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Parallel Fractures Model for Tracer Flow Through Geothermal Reservoirs - Preliminary Results

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ABSTRACT

A parallel fractures model, having equal width and spacing, has been developed to study the flow of tracers through naturally fractured geothermal reservoirs. The model is capable of handling either a single fracture or a system of two or more parallel fractures, interacting with associated porous bodies. The reservoir is treated as being composed of two regions; a mobile region where diffusion and convection are allowed and a stagnant or immobile region where only diffusion and adsorption are allowed. Both regions are interconnected by means of a very thin fluid film contained within the immobile region which controls the fluid and mass transfer between both regions. The mobile region represents the system of fractures, where tracer is free to flow reaching high velocities, whereas non-homogeneities of the reservoir rock, such as microfractures and dead-end fractures are represented by means of an equivalent porous body where fluid remains immobile. The boundary-value problem for the system is stated and its solution into Laplace's space is presented. Numerical inversion of this solution was performed by means of the Stehfest algorithm. Preliminary results showing results obtained from the proposed model are included. Further work is underway to apply the model for interpretation of actual tracer flow field data.

INTRODUCTION

In the past few years, reinjection of separated brine back into the producing formation for pressure maintenance purposes has been a major issue for an adequate development of liquid-dominated geothermal resources. To date, some practical experience on reinjection of geothermal brine into the reservoir has been gained. This experience seems to indicate that rapid inter-well movement of injected fluid has occurred, causing an undesirable early thermal breakthrough (Horne, 1982). Therefore, careful selection of injection-production wells schemes are required in order to reduce the risks of having this early temperature drop in the production wells (Rivera et al., 1982). Tracer flow testing is an appropriate technique for obtaining a good idea on how injected fluids travel in the reservoir and a correct interpretation of the tracer return curves at production wells provides with parameters that can be used to study the heat and mass transfer processes taking place in the reservoir.

Since geothermal reservoirs are known to be highly-naturally fractured reservoirs, standard tracer interpretation techniques developed for flow through porous media in the hydrogeology and petroleum technical literature are not applicable. Most of the models published to date for interpretation of tracer return curves in geothermal reservoirs allow mostly qualitative estimation of reservoir parameters (Fossum, 1983, Tester et al., 1982, Jensen, 1983) and only few of them allow quantitative determination of basic reservoir parameters (Walkup and Horne, 1985).

In this study a parallel fractures model was developed to quantitatively determine basic geothermal reservoir parameters from tracer return curves. This model seems to be simpler than that proposed by Walkup and Horne since it only requires one Laplace's inversion and four fitting parameters; meanwhile the former requires two numerical inversions and has five parameters to adjust.

PARALLEL FRACTURES MODEL

The proposed model is shown in Figure 1. The fractured heterogeneous medium is represented by means of a system of equally spaced parallel fractures alternated with porous blocks. As shown in Fig. 1, this system is made up of two connected regions; a mobile region constituted by the fracture itself, where diffusion and convection processes are taking place and an immobile region where only diffusion and adsorption are allowed. These later processes go first through the very thin stagnant fluid layer of δ thickness, spreading then into the porous body. The idea of dividing the flow system into two interconnected regions has already been used by several authors (Deans, 1963; Walkup and Horne, 1985, Maloszewski and Zuber, 1985 among others).

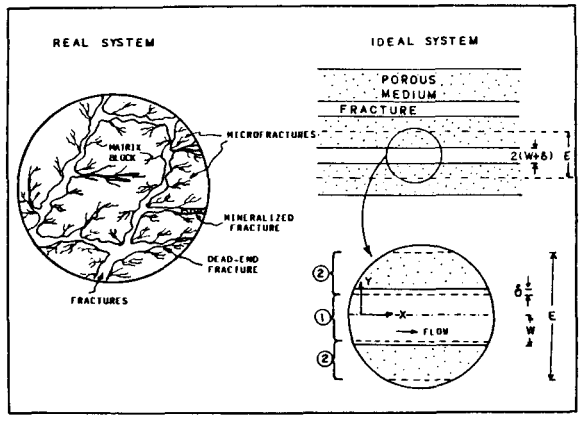

Figure 1: Proposed parallel fractures model.

Taking a mass balance on the mobile region and assuming the following: a) no production or reaction of the chemical species within the control volume; b) continuous injection of tracer into the fracture system; c) flow in the fractures is fast enough so that only steady convective flow in the x direction takes place; d) tracer distribution across the fracture width can be assumed constant due to efficient transverse diffusion and dispersion; e) constant density, the governing equation for the mobile region is as follows:

$$D_m \frac{\partial^2 C_m}{\partial x^2} - V_m \frac{\partial C_m}{\partial x} - \lambda (C_m - C_i) + \frac{\phi_e}{W-\delta} D_e \frac{\partial C_e}{\partial y}\bigg|_{W-\delta} - \frac{\partial C_m}{\partial t} = 0 \quad (1)$$

Taking a mass balance on the immobile region and assuming that only diffusion in the y– direction is important and that reversible adsorption with a linear adsorption isotherm is taking place, the governing equation for this region is given by

$$\frac{D_e}{1 + \frac{\rho k(1-\phi_e)}{\phi_e}} \frac{\partial^2 C_e}{\partial y^2} - \lambda (C_e - C_i) - \frac{\partial C_e}{\partial t} = 0 \quad (2)$$

In order to complete the problem, the following initial and boundary conditions are defined:

Initial conditions:

$$C_m (x,y,o) = C_i \quad (3)$$

$$C_e (x,y,o) = C_i \quad (4)$$

Boundary conditions:

$$C_m (o,t) = C_o \quad (5)$$

$$C_e (x,W-\delta,t) = C_m (x,t) \quad (6)$$

$$C_m (\infty,t) = C_i \quad (7)$$

$$\frac{\partial C_e}{\partial t}\bigg|_{(x,\frac{E}{2},t)} = 0 \quad (8)$$

Equations (1) through (8) can be simplified by means of the following dimensionless variables:

$$C_{D1} = \frac{C_m - C_i}{C_o - C_i} \quad (9)$$

$$C_{D2} = \frac{C_e - C_i}{C_o - C_i} \quad (10)$$

$$x_D = \frac{x}{W} \quad (11)$$

$$y_D = \frac{y}{W} \quad (12)$$

$$t_D = \frac{V_m}{W} t \quad (13)$$

Equations (1) and (2) can then be written as follows:

$$\frac{1}{P_{e1}} \frac{\partial^2 C_{D1}}{\partial x_D^2} - \frac{\partial C_{D1}}{\partial x_D} - \frac{W}{V_m} \lambda C_{D1} + \frac{\varepsilon}{P_{e2}} \frac{\partial C_{D2}}{\partial y_D}\bigg|_{\frac{W-\delta}{W}} - \frac{\partial C_{D1}}{\partial t_D} = 0 \quad (14)$$

$$\frac{\gamma}{P_{e2}} \frac{\partial^2 C_{D2}}{\partial y_D^2} - \frac{W}{V_m} \lambda C_{D2} - \frac{\partial C_{D2}}{\partial t_D} = 0 \quad (15)$$

where:

$$P_{e1} = \frac{V_m W}{D_m} \quad (16)$$

$$P_{e2} = \frac{V_m W}{D_e} \quad (17)$$

$$\varepsilon = \frac{W \phi_e}{W - \delta} \quad (18)$$

$$\gamma = \frac{1}{1 + \frac{\rho k(1-\phi_e)}{\phi_e}} \quad (19)$$

Equations (14) and (15) can be solved by means of the Laplace's transform. Solution for concentration distribution in the mobile and immobile regions in Laplace's space are as follows:

$$\overline{C}_{D1} = \frac{1}{s} \exp\left\{\left[\frac{P_{e1}}{2} - \left[\frac{P_{e1}^2}{4} + \left\{s + \frac{W\lambda}{V_m} + \frac{\varepsilon\sqrt{\beta}}{P_{e2}} \tanh\left[-\frac{\sqrt{\beta}}{2W}(E-2(W-\delta))\right]\right\} P_{e1}\right]^{1/2}\right] x_D\right\} \quad (20)$$

$$\overline{C}_{D2} = \left\{\frac{\exp\left[-\sqrt{\beta}\left(\frac{E}{W} - y_D\right)\right] + \exp(-\sqrt{\beta}\, y_D)}{\exp\left[-\frac{\sqrt{\beta}}{W}(E-W+\delta)\right] + \exp\left[-\frac{\sqrt{\beta}}{W}(W-\delta)\right]}\right\} \overline{C}_{D1} \quad (21)$$

where:

$$\beta = \frac{P_{e2}\left(s + \frac{\lambda W}{V_m}\right)}{\gamma} \quad (22)$$

Numerical inversion by means of the Stehfest algorithm is used in order to compute concentration distribution in real space from equations (20) and (21).

As it was pointed out by Walkup and Horne, the solution of an spike-input is the time derivative of a step input. Therefore, to obtain the solution for a spike-input all that is needed is to multiply the expression for the step

input in (x,y,s) space (eq. (20)) by s before it is inverted to (x,y,t) space. Thus:

$$(C_{D1})_{spike} = \frac{\partial C_{D1}}{\partial t} = L^{-1}\left\{ s\,\bar{C}_{D1} \right\} \qquad (23)$$

For a detailed derivation of equations (20) and (21) the reader should refer to the report by Ramirez (1987).

Preliminary Results

Further work to extend the model's applicability to include field data is still in progress. Thus far only preliminary results had been obtained from the model. Table 1 includes data used to obtain results shown on Figures 2 through 4. Figure 2 illustrates concentration profiles for the mobile region with dimensionless time going from 0.1 to 1000 for a velocity of 0.01 (m/day). Figures 3 and 4 show the effect of velocity on concentration profiles in the mobile region for two different times, 0.12 and 1.2 hours.

Table 1

Data used to test the model. (Some data were taken from Example 1, Hugakorn et al., 1983).

V_m = 0.01 (m/day)
λ = 0.000154 $(day)^{-1}$
W = 5×10^{-5} (m)
δ = 10^{-7} (m)
γ = 1
ε = 1.18775
D_e = 1.382×10^{-7} (m^2/day)
D_m = 1.975×10^{-4} (m^2/day)

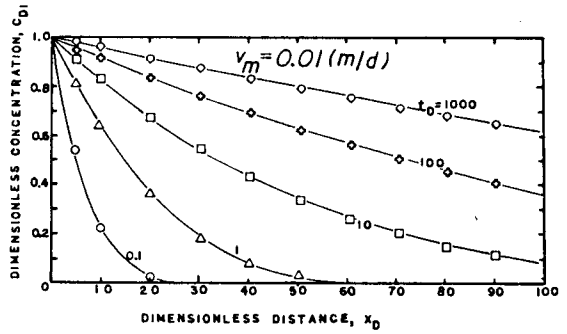

Figure 2: Dimensionless concentration profiles at t_D = 0.1, 10, 100, 1000. V_m = 0.01 (m/day).

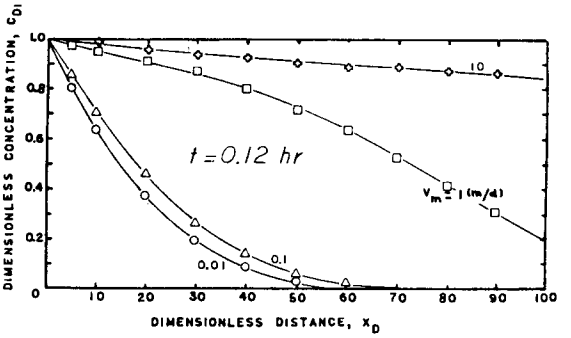

Figure 3: Effect on velocity on dimensionless concentration profile. V_m = 0.01, 0.1, 1, 10 (m/day). t = 0.12 hr.

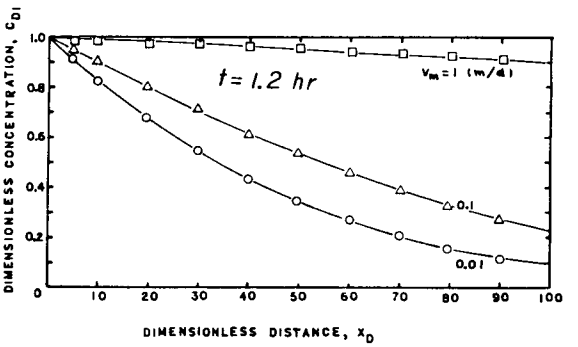

Figure 4: Effect on velocity on dimensionless concentration profile. V_m = 0.01, 0.1, 1, (m/day). t = 0.12 hr.

NOMENCLATURE

C = Concentration, (M/L^3)
D = Diffusion coefficient, (L^2/t)
E = Fracture spacing, (L)
k = Adsorption constant, (L^3/M)
L^{-1} = Inverse Laplace's operator
P_e = Peclet number, (dimensionless)
s = Laplace operator
t = Time, (t)
V = Velocity, (L/t)
W = Fracture half-width, (L)
x = Distance in x-direction, (L)
y = Distance in y-direction, (L)

Greek Symbols

β = Dimensionless group defined by eq. (22)
γ = Dimensionless group defined by eq. (19)
δ = Stagnant fluid film thickness, (L)
ε = Dimensionless group defined by eq. (18)

ϕ = Porosity, referred to total-bulk volume, (dimensionless)

λ = Radioactive decay constant, (t^{-1})

ρ = Density, (M/L^3)

Subscripts

D = Dimensionless variable (distance, time or concentration)

e = Refers to the immobile (stagnant) region

i = Refers to initial conditions

m = Refers to the mobile (fractured) region

o = Refers to inlet conditions

1 = Refers to mobile region

2 = Refers to immobile region

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