	50 m, 250 m
effective permeability	6.0×10^{-15}m^2
thickness	305 m
relative permeability: Corey curves with $S_{lr} = 0.30$, $S_{vr} = 0.05$	
initial temperature	300°C
initial liquid saturation	0.99
initial pressure	85.93 bar
production/injection	
pattern area	1 km^2
distance between producers and injectors	707.1 m
production rate (*)	30 kg/s
injection rate (*)	30 kg/s
injection enthalpy	500 kJ/kg

(*) full well basis

Fig. 3 shows temperature profiles along the line connecting a producer and an injector after 36.5 years, corresponding to injection of approximately 12.2 pore volumes. For both fracture spacings the agreement between the semi-analytical and the MINC simulations is excellent. The D = 50 m results are indistinguishable from the porous medium calculation, while at the larger fracture spacing of D = 250 m the thermal sweep is less complete and lower temperatures are obtained. Predicted total heat transfer from the impermeable rocks to the fluids agrees to better than 1% between the semi-analytical and MINC approaches at most times (see Table 5). The semi-analytical approach required the same amount of computing time as the porous medium case, while the MINC calculation was approximately five times slower.

DISCUSSION AND CONCLUSIONS

We have incorporated an analytical heat transfer model into a numerical simulator for fluid and heat flow to calculate heat exchange between impermeable blocks of rock and fluids migrating past these blocks in fractures. For problems in which rock matrix permeability is negligible this offers a means of simulating fluid and heat flow in fractured media with essentially no increase in computing work as compared to porous medium simulations. Detailed analysis of heat flow from a cube suggests that the semi-analytical method should provide acceptable engineering accuracy. We are presently investigating some variations on the form Eq. (1) which might achieve even better accuracy. However, simulations for a two-phase production/injection problem with phase change gave almost perfect agreement with the MINC method for two different fracture spacings, so that there may be little need for further improvement.

There are two reasons why the semi-analytical method performs better on a reservoir problem than might have been expected from the test results for an individual

Table 5. Cumulative heat transfer from rocks to fluids in 1/8 of five-spot.

Time (years)	Cumulative Conductive Heat Transfer (10^{14}J)			
	Fracture Spacing 50 m		Fracture Spacing 250 m	
	MINC	Semi-analytical	MINC	Semi-analytical
1	1.07	1.07	.83	.85
2	2.11	2.10	1.77	1.79
5	5.22	5.18	4.64	4.67
10	10.33	10.29	9.40	9.46
15	15.48	15.44	14.10	14.16
20	20.43	20.38	18.70	18.79
25	25.24	25.19	23.21	23.33
30	30.14	30.10	27.60	27.75
35	35.05	35.00	31.91	32.07

cube. Firstly, time discretization errors are not as severe in the reservoir problem, because time steps can be allowed to settle at $\Delta t = .5$ - 1 year after the initial rapid transient, while in the single-cube problem they were kept growing to cover the large time range from very early transient to late-time temperature equilibrium. Secondly, the aggregate response of many rock blocks in a reservoir problem tends to compensate for whatever inaccuracies may be present in the modeling of individual block response. To see how this comes about, suppose that because of discretization effects the blocks near the injection well do not deliver heat to the fluids as rapidly as they should.(This is what actually happens in the semi-analytical method; see Table 3.) As a consequence fluids will have lower temperatures when at later time they sweep past downstream blocks, and hence they will pick up more heat from those blocks. This compensation of inaccuracies in individual block response from global reservoir mechanisms is completely analogous to what was observed in analysis of waterfloods in fractured hydrocarbon reservoirs (Wu and Pruess, 1986). It indicates that satisfactory accuracy in reservoir flow problems should be attainable with rather modest accuracy requirements for individual blocks.

The semi-analytical method for heat transfer presented here should also be applicable to problems of chemical transport in fractured media, because chemical transport in low-permeability blocks of rock can be described in analogy to heat conduction. Work along these lines is in progress.

ACKNOWLEDGEMENT

This work was supported by the Geothermal Technology Division, U.S. Department of Energy, under Contract No. DE-AC03-76SF00098.

REFERENCES

Carslaw, H. S., and Jaeger, J. C. (1959), *Conduction of Heat in Solids,* Oxford University Press, Oxford, England, Second Edition.

Chase, C. A. and O'Dell, P. M. (1973), "Application of Variational Principles to Cap and Base Rock Heat Losses," *Trans. AIME, 225,* 200-210.

Peaceman, D. W. (1977), *Fundamentals of Numerical Reservoir Simulation,* Elsevier, Amsterdam.

Pruess, K. (1983a), "Heat Transfer in Fractured Geothermal Reservoirs with Boiling," *Water Resources Research, 19,* (1), 201-208, February.

Pruess, K. (1983b), "Development of the General Purpose Simulator MULKOM," Annual Report 1982, Earth Sciences Division, Report LBL-15500, Lawrence Berkeley Laboratory.

Pruess, K. (1988), "SHAFT, MULKOM, TOUGH," Lawrence Berkeley Laboratory Report LBL-24430, Berkeley, Ca., January, (submitted to Geotermia, Rev. Mex. Geoenergia).

Pruess, K. and Bodvarsson, G. S. (1984), "Thermal Effects of Reinjection in Geothermal Reservoirs with Major Vertical Fractures," *Journal of Petroleum Technology, 36,* (10) 1567-1578, September.

Pruess, K. and Karasaki, K. (1982), "Proximity Functions for Modeling Fluid and Heat Flow in Reservoirs with Stochastic Fracture Distributions," presented at Eighth Workshop on Geothermal Reservoir Engineering, Stanford University, Stanford, Ca., December.

Pruess, K. and Narasimhan, T. N. (1982), "On Fluid Reserves and the Production of Superheated Steam from Fractured, Vapor-Dominated Geothermal Reservoirs," *Journal of Geophysical Research, 87,* (B11) 9329-9339.

Pruess, K. and Narasimhan, T. N. (1985), "A Practical Method for Modeling Fluid and Heat Flow in Fractured Porous Media," *Society of Petroleum Engineers Journal, 25,* (1), 14-26, February.

Vinsome, P. K. W., and Westerveld, J. (1980), "A Simple Method for Predicting Cap and Base Rock Heat Losses in Thermal Reservoir Simulators," *J. Canadian Petroleum Technology,* 87-90, July-September.

Weinstein, H. G., (1972), "Semi-Analytic Method for Thermal Coupling of Reservoir and Overburden," *Society Petroleum Engineers Journal,* 439-447, October.

Weinstein, H. G., (1974), "Extended Semi-Analytical Method for Increasing and Decreasing Boundary Temperature," *Society Petroleum Engineers Journal,* 152-164, April.

Wu, Y. S. and Pruess, K. (1986), "A Multiple-Porosity Method for Simulation of Naturally Fractured Petroleum Reservoirs," paper SPE-15129, presented at 56th Annual California Regional Meeting of the SPE, Oakland, Ca., April (to appear in *Society Petroleum Engineers Journal).*

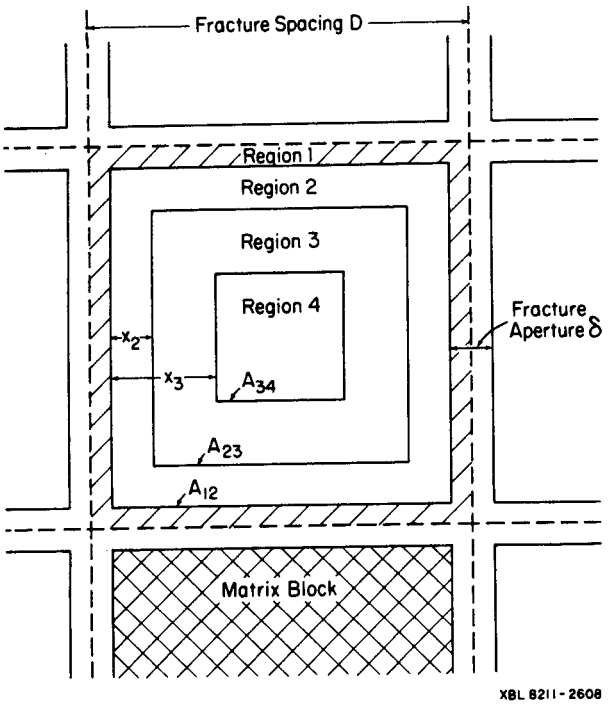

Figure 1. The concept of multiple interacting continua for an idealized fracture system.

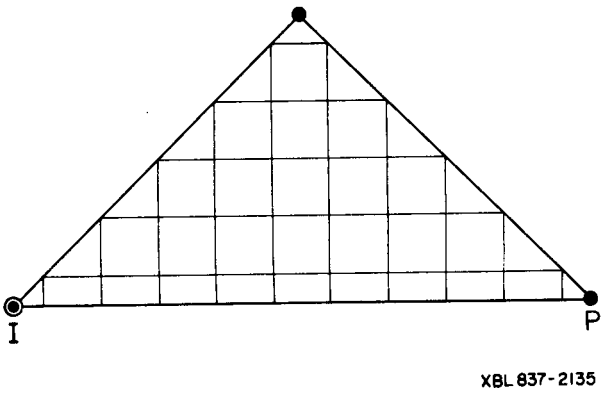

Figure 2. Computational grid for five-spot production/injection problem (I - injector, P - producer).

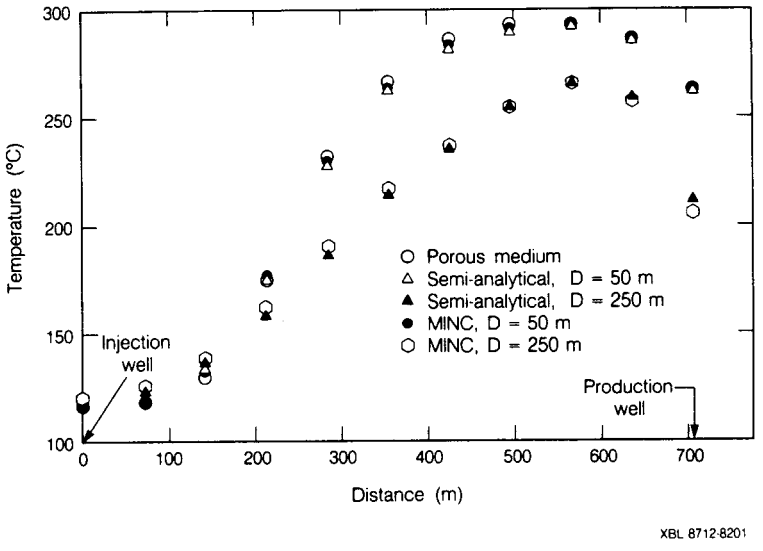

Figure 3. Temperature profiles for five-spot after 36.5 years.