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USE OF DERIVATE FOR DETECTING LINEAR IMPERMEABLE BARRIERS BY TRANSIENT PRESSURE TEST

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ABSTRACT

The goal of this paper is to show a numerical method for evaluate distance at an impermeable barrier, using buildup and drawdown pressure test analysis.

This technical needs of derivates evaluation in transition zone defined between two classics semilog straight lines with slopes "m" and "2m" that are present in these systems.

Complementary to shis study the authors propose type curves for application in alternatives analysis.

An illustrative example is showed using the proposed method.

INTRODUCTION

The identification of heterogeneities in geothermal reservoir takes an important place in evaluation of these. Accurate predictions from simulation works require of true conceptual models and propper caracterizations.

The transient well testing are very useful tools to evaluate the hydraulic parameters of reservoir and identify of heterogeneities of this, for example the impermeable barriers that are the main objective of this paper.

Horner (1951) was the first which pointed out that from buildup well testing is possible to detect impermeable boundaries, he reported that the graphic of Pws vs. log ($t/(t_p+ t)$) shows two straight lines with slopes "m" and "2m".

Davis and Hawkins (1963), Witherspoon et. al. (1976) and Erlougher (1977) gave methods to evaluate the distance to impermeable barriers using the profile of the straight lines. Nevertheless these methods are restricted if the lines are not developed completely.

Gray (1965) established that the difference between the extrapolated pressure data of the first semilog straight line gives a useful method to obtain the distance to barrier and Martinez and Cinco (1983) extended further this method.

The objective of this study to carry out an interpretation algorithm, oriented in the transition zone between one and other semilog straight using the numerical derivates of pressure. The method proposed does not require definition of the second straight line.

BASIC EQUATIONS

The dimensionless pressure drop caused by one well at constant rate producing in an infinite medium, homogeneous and isotropic, can be given by the line source solution of the diffusivity equation and is expressed in terms of exponential integral as:

$$P_D = -0.5Ei(-r_D^2 / (4t_D)) \qquad \text{..............(1)}$$

Where the dimensionless variables are defined with conventional form by:

$$P_D = (\rho kh\Delta p)/(\beta W\mu)$$

$$t_D = (\alpha kt)/(\emptyset \mu c_t r_w^2)$$

$$r_D = r/r_w \qquad \text{.............(2)}$$

The effect of one impermeable barrier can be simulated from the images theory, locating one production well at the other side of the barrier and simetric to the real well, this is showed in figure 1. The solution for this system is given by:

$$P_D=-0.5(Ei(-r_{D1}^2/(4t_D))+Ei(-r_{D2}^2/(4t_D)))\text{...(3)}$$

The r_{D1} and r_{D2} are dimensionless distance between the observation point (x,y) and the real well and image well respectively, and are defined as:

$$r_{D1} = r_1/r_W$$

$$r_{D2} = r_2/r_W \qquad \text{................(4)}$$

For the system of figure 1, the following relations are valid:

$$r_{D1}^2 = x_D^2 + y_D^2$$

$$r_{D2}^2 = (2d_D - x_D)^2 + y_D^2 \qquad \text{................(5)}$$

Where dD is dimensionless distance between real well and impermeable barrier and is expressed in the form of:

$$d_D = d/r_w \quad \text{...............(6)}$$

The right hand term on equation (3) is related to the drop pressure in the acting well, also the second one, describes the drop pressure due to the barrier (Image Well).

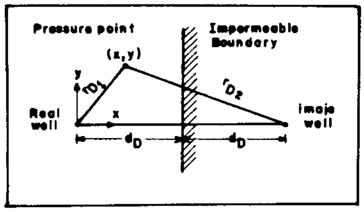

Fig.1: Use the image well method to simulated a non flow boundary.

DRAWDOWN TEST

On the drawdown test, the equation (3) is evaluated at $r_{D1}=1$ and $r_{D2}=2d_D$ for larger value times, so the dimensionless drop pressure in the wellbore could be written as:

$$P_D=0.5(\ln(t_D)-Ei(-d_D^2/t_D)+0.8097) \quad \text{.......(7)}$$

Getting the derivate of equation (7) with time and rearranging:

$$t_D P_D' = 0.5(1+EXP(-d_D^2/t_D)) \quad \text{.............(8)}$$

Figure (2) shows the type curve for equation (8). Two straight lines are defined here: in the first one, the behavior of the pressure in the wellbore is according to one well producing into an infinite medium; in the second one, the behavior of the pressure is affected by the impermeable boundary effects.

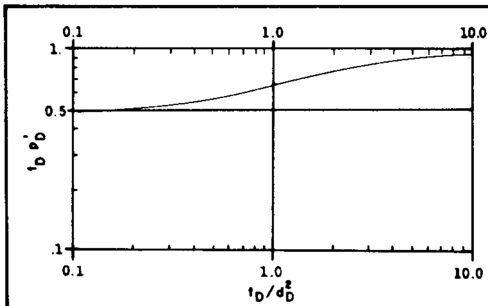

Fig. 2: Type curve for linear barrier detection by pressure drawdown test using derivate method.

Type curve showed at figure 2, has no problem of unicity, making easy the analysis since it can be moved just horizontally.

As a matter of fact, we need just a correct definition of the first horizontal line a short part of the transition zone to make the match.

Analysis Technique. Equation (8) can be written as:

$$2t_D P_D'-1=EXP(-d_D^2/t_D) \quad \text{...............(9)}$$

so, taking natural logarithm and writing it in real variables we get:

$$\ln(t(dP_w/dt)/m_1' - 1) = -d^2/(\eta t) \text{......(10)}$$

where:

η = hydraulic diffusivity coefficient.

$m_1' = m_1/\ln(10)$.

m_1 = slope of the first straight line in a P_w vs. log(t) plot.

Equation (10) suggests that an $\ln(t(dP_w/dt)/m_1'-1)$ vs. 1/t plot could be a straight line with a slope proportional to the square distance to the fault.

Another analysis technique can be proposed, if we get the product of equation (11) by $-\eta t$, this is;

$$d^2=-\eta t \ln(t(dP/dt)/m_1'-1) \quad \text{............(11)}$$

the above equation says that in the transition zone the right hand term is a constant equal to the square distance to the barrier.

BUILDUP TEST

In a buildup test, formaly trated by Horner (1951), Mathews (1967) and Earlougher (1977), the behavior of the dimensionless pressure in the wellbore is given by:

$$P_D=0.5(\ln((t_{pD}+\Delta t_D)/\Delta t_D)-Ei(-d_D^2/(t_{pD}+\Delta t_D))+Ei(-d_D^2/\Delta t_D)) \quad \text{...............(12)}$$

Derivating by time in both sides of the equation and rearranging:

$$((t_{pD}+\Delta t_D)/t_{pD})\Delta t_D P_D' = 0.5(1+EXP(-d_D^2/\Delta t_D)-(\Delta t_D/t_{pD})(EXP(-d_D^2/t_{pD}+\Delta t_D)-EXP(-d_D^2/\Delta t_D)) \quad \text{...............(13)}$$

The type curve showed in figure (3) is obtained of equation (13), for different relation values of dimensionless variables t_{pD}/d_D^2.

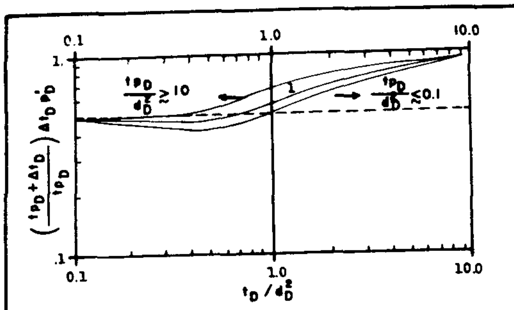

Fig.3: Type curve for linear barrier detection by pressure buildup test using derivate method.

This type curves shows that the shape of transition zone defined between the two horizontal lines is determinated by t_{pD}/t_D^2, and for values of $0.1 > t_{pD}/d_D^2 > 10$ all curves trend to form a group of one curve only.

Also, values of $t_{pD}/d_D^2 > 10$ are required to avoid desviations from the pressure data below of the first semilog straight line.

More over, as in the case of drawdown test, this type curve does not present any unicity problems.

Analysis Technique: Rearranging equation (13), we get:

$$2((t_{pD}+\Delta t)/\ t_{pD})\Delta t_D P_D' - 1 + f) = EXP(-d_D^2/\Delta t) \qquad \ldots\ldots\ldots\ldots\ldots(14)$$

Where:

$$f = (\Delta t_D/t_{pD})(EXP(-d_D^2/(t_{pD}+\Delta t_D) - EXP(-d_D^2/\Delta t_D))$$

and taking the logarithms in both sides of equation (14) and multiplying by $-\eta t_D$ the equation stablished is:

$$-\eta t_D \ln(2((t_{pD}+\Delta t_D)/t_{pD})\Delta t_D P_D' - 1 + f) = d_D^2 \qquad \ldots\ldots\ldots\ldots\ldots(15)$$

in terms of real variables:

$$d^2 = -\ \eta t \ln(2(t_p+\Delta t)/t_p \Delta t(dp_{ws}/d\Delta t)/m_1' - 1 + f) \qquad \ldots\ldots\ldots\ldots\ldots(16)$$

An iterative procces of solution for equation (16) can be writing assuming an initial valve of $d(d^k)$ in equation "f" and solving for a new valve of $d\ (d^{k+1})$, and so on, until we reach the convergence criterian as:

$$\left|\frac{d^{k+1} - d^k}{d^k}\right| < \text{Tol.}$$

APPLICATION EXAMPLE

To show proposed technique in present paper a theoretical example for a buildup pressure test is generated, locating an impermeable barrier at 20 m. of distance, and considering a production time of 100,000 seconds, the remaining information for this example is the next:

$N = 60\ Kg/q$

$P_i = 8.0E6Pa$

$kh = 5.0E\text{-}13\ m^3$

$\phi C_t h = 3.0E\text{-}7m/Pa$

$\rho = 722.Kg/m^3$

Figure (4) shows the Horner plot of this example, in this it can be seen that second straight line does not appear. Figure (5) presents log-log plot of $(t_p+\Delta t)/t_p)\ \Delta t\ dp/dt$ versus Δt. The value of $m_1 = m_1'/\ln(10) = 610{,}000$ can be obtained from both graphics.

From graphics were selected three points in transition zone, as follows the test data are presented:

	Δt	p_{ws}	$(t_p+\Delta t)/t_p)\Delta t(dp_w/dt)$
1	0.63E4	6.7E6	0.38E6
2	0.10E5	6.8E6	0.42E6
3	0.20E5	7.1E7	0.46E6

The initial value of "d" was 10 m. and taking a tolerance of 0.001 the calculated values of "d" were; 20.04, 20.02 and 19.89, which are very similar to value used for generating this theoretical test.

Complementary it is mentioned that the convergency of this method is to speedy.

As an alternative, these data were matched with type curve of figure (3), obtaining a value of distance to a barrier of 19.5 m.

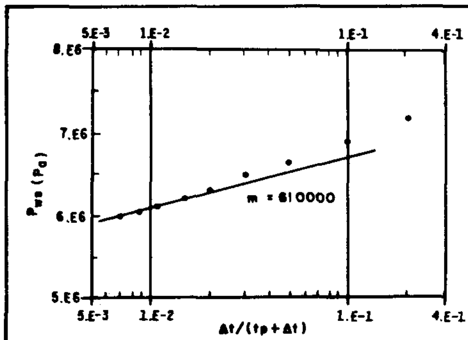

Fig.4: Honer plot for example.

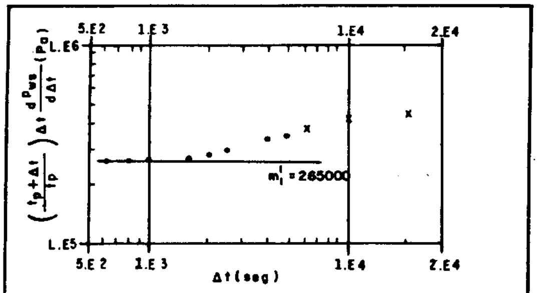

Fig.5: Log-log plot for example.

CONCLUSIONS

Analysis techniques were presented in this paper for evaluating distance from one well to an impermeable barrier, as well as in drawdown buildup tests. These do not need development of second straight line, because they are focussed in a transition zone that is defined between two classic straight lines.

At least, one point is needed in order to use this method, to verify distances, a good collection pressure data is suggested. Only, for buildup tests the technique is iterative and its convergency is to speedy. As an alternative type curves are proposed, mentioning that total development of transition zones are not needed. It was also founded that for values of t_{pD}/r_D^2 10 there are no irregular behaviors of graphics for buildup tests.

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NOMENCLATURE

c_t : Total compressibility (Pa^{-1}).
d : Distance between source well and impermeable barrier (m).
d_D : Dimensionless distance between source well and impermeable barrier.
h : Formation thickness (m).
k : Permeability (m^2).
m_1 : Slope of the first straight line.
m_1' : m_1 ln(10).
P_D : Dimensionless pressure.
P_i : Initial reservoir pressure (Pa).
p_w : Wellbore pressure (Pa).
p_{ws} : Shut-in wellbore pressure (Pa).
r_D : Dimensionless radial distance, r/rw.
r_1 : Distance between source well and observation well (m).
r_2 : Distance between image well and observation well (m).
r_w : Radius of well (m).
t_D : Dimensionless time.
t : Time (s).
t_{pD} : Dimensionless production time.
t_p : Production time (s).
t : Shut-in time during buildup (s).
W : Mass flow rate (Kg/s).
α : Constant convertion (=1).
β : Constant convertion ($=1/2\pi$).
ρ : Density (kg/m^3).
η : Hydraulic diffusivity.
ϕ : Porosity (fraction).
μ : Viscosity (Pa-s).

SPECIAL FUNCTION

$E_i(-x)$ = Exponential integral, $-\int_x^\infty (EXP(-u)/u)du$.
EXP(x) = Exponential of x.

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