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Well-Test Analysis for a Well in a Finite, Circular Reservoir

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ABSTRACT

This study presents drawdown and buildup pressure derivative type-curves for a well producing at a constant rate from the center of a finite, circular reservoir. Early time response (wellbore storage and skin effects) is correlated by C_De^{2s}, and late time response (outer boundary effects) by r_{eD}^2/C_D. The outer boundary may be closed, or at a constant pressure. Design relations are developed for the time to the beginning and the end of infinite-acting radial flow. Producing time effects on buildup responses are also discussed.

INTRODUCTION

Transient pressure response for a well producing from a finite reservoir of circular, square, and rectangular drainage shapes has been studied by *van Everdingen* and *Hurst* (1949); *Miller et al.* (1954); *Aziz* and *Flock* (1963); *Earlougher et al.* (1968); *Ramey* and *Cobb* (1971); *Kumar* and *Ramey* (1974); *Cobb* and *Smith* (1975); and *Chen* and *Brigham* (1978) among others. *Mishra* and *Ramey* (1987) presented a buildup derivative type-curve for a well with storage and skin, and producing from the center of a closed, circular reservoir. Their type-curve applies for large producing times such that $t_{pD} > t_{Dpss}$. This work presents drawdown and buildup pressure derivative type-curves for a well producing at a constant rate from the center of a finite, circular reservoir. The outer boundary may be closed, or at a constant pressure. The differences between the responses for a well in a closed, circular reservoir (fully developed field), and a well in a circular reservoir with a constant-pressure outer boundary (active edgewater drive system, or developed five-spot fluid-injection pattern) are discussed. Design relations are developed to estimate the time period which corresponds to infinite-acting radial flow, or to a semi-log straight line on a pressure vs. logarithm of time graph. Producing time effects on buildup responses are studied using the slope of a dimensionless *Agarwal* (1980) buildup graph.

THEORY

The dimensionless wellbore pressure drop for a constant-rate well with storage and skin may be expressed as (*van Everdingen* and *Hurst*, 1949):

$$p_{wD}(t_D) = \frac{2\pi kh\,(p_i - p_{wf})}{qB\mu}$$

$$= L^{-1}\left[\frac{1}{C_Dl^2 + \dfrac{l}{s + l\bar{p}_D}}\right] , \qquad (1)$$

where L^{-1} is the inverse Laplace transform operator. In Eq. (1), $\bar{p}_D$ refers to the dimensionless wellbore pressure drop in Laplace space without storage or skin. For the case of a constant-rate well producing from the center of a closed circle, the expression for $\bar{p}_D$ is (*van Everdingen* and *Hurst*, 1949):

$$\bar{p}_D = \frac{1}{l^{3/2}}\left[\frac{K_0(\sqrt{l})\,I_1(r_{eD}\sqrt{l}) + I_0(\sqrt{l})\,K_1(r_{eD}\sqrt{l})}{K_1(\sqrt{l})\,I_1(r_{eD}\sqrt{l}) - I_1(\sqrt{l})\,K_1(r_{eD}\sqrt{l})}\right] \qquad (2)$$

For the case of a constant-rate well producing from the center of a circular reservoir with a constant-pressure outer boundary, the expression for $\bar{p}_D$ is (*van Everdingen* and *Hurst*, 1949):

$$\bar{p}_D = \frac{1}{l^{3/2}}\left[\frac{K_0(\sqrt{l})\,I_0(r_{eD}\sqrt{l}) - I_0(\sqrt{l})\,K_0(r_{eD}\sqrt{l})}{K_1(\sqrt{l})\,I_0(r_{eD}\sqrt{l}) + I_1(\sqrt{l})\,K_0(r_{eD}\sqrt{l})}\right] . \qquad (3)$$

The dimensionless wellbore pressure drop from Eq. (1) was obtained by inverting the Laplace space solution numerically with the *Stehfest* (1970) algorithm.

DRAWDOWN RESPONSE

Table 1 shows the dimensionless wellbore pressure drop and the semi-log pressure derivative expressions for a well in a finite, circular reservoir during specific flow periods. All expressions in Table 1 may be written as combinations of t_D/C_D, C_De^{2s}, and r_{eD}^2/C_D. For example,

$$\ln(r_{eD}) + s = \frac{1}{2}\ln\left[\frac{r_{eD}^2}{C_D}\,C_De^{2s}\right] \text{ , and} \qquad (4)$$

$$C_2 = \frac{1}{2}\ln\left[\frac{2.2458\,A}{C_Ar_w^2}\right] + s = \frac{1}{2}\ln\left[\frac{2.2458\,\pi}{C_A}\,\frac{r_{eD}^2}{C_D}\,C_De^{2s}\right] \qquad (5)$$

Thus, if the dimensionless drawdown pressure and the pressure derivative responses are graphed against t_D/C_D, the parameters C_De^{2s} and r_{eD}^2/C_D may be selected as the correlating parameters. The verification of C_De^{2s} and r_{eD}^2/C_D as the

Table 1 - Dimensionless wellbore drawdown pressure and derivative expressions for a well in a finite, circular reservoir

Flow period	p_{wD}	$p'_{wD} = dp_{wD}/d \ln t_D$
Wellbore storage	t_D/C_D	t_D/C_D
Infinite-acting radial flow	$0.5\ [\ln (t_D/C_D) + C_1]$	0.5
Pseudosteady state (No wellbore storage, and closed reservoir)	$2\pi\ t_{DA} + C_2$	$2\pi\ t_{DA}$
Steady state (Constant-pressure outer boundary)	$\ln (r_{eD}) + s$	0

$$C_1 = \ln (C_D e^{2s}) + 0.80907 \quad \text{, and} \qquad C_2 = 0.5 \ln \left[\frac{2.2458\ A}{C_A r_w^2}\right] + s$$

correlating parameters is also shown in Fig. 1 for both closed and constant-pressure outer boundary cases. The individual values of C_D, s, and r_{eD} used to generate the pressure derivative responses are shown on Fig. 1.

Figure 2 shows the drawdown pressure derivative type-curve developed in this study. Both closed and constant-pressure outer boundary cases are shown. The dimensionless times by which the semi-log pressure derivative is within 2% of 0.5 are:

$$\left.\frac{t_D}{C_D}\right]_{begin} \approx 150 + 200 \log (C_D e^{2s}) \quad \text{, and} \qquad (6)$$

$$\left.\frac{t_D}{C_D}\right]_{end} \approx \frac{0.175\ r_{eD}^2}{C_D} \quad . \qquad (7)$$

Design Eqs. (6) and (7) apply for both closed and constant-pressure outer boundaries. Equations (6) and (7) yield the condition for the development of at least half a log cycle of semi-log straight line as:

$$r_{eD}^2/C_D > 2710 + 3600 \log (C_D e^{2s}) \quad . \qquad (8)$$

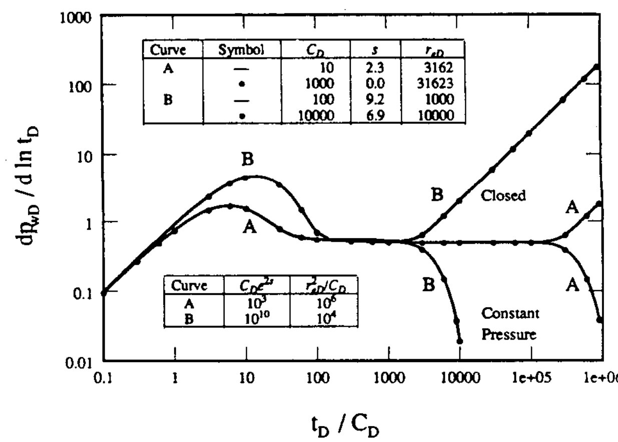

Fig. 1: Verification of $C_D e^{2s}$ and r_{eD}^2/C_D as the correlating parameters for the drawdown responses.

BUILDUP RESPONSE

The dimensionless buildup pressure is:

$$p_{wDs}(\Delta t_D) = \frac{2\pi kh\ (p_{ws} - p_{wf})}{qB\mu}$$

$$= p_{wD}(t_{pD}) + p_{wD}(\Delta t_D) - p_{wD}(t_{pD} + \Delta t_D) \quad (9)$$

and the slope of a dimensionless *MDH* (*Miller, Dyes,* and *Hutchinson,* 1950) buildup graph is:

$$MDH\ Slope = \frac{dp_{wDs}}{d \ln (\Delta t_D)} = \Delta t_D \frac{dp_{wDs}(\Delta t_D)}{d(\Delta t_D)} \quad . \qquad (10)$$

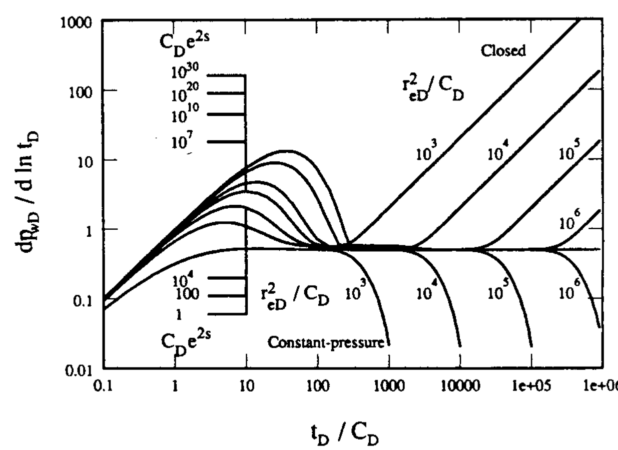

Fig. 2: Drawdown pressure derivative type-curve.

For large producing times such that $t_{pD} > t_{Dpss}$, *Mishra* and *Ramey* (1987) presented a type-curve as a log-log graph of *MDH* slope vs. $\Delta t_D/C_D$ with the correlating parameters as C_De^{2s} and r_{eD}^2/C_D. Their type-curve applies for a well in the center of a closed, circular reservoir. For large producing times such that $t_{pD} > t_{Dss}$, Fig. 3 shows the verification of the correlating parameters C_De^{2s} and r_{eD}^2/C_D for the buildup pressure derivative responses of a well in the center of a circular reservoir with a constant-pressure outer boundary. Figure 4 presents a buildup derivative type-curve for a well in the center of a circular reservoir with a constant-pressure outer boundary. The dimensionless times by which the semi-log buildup pressure derivative is within 2% of 0.5 on Fig. 4 are:

$$\left.\frac{\Delta t_D}{C_D}\right]_{begin} \approx 150 + 200 \log (C_De^{2s}) \text{ , and} \qquad (11)$$

$$\left.\frac{\Delta t_D}{C_D}\right]_{end} \approx \frac{0.175\, r_{eD}^2}{C_D} \quad . \qquad (12)$$

Equations (11) and (12) yield the condition for the development of at least half a log cycle of semi-log straight line as:

$$r_{eD}^2/C_D > 2710 + 3600 \log (C_De^{2s}) \quad . \qquad (13)$$

which is the same as Eq. (8).

Figure 5 shows the buildup derivative responses for a well in a circular reservoir with two different outer boundary conditions: closed and constant-pressure. Figure 5 applies for $C_De^{2s} = 1000$ and $r_{eD}^2/C_D = 10^6$. Figure 5 shows that for the same values of C_De^{2s} and r_{eD}^2/C_D, the semi-log straight line is longer for a well in a circular reservoir with a constant-pressure outer boundary than for a closed outer boundary.

The dimensionless times by which the slope of a dimensionless *MDH* buildup graph for a well in a closed reservoir is within 2% of 0.5 are:

$$\left.\frac{\Delta t_D}{C_D}\right]_{begin} \approx 150 + 200 \log (C_De^{2s}) \quad , \qquad (14)$$

$$\left.\frac{\Delta t_D}{C_D}\right]_{end} \approx \frac{0.01\, r_{eD}^2}{C_D} \quad \text{for } r_{eD}^2/C_D < 10^5 \text{ , and}$$

$$\approx \frac{0.005\, r_{eD}^2}{C_D} \quad \text{for } r_{eD}^2/C_D \geq 10^5 \quad . \qquad (15)$$

Equation (14) is the same as Eq. (11). The criterion for $\Delta t_D/C_D]_{begin}$ presented by *Mishra* and *Ramey* (1987) corresponds to the dimensionless time by which the slope of a dimensionless *MDH* buildup graph is approximately within 14% of 0.5. A comparison of Eqs. (12) and (15) shows that a semi-log straight line on a *MDH* buildup graph for a constant-pressure outer boundary is about one to one-and-a-half log cycles longer than a semi-log straight line on a *MDH* buildup graph for a closed reservoir, with all other conditions being same. Thus, if the buildup pressure derivative data for a well in a circular reservoir with a constant-pressure outer boundary is matched on a type-curve for a closed reservoir (Fig. 2 of *Mishra* and *Ramey*, 1987), the value for r_{eD}^2/C_D may be overestimated. Similarly, if the buildup pressure derivative data for a well in a closed reservoir is matched on a type-curve shown in Fig. 4, r_{eD}^2/C_D may be underestimated.

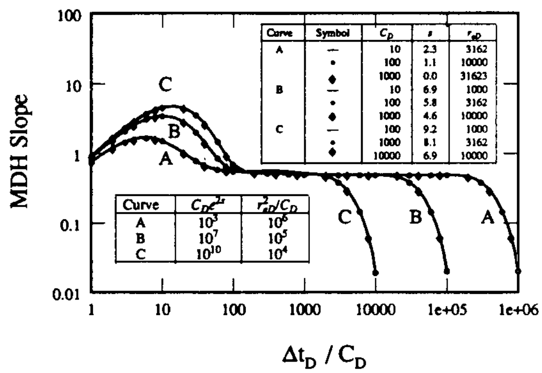

Fig. 3: Verification of C_De^{2s} and r_{eD}^2/C_D as the correlating parameters for the buildup responses (Constant-pressure outer boundary).

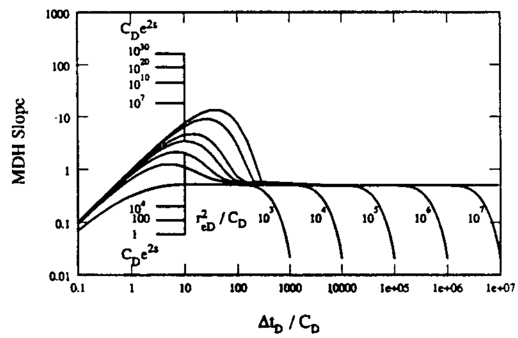

Fig. 4: Buildup pressure derivative type-curve (Constant-pressure outer boundary).

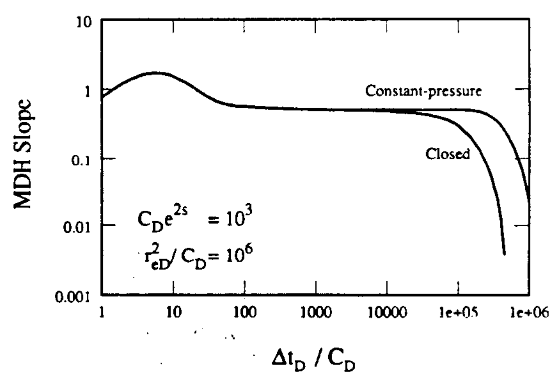

Fig. 5: Comparison of buildup derivative responses.

PRODUCING TIME EFFECTS ON BUILDUP RESPONSE

The *Horner* (1951) method has been recommended in the literature to analyze buildup data obtained after short producing times. The slope of a dimensionless *Horner* (1951) graph is:

$$Horner\ Slope = \frac{dp_{wDs}}{d \ln \left[\frac{t_{pD} + \Delta t_D}{\Delta t_D} \right]}$$

$$= - \frac{(t_{pD} + \Delta t_D)\Delta t_D}{t_{pD}} \cdot \frac{dp_{wDs}\,(\Delta t_D)}{d\,(\Delta t_D)} \quad (16)$$

Agarwal (1980) developed the concept of an equivalent drawdown time to analyze buildup data using drawdown type-curves for a well in an infinite reservoir. The dimensionless equivalent drawdown time is:

$$\Delta t_{eD} = \frac{t_{pD}\,\Delta t_D}{t_{pD} + \Delta t_D} \quad . \quad (17)$$

Agarwal (1980) showed that a graph of p_{wDs} vs. Δt_{eD} correlated buildup responses of a well in an infinite reservoir with the drawdown response. The correlation was good for all producing times larger than the time for storage effects to become negligible. For producing times less than the time for storage effects to become negligible, early time buildup responses did not correlate well. Also, the slope of a dimensionless *Agarwal* (1980) buildup graph is:

$$Agarwal\ Slope = \frac{dp_{wDs}}{d \ln (\Delta t_{eD})}$$

$$= \frac{(t_{pD} + \Delta t_D)\Delta t_D}{t_{pD}} \cdot \frac{dp_{wDs}\,(\Delta t_D)}{d\,(\Delta t_D)} \quad (18)$$

Equations (16) and (18) show that the *Horner* slope is equal, but opposite in sign to the *Agarwal* slope. Thus, producing time effects on buildup responses may be studied by using either the *Agarwal* or the *Horner* slope.

Aarstad (1987) presents the *Agarwal* (1980) slope as a function of the dimensionless shut-in time, Δt_{DA}, for several producing times, t_{pDA}, for wells without storage or skin, and located in a square or a rectangle. *Aarstad* (1987) shows that a graph of the *Agarwal* slope vs. Δt_{DA} does not result in a single curve for all producing times, if a well is located in a square or a rectangle. Therefore, *Aarstad* (1987) used t_{pDA} as a parameter to present the producing time effects on buildup responses for a well in a square or a rectangle.

Figure 6 shows an investigation of t_{pDA} as a correlating parameter for the buildup behavior of a well in the center of a closed, circular reservoir. Figure 6 applies for $C_D e^{2s} = 10^4$ and $r_{eD}^2/C_D = 10^6$. The values of C_D, s, t_{pD}, and r_{eD} used for various responses are shown on Fig. 6. Figure 6 shows that the early time responses for $t_{pDA} \leq 10^{-5}$ do not form a single curve with the responses for $t_{pDA} \geq 10^{-4}$. For $t_{pDA} \leq 10^{-5}$, the producing time is less than the time for storage effects to

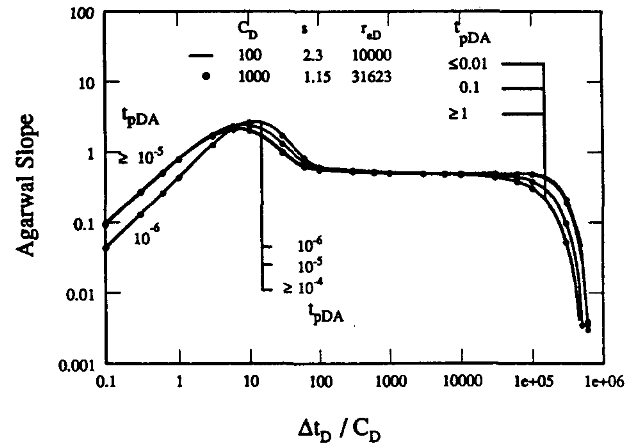

Fig. 6: Producing time effects on the buildup responses for a well in a closed reservoir ($C_D e^{2s} = 10^4$, and $r_{eD}^2/C_D = 10^6$).

become negligible. Thus, the lack of correlation at early times is consistent with *Agarwal's* (1980) finding. At late times, the buildup responses for all producing times do not form a single curve which is consistent with the work by *Aarstad* (1987). The lack of correlation at late times is due to the finite reservoir size.

For buildup derivative data analysis, a log-log graph of $d\,(p_{ws} - p_{wf})/d \ln (\Delta t_e)$ vs. Δt may be matched with a type-curve such as Fig. 2 of *Mishra* and *Ramey* (1987). But Fig. 6 shows that a type-curve matching without considering producing time effects may yield an overestimated r_{eD}^2/C_D for smaller producing times.

Figure 7 shows an investigation of t_{pDA} as a correlating parameter for the buildup behavior of a well in the center of a circular reservoir with a constant-pressure outer boundary. Figure 7 applies for $C_D e^{2s} = 10^4$ and $r_{eD}^2/C_D = 10^6$. The remarks for Fig. 6 also apply to Fig. 7. Thus, producing time effects may not be ignored in a type-curve matching analysis of buildup derivative data obtained from a well in a finite, circular reservoir.

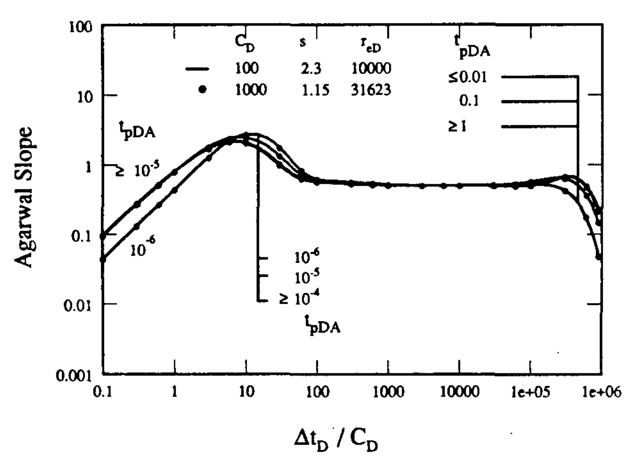

Fig. 7: Producing time effects on the buildup responses for a well in a reservoir with a constant-pressure outer boundary ($C_D e^{2s} = 10^4$, and $r_{eD}^2/C_D = 10^6$).

SUMMARY

New drawdown and buildup derivative type-curves for a well with storage and skin, and located in the center of a finite, circular reservoir have been presented. Design equations for the time to the beginning and the end of the semi-log straight line have been developed. The drawdown and the buildup responses for a well in a closed reservoir are compared with the responses for a well in a reservoir with a constant-pressure outer boundary. Producing time effects and outer boundary condition should be considered for a proper type-curve matching analysis of buildup derivative data obtained from a well in a finite, circular reservoir.

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NOMENCLATURE

A	Area
B	Formation volume factor
C	Wellbore storage coefficient
C_A	Shape factor
C_D	Dimensionless wellbore storage coefficient = $C/2\pi\phi c_t h r_w^2$
c_t	Total system compressibility
h	Thickness
I_n	Modified Bessel function of first kind of order n
K_n	Modified Bessel function of second kind of order n
k	Permeability
l	Laplace transform variable
p	Pressure
$\bar{p}_D$	Dimensionless pressure drop in the Laplace space
q	Well flow rate
r	Radius
r_{eD}	Dimensionless exterior radius = r_e/r_w
s	Skin = $2\pi kh(\Delta p_s)/qB\mu$
t	Time
t_D	Dimensionless time = $kt/\phi\mu c_t r_w^2$
t_{Dpss}	Dimensionless time to reach pseudosteady state = $kt_{pss}/\phi\mu c_t r_w^2$
t_{Dss}	Dimensionless time to reach steady state = $kt_{ss}/\phi\mu c_t r_w^2$
t_{pD}	Dimensionless producing time = $kt_p/\phi\mu c_t r_w^2$
t_{pDA}	Dimensionless producing time based on area = $kt_p/\phi\mu c_t A$
Δp_s	Pressure drop due to skin
Δt	Shut-in time
Δt_D	Dimensionless shut-in time = $k\,\Delta t/\phi\mu c_t r_w^2$
Δt_{DA}	Dimensionless shut-in time based on area = $k\,\Delta t/\phi\mu c_t A$
Δt_e	Equivalent drawdown time = $t_p\,\Delta t/(t_p + \Delta t)$
Δt_{eD}	Dimensionless equivalent drawdown time [Eq. (17)]

Greek symbols

ϕ	Porosity
μ	Viscosity

Subscripts

D	Dimensionless
e	Exterior, or equivalent
f	Flowing
i	Initial
p	Producing
pss	Pseudosteady state
s	Shut-in
ss	Steady state
t	Total
w	Wellbore

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