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RADIAL DISPERSION IN A DOUBLE-POROSITY SYSTEM WITH FRACTURE SKIN

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ABSTRACT

The problem of dispersion, advection and adsorption of a tracer in a double-porosity reservoir due to tracer injection in a well with a steady, radially divergent flow field was solved for the case of constant tracer concentration in the injection well. Longitudinal dispersion and advection was assumed to dominate transport in the fracture system and tracer diffusion and adsorption was assumed to dominate movement of the tracer in the matrix blocks. The blocks were assumed to be sphere shaped and covered with a thin skin of material that provides resistance to the diffusion of tracer into the blocks. Values of dimensionless concentration in the fracture system versus dimensionless time were computed by numerical inversion of the Laplace transform solution to the Airy equation. Type curves demonstrate effects of changing reservior characteristics and show the usefulness of the concept of fracture skin in understanding dispersive processes in fractured porous media.

INTRODUCTION

There has been a recent surge of interest in analytical solutions to problems of radial dispersion in porous media. Such analytical solutions can be used in tracer injection tests to evaluate dispersive and adsorptive properties of groundwater and geothermal aquifers, and can be used to verify the accuracy of numerical, solute-transport codes. Hsieh (1986) pointed out that the radial dispersion problem is of particular interest because it is perhaps the simplest case involving a spatially varying velocity field.

Unfortunately, because the coefficient of longitudinal hydrodynamic dispersion is linearly related to velocity, solutions to the one-dimensional, advection dispersion problem in radial coordinates are difficult to obtain. Ogata (1958) appears to have been the first to obtain a closed-form analytical solution to the radial dispersion problem. His solution is based upon the assumption that a steady, radially divergent flow field has been established around the injection well prior to the establishment of a step change in tracer concentration in the well bore.

Because of the form of the Ogata solution, it is difficult to evaluate and alternative forms have appeared in the literature (Tang and Babu, 1979; Hsieh, 1986).

In a fractured porous medium it is believed that a tracer may become dispersed not only by hydrodynamic dispersion but also by diffusion into the porous matrix. Feenstra _et al._ (1984) proposed a radial flow model for a single, horizontal fracture that accounts for matrix diffusion. Their model is simplified considerably by neglecting effects of longitudinal dispersion in the fracture. Chen (1985, 1986) proposed radial flow models that account for both matrix diffusion and longitudinal dispersion. Both Chen and Feenstra _et al._ assume that the aquifer or fracture is bounded by porous blocks of infinite thickness.

In this paper a new dimension of complexity, and therefore versatility, is added to the radial dispersion problems. The aquifer or geothermal reservoir is assumed to be composed of highly fractured, porous rock that might be characterized as a double-porosity system (Barenblatt, 1960). Longitudinal dispersion and advection is assumed to dominate tracer transport in the fractures and diffusion and adsorption is assumed to dominate tracer movement in the matrix blocks. Blocks are assumed to be sphere shaped for mathematical simplicity and coated with a thin skin of material that may provide resistance to the diffusion of tracer into the blocks. This skin may be the result of the deposition of minerals or the alteration of minerals due to the natural circulation of geothermal fluids. For a literature review of flow to a well in a double-porosity system and for a description of fracture skin as it relates to the flow problem see Moench (1983, 1984). As regards to diffusion in sphere-shaped blocks with skin, the proposed model is similar to the model of Rasmusen and Neretnieks (1980). It differs in that the more complicated case of radial flow in the fracture system is considered instead of one-dimensional, planar flow. Also, Rasmusen and Neretnieks did not present any computational results showing the effects of the diffusion barrier or skin.

MATHEMATICAL MODEL

The model is developed under the following general assumptions: (1) As depicted in Figure (1) a vertically oriented injection well of finite diameter fully penetrates a horizontal, confined, double-porosity aquifer of constant thickness and of infinite radial extent. (2) A steady state flow field, which is radially divergent and axially symmetric with respect to the injection well, is present in the fracture system as a result of the constant-rate injection of tracer-free fluid. (3) Advection in the blocks is negligible. (4) At the start of a test a step change in concentration occurs in the injection well. (5) Tracer is transported in the fracture system by radial advection and longitudinal mechanical dispersion: transverse mechanical dispersion and molecular diffusion in the fractures are negligible. Mechanical dispersion is assumed to be linearly related to velocity and is therefore a function of radial position. (6) Tracer diffuses in the sphere-shaped blocks (see Figure 2) in accordance with Fick's law. (7) As depicted in Figure 2, the blocks are coated with a thin layer (skin) of material that impedes the diffusion of tracer at the block-fracture interface and does not allow for the storage of tracer. (8) Tracer is attenuated in the porous blocks by adsorption, which is described by an equilibrium, adsorption isotherm. (9) Adsorption on the fracture surfaces is negligible.

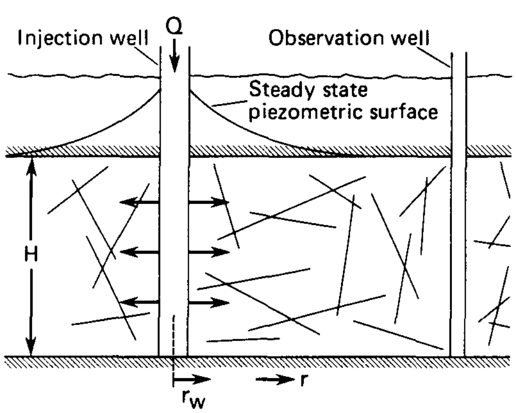

Figure 1. Schematic diagram of a double-porosity aquifer of thickness H with a steady state flow field established around an injection well.

The advection-dispersion equation for plane radial flow in a porous medium is given by Bear (1979, p. 247). For a double-porosity system it may be written as,

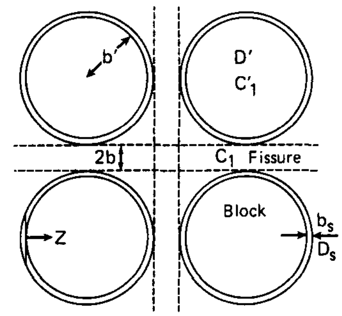

Figure 2. Geometrical configuration for sphere-shaped blocks with fracture skin.

$$\frac{\partial}{r\,\partial r}\left[rD_L \frac{\partial C_1}{\partial r} \right] - \frac{\partial}{r\,\partial r}[rVC_1] - q = \frac{\partial C_1}{\partial t} \qquad r > r_w \qquad (1)$$

where $q = -\frac{3D'}{b'}\left[\frac{\partial C_1'}{\partial z}\right]_{z=0}$

for sphere-shaped blocks (see Moench, 1984, equations 29-31). The symbols are defined in the Nomenclature.

The first two terms on the left hand side of equation (1) represent, in order, the transport of solute by dispersion and advection. The third term represents the exchange of solute at the block-fracture interface by matrix diffusion. The term on the right hand side of equation (1) represents the accumulation of tracer in the fracture system.

The coefficient of longitudinal hydrodynamic dispersion is assumed to take the form

$$D_L = \alpha_L V \qquad (2)$$

where molecular diffusion has been neglected. The longitudinal dispersivity, α_L, is assumed to be a characteristic property of the fracture system. The velocity, V, is described mathematically as

$$V = A/r \qquad (3)$$

where $A = 3Q/2\pi\phi_f H$.

In order to solve equation (1) the following boundary conditions and initial conditions are used:

$$C_1(r_w,t) = C_0 \qquad (4)$$

$$C_1(r\to\infty,t) \to 0 \qquad (5)$$

$$C_1(r,0) = 0 \qquad (6)$$

The diffusion equation for the sphere-shaped blocks, allowing for linear equilibrium sorption, is

$$D'\frac{\partial^2(zC_1')}{\partial z^2} = \phi'R'\frac{\partial(zC_1')}{\partial t} \qquad 0 \le z \le b' \qquad (7)$$

The boundary conditions used to solve equation (7) are,

$$\frac{\partial C'_1(b',t)}{\partial z} = 0 \qquad (8)$$

$$D_s\frac{[(C'_1)_{z=0} - C_1]}{b_s} = D'(\frac{\partial C'_1}{\partial z})_{z=0} \qquad (9)$$

Equation (9) represents continuity of diffusive flux across the skin and derives from heat flow theory (see Carslaw and Jaeger, 1959, p. 20). It is assumed that the skin is negligibly thin and does not accomodate the accumulation of mass.

Using the dimensionless parameters defined in the Nomenclature, the controlling equations and boundary conditions are rewritten in dimensionless form. The coupled, dimensionless boundary-value problems become, for the fracture system,

$$\frac{\partial C^2}{\rho\,\partial\rho^2} - \frac{\partial C}{\rho\,\partial\rho} - q_D = \frac{\partial C}{\partial t_D} \qquad \rho > \rho_0 \qquad (10)$$

where $q_D = -3\gamma(\frac{\partial C'}{\partial z_D})_{z_D=0}$

$$C(\rho_0,t_D) = 1 \qquad (11)$$

$$C(\rho\to\infty,t_D) \to 0 \qquad (12)$$

$$C(\rho,0) = 0 \qquad (13)$$

and, for the block system,

$$\frac{\partial^2(z_D C')}{\partial z_D^2} = \frac{\sigma}{\gamma}\frac{\partial(z_D C')}{\partial t_D} \qquad 0 \le z_D \le 1 \qquad (14)$$

$$C'(0,t_D) - C = S_F(\frac{\partial C'}{\partial z_D})_{z_D=0} \qquad (15)$$

$$\frac{\partial C'(1,t_D)}{\partial z_D} = 0 \qquad (16)$$

$$C'(z_D,0) = 0 \qquad (17)$$

Equations (10)-(17) are solved by the method of Laplace transformation.

LAPLACE TRANSFORM SOLUTIONS

The Laplace transform solutions are, for the fracture system,

$$\bar{C} = \frac{1}{p}\exp(\frac{\rho - \rho_0}{2})\frac{Ai(\beta^{1/3}y)}{Ai(\beta^{1/3}y_0)} \qquad (18)$$

where $y = \rho + (4\beta)^{-1}$

$y_0 = \rho_0 + (4\beta)^{-1}$

$\beta = p + \bar{q}_D$

$$\bar{q}_D = \frac{3\gamma[m\coth(m) - 1]}{1 + S_F[m\coth(m) - 1]}$$

$$m = (\sigma p/\gamma)^{1/2}$$

and, for the block system,

$$\bar{C}' = \bar{C}\,\frac{\sinh[m(1-z_D)]\,\operatorname{csch}(m)}{(1-z_D)\{1 + S_F[m\coth(m) - 1]\}} \qquad (19)$$

The Laplace transform variable, p, is inversely related to the dimensionless time, t_D. The bar over C, C' and q_D designates their Laplace transforms.

BREAKTHROUGH CURVES

The Laplace transform solutions given by equations (18) and (19) are easily inverted with the Stehfest (1970) algorithm to produce dimensionless breakthrough curves. Figure 3 shows dimensionless breakthrough curves, due to tracer injection, for the indicated values of the parameters, comparing the case of zero matrix diffusion with finite matrix

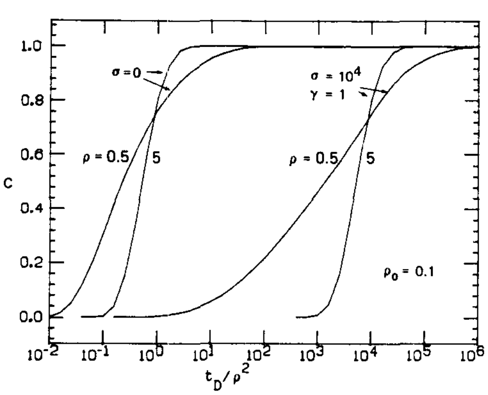

Figure 3. Dimensionless concentration breakthrough curves for the case of zero matrix diffusion (σ=0) compared with a case of finite matrix diffusion ($\sigma=10^4$, γ=1).

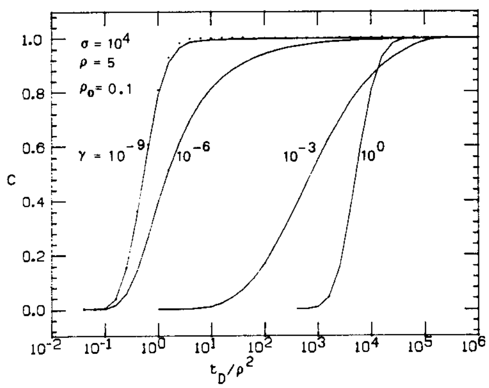

Figure 4. Dimensionless concentration breakthrough curves for the case of zero matrix diffusion (plotted points) compared with finite matrix diffusion for various values of γ and fixed σ (solid lines).

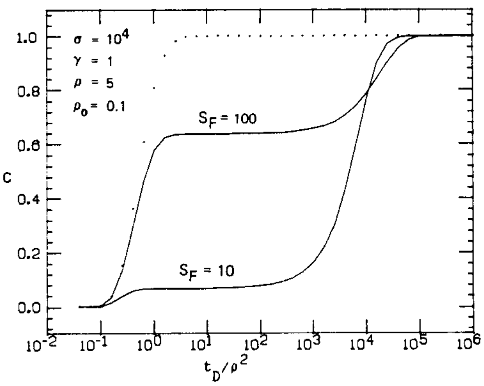

Figure 5. Dimensionless concentration breakthrough curves for the case of zero matrix diffusion (plotted points) compared with finite matrix diffusion with fracture skin for ρ=5.0 and ρ_0=0.1 (solid lines).

diffusion. As expected the effect of matrix diffusion is such as to delay the arrival of the breakthrough curve at a given radial distance from the injection well. The amount of delay is directly related to the magnitude of the parameter σ, which is dependent upon the retardation factor and the porosity of the block system and the porosity of the fracture system (see Nomenclature). Also the greater the dispersivity of the aquifer system, α_L, the greater is the spreading of the tracer.

Figure 4 shows tracer breakthrough curves for various values of γ given fixed values of ρ, ρ_0 and σ. The parameter γ is proportional to the diffusion coefficient for the block system (see Nomenclature). This shows that as γ decreases, due to reduced diffusion coefficient, effects of matrix diffusion diminish and the response approaches that expected for no matrix diffusion. It is of interest to note the steepening of the curve for $\gamma=10^0$ in Figure 4. This curve corresponds to the case where, because of a large diffusion coefficient, the tracer is taken up by the blocks almost instantaneously causing a long delay in the appearance of the breakthrough curve. It is as though there is enhanced storage of tracer in the fracture system. The response for $\gamma=10^0$ is about the same as that for $\gamma=10^{-9}$ except that it is shifted to the right by a factor of $1+\sigma$.

Figure 5 shows effects of fracture skin upon tracer breakthrough curves. A concentration plateau separates the breakthrough curve for zero matrix diffusion from the breakthrough curve for finite matrix diffusion. For S_F=100 the concentration buildup faithfully follows the case of zero matrix diffusion at early time. For S_F=10 the concentration buildup follows the case of finite matrix diffusion at late time. Similar responses are shown in Figure 6 using a larger value of α_L.

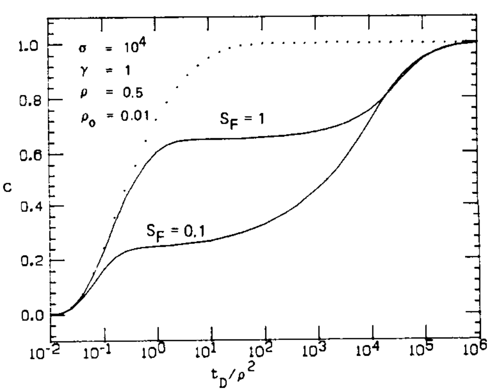

Figure 6. Dimensionless concentration breakthrough curves for the case of zero matrix diffusion (plotted points) compared with finite matrix diffusion with fracture skin for ρ=0.5 and ρ_0=0.01 (solid lines).

CONCLUSIONS

The figures showing hypothetical dimensionless breakthrough curves in observation wells illustrate tracer spreading due to dispersion in the fracture system and matrix diffusion in sphere-shaped blocks. The barrier to diffusion (or fracture skin) located on block surfaces causes a concentration "plateau" to occur in the breakthrough curves separating the response for zero matrix diffusion from that for finite matrix diffusion. The magnitude of the separation depends upon the retardation factor for the blocks and the block and fracture porosity. Because this model includes the effects of a radially varying velocity field in the fractures, the diffusion of tracer in the matrix blocks, and the barrier to diffusion at the fracture block interfaces, it should be useful in helping to validate large numerical models for chemical transport problems.

NOMENCLATURE

- A — [= $3Q/2\pi\phi_f H$] advection parameter, L^2/T.
- Ai(x) — Airy function.
- b — half thickness of a representative fracture, L.
- b' — radius of a representative, sphere-shaped block, L.
- b_s — average thickness of fracture skin, L.
- C_0 — input concentration of the tracer, M/L^3.
- C_1 — concentration of tracer in the fracture system, M/L^3.
- C'_1 — concentration of tracer in block system, M/L^3.
- C — [= C_1/C_0] dimensionless concentration in fracture system.
- C' — [= C'_1/C_0] dimensionless concentration in block system.
- D_L — longitudinal hydrodynamic dispersion at a point in the fracture system, L^2/T.
- D' — diffusion coefficient for block system, L^2/T.
- D_s — diffusion coefficient for fracture skin, L^2/T.
- H — aquifer thickness, L
- K_d — distribution coefficient for porous blocks, L^3/M.
- p — Laplace transform variable.
- Q — rate of fluid injection in the well, L^3/T.
- q — source term for tracer diffusion at block-fracture interface, M/L^3T.
- q_D — dimensionless form of q.
- r — radial distance from center line of injection well, L.
- r_w — radius of injection well, L.
- R' — [= $1 + \rho_b K_d/\phi'$] retardation factor for blocks.
- S_F — [= $D'b_s/D_s b'$] dimensionless skin factor for diffusion.
- t — time, T.
- t_D — [= At/α_L^2] dimensionless time.
- V — average radial velocity at a point in the fracture system, L/T.
- z — radial distance in sphere-shaped blocks, directed inward from skin-block interface, L.
- z_D — [= z/b'] dimensionless radial distance in sphere-shaped blocks.
- α_L — longitudinal dispersivity for the fracture system, L.
- ρ — [= r/α_L] dimensionless radial distance in fracture system.
- ρ_0 — [= r_w/α_L] dimensionless dispersivity.
- ρ_b — bulk density of porous blocks, M/L^3.
- ϕ' — porosity of blocks.
- ϕ_f — [= $3b/b'$] fracture porosity.
- σ — [= $3\phi'R'/\phi_f$] dimensionless grouping for porosity and tracer sorption.
- γ — [= $\alpha_L^2 D'/Abb'$] dimensionless grouping for dispersion and diffusion.

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