

Analysis of Injection-Backflow Tracer Tests in Fractured Geothermal Reservoirs

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ABSTRACT

Tracer tests have been an important technique for determining the flow and reservoir characteristics in various rock matrix systems. While the interwell tracer tests are aimed at the characterization of the regions between the wells, single-well injection-backflow tracer tests may be useful tools of preliminary evaluation, before implementing long term interwell tracer tests.

This work is concerned with the quantitative evaluation of the tracer return profiles obtained from single well injection-backflow tracer tests. First, two mathematical models of tracer transport through fractures, have been reviewed. These two model are based on two different principles: Taylor Dispersion along the fracture and simultaneous diffusion in and out of the adjacent matrix. Then the governing equations for the transport during the injection-backflow tests have been solved. Finally the results were applied to field data obtained from Raft River and East Mesa geothermal fields. In order to determine the values of the parameters of the models that define the transport mechanisms through fractures a non-linear optimization technique was employed.

INTRODUCTION

Reinjection of waste hot water has been commonly practiced in many geothermal reservoirs either as a means of disposal or as a way to maintain the reservoir pressure and liquid volume. In some cases, however, it has been observed that the process had detrimental effects such as early breakthrough of the injected fluids and reduction in the enthalpy (Horne 1982).

Since both beneficial and detrimental effects are possible, for the design of a successful reinjection program, the mechanisms of the fluid flow in the reservoir have to be understood. Tracer tests have been one of the important tools of studying the flow characteristics of various rock matrix systems. The quantitative interpretation of the test results can be accomplished through studying the mixing curves by using mathematical models describing the transport mechanism in the reservoir.

As far as the flow of tracer through porous media is concerned, a fairly large number of research results have been published (Ogata and Banks, 1961; Deans, 1963; Perkins and Johnston, 1963; Coats and Smith, 1964; Lenda and Zuber, 1970; Brigham, 1974; van Genuchten and Wieranga, 1976; Ivanovich and Smith, 1978; Antunez and Brigham, 1984). The models developed for the porous

media flow, however, are not necessarily applicable to geothermal reservoirs which are usually highly fractured in nature. Recent studies on flow through fractured media led to the development of new models describing the physics of the tracer transport through fractures.

Grisak and Pickens (1980) formulated a double porosity model combining a convective-dispersive transport in the fractures and a diffusive transport in the unfractured matrix. A finite element method was developed for simulating non-reactive and reactive solute transport by convection, mechanical dispersion and diffusion in a unidirectional flow field.

In 1981, Rodriguez and Horne presented a theoretical study of the one-dimensional convective-dispersive flow through fractures. In that work, they derived an expression for the dispersivity in flow through a fracture. Fossum and Horne (1982) applied this model to interwell tracer test data obtained from Wairakei, New Zealand with some success. They used a non-linear optimization technique to match the model to field data.

In 1982, from the studies of migration of radionuclides in bedrock surrounding nuclear waste repositories, Neretnieks, Eriksen and Tahtinen developed a matrix diffusion model describing the tracer movement in a single fissure in granitic rock. Using this matrix diffusion model Jensen and Horne (1983) were able to obtain a better match to the data obtained from Wairakei, New Zealand as compared to the dispersion model used by Fossum and Horne (1982).

So far, interwell tracer tests have been a useful technique in determination of the interconnections between the injectors and the producers. The single well (injection-backflow) tests, on the other hand, have been proposed as tools for characterizing the flow field within the radius of influence around the injectors. Even though the injection-backflow tests are proposed as preliminary evaluation tools before the employment of long term interwell tests, the amount of information that can be recovered from these tests is potentially as much as that can be obtained from interwell tracer tests.

In 1982, Downs and his coworkers presented a preliminary study of the injection-backflow tests conducted at Raft River Geothermal Field. Later in 1983, Capuano et al. presented the qualitative analysis of the tests conducted at both Raft River and East Mesa fields. It was concluded that the injection-backflow tracer tests can be successfully used to characterize the flow in the near well-bore environment.

In this work, a theoretical study of the return profiles from injection-backflow tracer tests is presented. Both convection-dispersion and matrix diffusion models are employed in the analysis of the return profiles of both continuous injection and spike injection cases. In addition, the theoretical results of the continuous injection case are applied to the field data from Raft River and East Mesa geothermal fields by using a non-linear least squares optimization technique, in order to determine the effective parameters of the tracer transport.

MATHEMATICAL DEVELOPMENTS

The injection-backflow tests can be divided into three stages (*Antunez and Brigham, 1984*): a) Injection period, b) Shut-in period, c) Backflow period. However, if the test is not aimed at the determination of regional flow beyond the test well's radius of influence, it may be completed in only two stages, injection and immediate backflowing.

Since the transport of tracers through geothermal reservoirs is primarily through fractures, the success of interpretive analysis of the return curves depends on the understanding the physics of the mixing process during the flow. In this work, two mathematical models based on two principal mechanisms, dispersion in fracture and the diffusion into the matrix, were employed to analyze the tracer return profiles from injection and immediate backflowing tests.

The injection-backflow tests can be conducted by either injecting a tracer fluid of concentration C_o continuously during the injection period (continuous injection case) or injecting a tracer slug followed by the untraced fluid (slug or spike injection case). Here, both continuous injection and spike injection cases for both convection-dispersion and matrix diffusion models will be considered.

A- CONVECTION DISPERSION MODEL

In a fracture, under either laminar or turbulent flow the fluid will be transported faster in the center of the fracture than near the walls. The result of this non-uniform "convective" transport is the dispersion of the tracer over the region of the transport. Although this convective smearing of the tracer gives rise to large concentration gradients across the narrow width of the fracture, molecular diffusion tends to rapidly equalize the tracer concentration across the fracture, thus counteracting the effect of convective dispersion (*Horne and Rodriguez, 1983*).

The combination of the transverse diffusion and convective dispersion in the flow channel is known as "Taylor Dispersion" and was derived by *Taylor (1953)* for pipe flow. The net result of the Taylor Dispersion is that the tracer front propagates with the mean speed of the flow. The expression for the net longitudinal dispersivity for the flow in a fracture is given by *Horne and Rodriguez (1983)*.

A.1- Continuous Injection Case

Taylor (1953) presents the equation governing the effective longitudinal dispersion in an infinite medium.

$$\eta \frac{\partial^2 C}{\partial z^2} = \frac{\partial C}{\partial t} \quad (1)$$

The boundary and initial conditions are:

$$\begin{aligned} C &= 0 & \text{at} & \quad t = 0 \\ C &\rightarrow C_o & \text{as} & \quad z \rightarrow -\infty \\ C &\rightarrow 0 & \text{as} & \quad z \rightarrow \infty \end{aligned}$$

The solution to Eq.1 with the above boundary and initial conditions is given by *Taylor (1953)*:

$$C_D = \frac{C}{C_o} = \frac{1}{2} \operatorname{erfc} \left[\frac{x - ut}{2\sqrt{\eta t}} \right] \quad (2)$$

Eq.2 represents the concentration profile during the injection period and it is symmetric about $x = ut$ which is the average distance traveled by the front. Also the point $x = ut$ corresponds to position of the 50 percent concentration contour. If L_t is the zone of transition in which C_D changes from 0.9 C_D to 0.1 C_D , the expression for L_t is given by *Taylor (1953)*:

$$L_t = 3.62 \sqrt{\eta t} \quad (3)$$

Taylor also mentioned that as t increases L_t increase proportionally to \sqrt{t} whereas the distance traveled by the particles of fluid are proportional to t . Eventually as t increases L_t become small compared with $L = ut$ which is the distance traveled by the moving plane traveling with the mean speed of flow, u . Therefore there is a minimum injection time requirement for the theory to be applicable. To obtain the concentration profile at the end of the injection period, the variable t in Eq. 2 is replaced by t_f .

The analysis of the injection-backflow tests is not a simple one-dimensional problem, because of the change in flow direction during the backflow period. There is also a possibility of change in the average flow velocity during the backflow period. For these reasons, to obtain the backflow period profile the governing equations have to be solved with appropriate initial and boundary conditions. However, for this specific problem, we will apply a simple technique developed by *Antunez and Brigham (1983)*, to obtain the solutions.

In this case, first with the assumption of equal average flow velocities during the injection and backflow periods, the problem is simplified. The concentration profile at the end of the injection period is given by the middle curve in Fig. 1.

The 50 percent concentration point is at a distance of $x = L$ to the injection point $x = 0$, and the profile is symmetric about this point. At this point, first of all, we have to remember that the front propagates with the mean speed of the flow. To obtain the backflow period solution we utilize both the equal injection and backflow average velocities and the symmetry of the profile. During the backflow period we imagine a pseudo-front going away from the injection point as if the injection period is continuing, while the real front approaches the well. Since the injection and backflow velocities are imagined to be equal for both real and pseudo fronts, the distance traveled by them will also be equal as it is seen in Fig. 1.

Then the concentration of the pseudo-profile at any distance x is given by:

$$\frac{C_p}{C_o} = \frac{1}{2} \operatorname{erfc} \left[\frac{x - u(t_f + t_p)}{2\sqrt{\eta(t_f + t_p)}} \right] \quad (4)$$

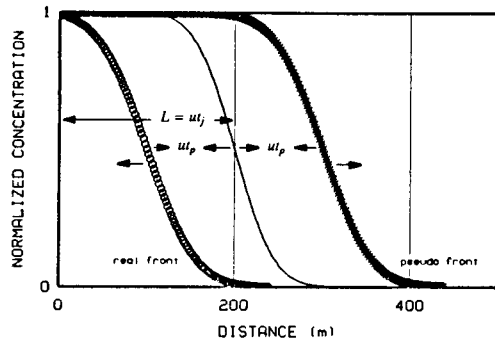


Figure 1: Dispersion of a sharp interface caused by Taylor dispersion.

If the concentrations of the pseudo-profile are to be calculated at $x = 2L = 2ut_j$, then when the 50 percent concentration of the pseudo-front reaches to $x = 2L$, the same concentration of the real front will reach to the well which is taken as the measurement point. Therefore, the concentrations measured at the well can be calculated by using the pseudo-front concentrations evaluated at $x = 2L$. The pseudo concentration C_p is related to the actual concentration C_r by:

$$C_D = \frac{C_r}{C_o} = 1 - \frac{C_p}{C_o} = \frac{1}{2} + \frac{1}{2} \operatorname{erf} \left[\frac{\alpha(t_j - t_p)}{2\sqrt{t_j + t_p}} \right] \quad (5)$$

where

$$\alpha = \frac{u}{\sqrt{\eta}}$$

A.2- Spike Injection Case

Using the assumption of equal injection and backflow velocities and the symmetry of the injection period profile, we can find the spike injection case solution as:

$$C_D = \frac{C}{C_o} = \frac{\alpha t_i}{2\sqrt{\pi(t_j + t_p)}} e^{-\frac{\alpha^2(t_j - t_p)^2}{4(t_j + t_p)}} \quad (6)$$

B- MATRIX DIFFUSION MODEL

When a tracer fluid flows in a fracture, the tracer will diffuse into the porous matrix adjacent to the fracture. Assuming the porosity ϕ , and the apparent diffusion coefficient D_a are constant throughout the matrix contacted by the fluid the one-dimensional form of the equation of the diffusion into the porous matrix is given by:

$$D_a \frac{\partial^2 C_p}{\partial y^2} = \frac{\partial C_p}{\partial t} \quad (7)$$

When the source of the tracer fluid is discontinued the effect will be to flush the fracture and reverse the concentration gradient causing tracer to migrate from the matrix into the fracture.

Assuming the concentration profile across the fracture is evened out due to molecular diffusion, the flow and sorption from the water in fracture is represented by:

$$\frac{\partial C_f}{\partial t} + u \frac{\partial C_f}{\partial x} = 2 \frac{D_a}{\delta} \frac{\partial C_p}{\partial y} \Big|_{y=0} \quad (8)$$

The two diffusion coefficients D_a and D_s in Eq. 7 and Eq. 8 are related by:

$$D_s = \phi D_a \quad (9)$$

Equations 7 and 8 are the system of equations describing the physical situation of one-dimensional convective flow through a fracture with simultaneous tracer diffusion into the surrounding porous matrix (Neretnieks, 1980; Neretnieks et al. 1982; Jensen, 1983).

B.1- Continuous Injection Case

For a constant solute source of C_o at $x = 0$, initially the media are saturated with fluids free from the tracer, the boundary and initial conditions are given as:

$$\begin{aligned} C_f = C_p = 0 & \quad \text{at} \quad t = 0 \\ C_f = C_o & \quad \text{at} \quad x = 0 \\ C_p = C_f & \quad \text{at} \quad y = 0 \\ C_p \rightarrow 0 & \quad \text{as} \quad y \rightarrow \infty \end{aligned}$$

If we assume $C_o = 1$ then the solutions obtained will be the concentrations normalized by C_o . At the end of the injection period of time t_j , the solutions representing the profiles in the fracture and in the porous matrix respectively are:

$$C_f = \operatorname{erfc} \left\{ \frac{\sqrt{D_a \phi}}{\delta} \frac{x}{u} \frac{1}{\sqrt{t_j - \frac{x}{u}}} \right\} \quad \text{for } t_j \geq \frac{x}{u} \quad (10)$$

$$C_f = 0 \quad \text{for } t_j < \frac{x}{u}$$

$$C_p = \operatorname{erfc} \left\{ \left[2 \frac{\sqrt{D_a \phi}}{\delta} \frac{x}{u} + \sqrt{\frac{\phi}{D_a}} y \right] \frac{1}{2\sqrt{t_j - \frac{x}{u}}} \right\} \quad \text{for } t_j \geq \frac{x}{u} \quad (11)$$

$$C_p = 0 \quad \text{for } t_j < \frac{x}{u}$$

The injection period profile in the fracture will be as in Fig. 2.

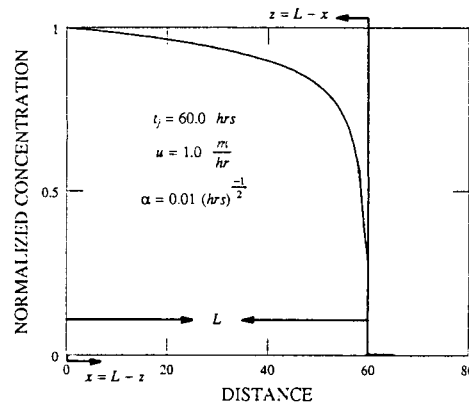


Figure 2: Dispersion of a sharp interface caused by matrix diffusion.

To account for the change in the direction of the velocity vector in the backflow period, the governing differential equations have to be modified. The modification will be done by utilizing the nature of the injection period solution and making a simple coordinate change.

Looking at the profile at the end of the injection period we see that the concentration in the fracture is zero after a distance of $x = u_f = L$, (see Fig. 2). When the origin of the new coordinate system is chosen at $x = L$ and the new space variable is defined as z , the injection period profile can be expressed in terms of z , by simply replacing x by $L - z$.

Now with the new coordinate system, the governing differential equation of flow in the fracture becomes:

$$\frac{\partial C_f}{\partial \tau} + u \frac{\partial C_f}{\partial z} = \frac{2D_a}{\delta} \frac{\partial C_p}{\partial y} \Big|_{y=0} \quad (12)$$

where τ is the time coordinate, starting from the beginning of the backflow period.

As far as the diffusion of tracer into or out of the matrix is concerned, there is no change in the conditions for constructing the governing equation. Hence, the equation remains the same, except for the time variable.

$$D_a \frac{\partial^2 C_p}{\partial y^2} = \frac{\partial C_p}{\partial \tau} \quad (13)$$

The boundary conditions are:

$$\begin{array}{lll} C_f = 0 & \text{at } z = 0 & \tau \geq 0 \\ C_p = C_f & \text{at } y = 0 & \tau \geq 0 \\ C_p \rightarrow \infty & \text{as } y \rightarrow \infty & \tau \geq 0 \end{array}$$

and the initial conditions are given by Eq. 10 and Eq. 11

However, attempts to obtain the real space solutions to Equations 12 and 13 failed because of the complexity introduced by the initial conditions. Hence, the Laplace transformed forms of the initial conditions will be preserved, and a solution in the Laplace space will be obtained.

The initial conditions in Laplace space are:

$$\bar{C}_f = \frac{1}{s} e^{-\frac{zs}{u}} e^{-\frac{2\sqrt{D_a}\phi x}{\delta u} \sqrt{s}} \quad (14)$$

$$\bar{C}_p = \frac{1}{s} e^{-\frac{zs}{u}} e^{-\left[\frac{2\sqrt{D_a}\phi}{\delta} \frac{x}{u} + \sqrt{\frac{s}{D_a}} y\right] \sqrt{s}} \quad (15)$$

where

$$x = L - z$$

Now the initial conditions are given by Equations 14 and 15. Taking the Laplace transform of Eq. 13 with respect to the time variable, τ , we obtain :

$$\frac{\partial^2 \bar{C}_p}{\partial y^2} - \frac{p}{D_a} \bar{C}_p = -\frac{1}{s D_a} e^{-\beta \sqrt{s}} e^{-\frac{zs}{u}} e^{-\sqrt{\frac{s}{D_a}} y} \quad (16)$$

where

$$\beta = \frac{2\sqrt{D_a}\phi x}{\delta u}$$

Eq. 16 is a linear nonhomogeneous differential equation and the solution of Eq. 16 is given by:

$$\bar{C}_p = \bar{C}_f e^{-\sqrt{\frac{p}{D_a}} y} - e^{-\frac{zs}{u}} \frac{e^{-\beta \sqrt{s}}}{(p-s)} \left[e^{-\sqrt{\frac{p}{D_a}} y} - e^{-\sqrt{\frac{s}{D_a}} y} \right] \quad (17)$$

Laplace transform of Eq. 12 yields a term containing the derivative of Eq. 17 with respect to y . Therefore differentiating Eq. 17 with respect to y at $y = 0$, and substituting its derivative into the Laplace transform of Eq. 12, we obtain :

$$p \bar{C}_f - \bar{C}_f + u \frac{\partial \bar{C}_f}{\partial z} =$$

$$\frac{2D_a}{\delta} \left\{ -\bar{C}_f \sqrt{\frac{p}{D_a}} - \frac{\sqrt{s} - \sqrt{p}}{\sqrt{D_a}(p-s)} e^{-\frac{zs}{u}} e^{-\beta \sqrt{s}} \right\} \quad (18)$$

Eq. 18 is also a linear nonhomogeneous differential equation representing the flow in the fracture, and the solution of Eq. 18 is given by:

$$\bar{C}_f = \frac{1}{s} \left[1 + \frac{2\alpha}{\sqrt{p} + \sqrt{s}} \right] \left[\frac{1}{s + p + 2\alpha(\sqrt{p} + \sqrt{s})} \right] \quad (19)$$

where

$$\alpha = \frac{\sqrt{D_a}\phi}{\delta}$$

B.2- Spike Injection Case

Applying the same technique that was used for the continuous injection case, we can find the spike injection case solution as:

$$\bar{C}_f = \frac{M}{Q} \left[1 + \frac{2\alpha}{\sqrt{p} + \sqrt{s}} \right] \left[\frac{1}{s + p + 2\alpha(\sqrt{p} + \sqrt{s})} \right] \quad (20)$$

APPLICATIONS

In the previous section, tracer transport models through fractures have been discussed and extended for the analysis of the return profiles from the single well injection-backflow tracer tests.

Now, the results from the application of the models to field data will be presented. Both models were curve fitted to four sets of data obtained from two wells in two different geothermal fields. The curve fitting was done by using a non-linear optimization program based on the work of Golub and Pereya (1973). Each model has only one non-linear parameter to be determined through curve fitting. Equations 18 and 20 are analytic only in (z, s, p) -space

which is two Laplace transformations away from the real space. Therefore, three function subprograms were utilized to perform a double numerical inversion process by using (Stehfest, 1970) algorithm.

The first two sets of data were from a well in Raft River geothermal field. One of the sets of data was a 4-hour injection test and the other was a 48.5-hour injection test. The other two sets of data were from a well in East Mesa geothermal field. The first one of these sets was a 7.22-hour injection test and the injection period for the second one was 14.22 hours. From the analysis of these sets we were able to compare the ability of the models to represent the flow in the reservoir, as well as analyze the effect of the injection period on the return profiles.

The results of the curve fittings are shown in figures from Fig. 3 to Fig. 10 at the end of this paper.

Observe that the matrix diffusion model gives better fits than the convection-dispersion model does. Also note that the convection-dispersion model gives far better results on the small injection period tests than it does on the long injection period tests. The matrix diffusion model, on the other hand, fits all sets of the data equally well and the fits are excellent.

One important point to note is that the values of the non-linear parameters differed even though they were recovered from data on the same well. The non-linear parameter of the matrix diffusion model is given by:

$$\alpha = \frac{\sqrt{D_e} \phi}{\delta} \quad (21)$$

The effective diffusivity D_e is a function of temperature, porosity, molecular diffusivity and the geometry of the rock. We assumed that the temperature, the porosity and the fracture aperture are constant along the path traveled by the tracer fluid. Therefore, the values of the non-linear parameters obtained from the analysis of the data sets of the same well have to be the same. The reason for the differing numerical values can be found in the effects of the injection periods. Since the assumption of constant fracture aperture and uniform porosity is not absolutely true, the non-linear parameter, α , recovered from the fits represents an average value over the distance traveled by the fluid. Therefore, the longer the injection period the longer the distance traveled by the tracer fluid and, of course, the closer the results to the average values of the whole domain.

The poor fits obtained from the application of the convection-dispersion model may be explained as follow. If the injection time is short, then the amount of the tracer diffusing into the fracture will not be high, so the length of diffusion. Hence, the main contribution to the dispersion within the fracture will come from Taylor Dispersion. As the injection period increases, the effect of the interaction between the adjacent matrix and the fluid in the fracture becomes the dominant mechanism of dispersion. Hence, the convection-dispersion model fails to give a good fit to the data obtained from the long injection period tests.

The last point to be considered is the non-unit normalized concentration value even at the beginning of the backflow period, $t_p = 0$, for the fit of convection-dispersion

model to the data of well 2C which is shown in Fig. 6. In order to explain this we need to check the injection time constraint explained in section A.1 of the mathematical developments. Now let's look at the condition for the injection time to be satisfied so that the infinite medium solution is applicable. From the point of view of numerical calculations the argument of the error function must be greater than or equal to 2, for the value of the error function to be 1. Therefore, to be able to get a unit C_D at the injection point, the following has to be satisfied.

$$\alpha \sqrt{t_i} \geq 4 \quad (22)$$

If we look at the values given in Fig. 6 we see that:

$$\alpha \sqrt{t_i} = 1.33 < 4 \quad (23)$$

Now there is two possible explanations can be given for this result:

- 1) The injection time is not enough for the theory to be applicable
- 2) The model itself is not applicable.

Of course, the second explanation is the logical one because of the inability of the model to represent the long injection period tests as explained above.

CONCLUSIONS

This study showed that the injection-backflow tracer tests can be used in determining the dispersion characteristics of the area within the radius of influence of the test well. In this study, two mathematical models describing the tracer transport in fractured medium were extended to suit for the quantitative analysis of the return profiles of injection-backflow tracer tests. Then the models were used to match six sets of field data. From these fits it was seen that most of the profiles can be successfully matched by the matrix diffusion model, whereas only short term injection tests could be fitted by the convection-dispersion model. In a short injection-backflow test, the time might not be sufficient for the tracer to diffuse far enough into the porous matrix to produce an asymmetric and long tailed profile. Hence, return profiles of the short injection period tests can be matched by the convection-dispersion model as well as by the matrix diffusion model. In the case of relatively high porosity or long injection periods, however, the return profiles are expected to be asymmetric and long tailed, therefore, can be matched well only by the matrix diffusion model.

It was observed that the non-linear parameters of models determined from fits to different data sets obtained from the same well gave different results. The differences between the two values of the same parameter were small if the differences between the injection periods were not large. In the long injection period tests, the information is obtained from a larger domain than in the short injection period tests. Therefore, the fits to the long injection period test data are expected to give better estimations of the governing parameters of the transport models.

NOMENCLATURE

Convection-Dispersion Model

δ : average velocity
 D : molecular diffusivity
 η : dispersivity coefficient
 C : concentration (mass per unit volume)
 C_D : normalized concentration
 x : distance along the flow direction
 t_j : total injection time
 t_p : time variable of production period
 t_i : slug injection time
 $z = x - ut$: moving space coordinate
 α : nonlinear parameter of the solution equation

Matrix Diffusion Model

C_p : concentration in matrix adjacent to fracture
 C_f : concentration in fracture
 C_o : initial concentration of the traced fluid
 D_a : apparent diffusion coefficient
 D_e : effective diffusion coefficient
 ϕ : porosity of adjacent matrix
 x : distance in the flow direction in injection period
 y : distance normal to the flow direction
 z : distance in the flow direction in backflow period
 $L = ut_j$: distance of the front from injection point at the end of the injection period
 M : mass of tracer material
 A : area open to flow
 Q : volumetric injection rate of traced fluid
 t_j : total injection time
 τ : time variable of production period
 p : Laplace parameter corresponding to τ
 s : Laplace parameter corresponding to t_j
 α : nonlinear parameter of the solution equation

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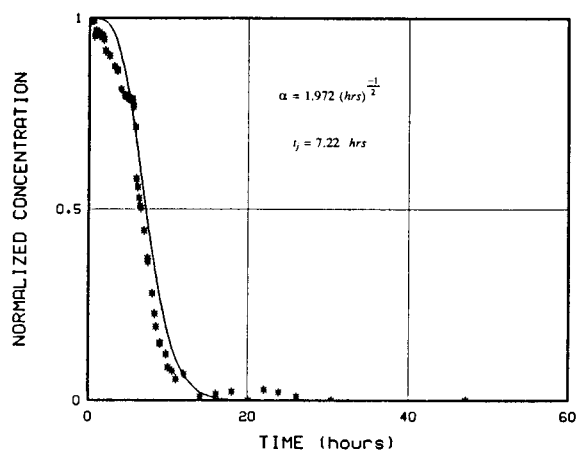


Figure 3: The result of curve fitting the convection-dispersion model to the data from Well #19 - Test 4.

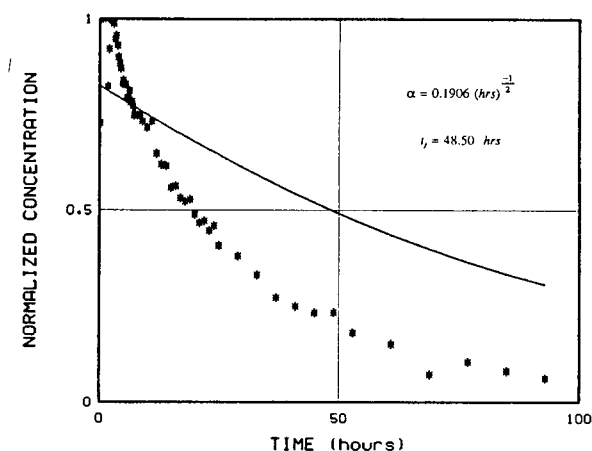


Figure 6: The result of curve fitting the convection-dispersion model to the data from Well #2 - Test c.

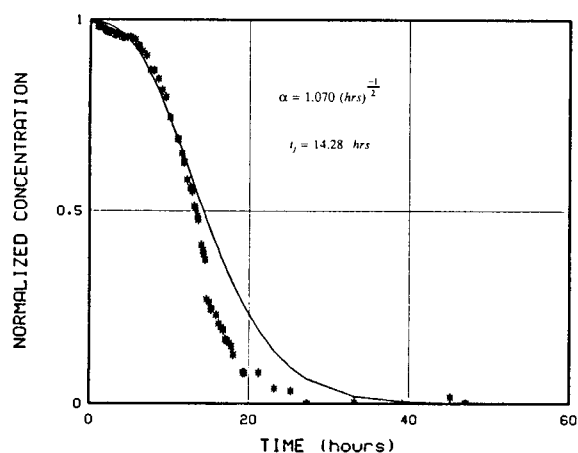


Figure 4: The result of curve fitting the convection-dispersion model to the data from Well #19 - Test 6.

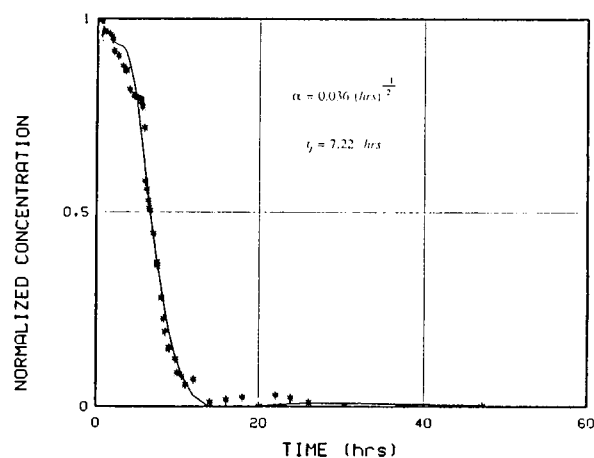


Figure 7: The result of curve fitting the matrix diffusion model to the data from Well #19 - Test 4.

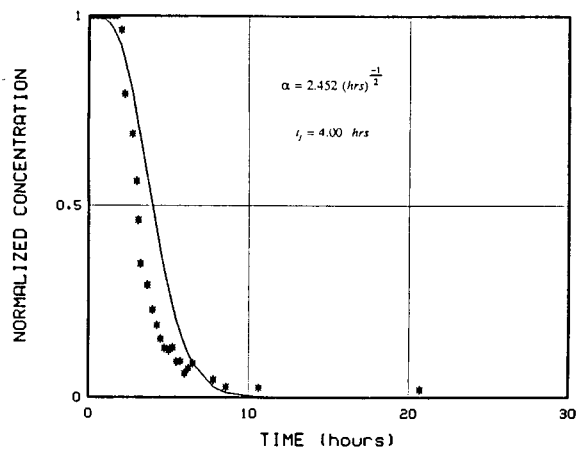


Figure 5: The result of curve fitting the convection-dispersion model to the data from Well #2 - Test a.

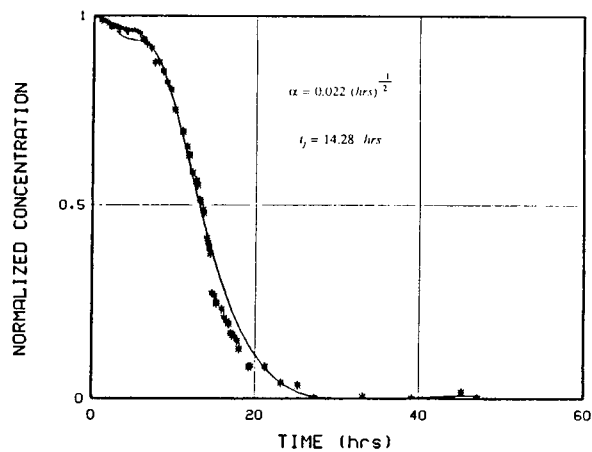


Figure 8: The result of curve fitting the matrix diffusion model to the data from Well #19 - Test 6. Correlation between dimensionless boundary distance and dimensionless departure time.

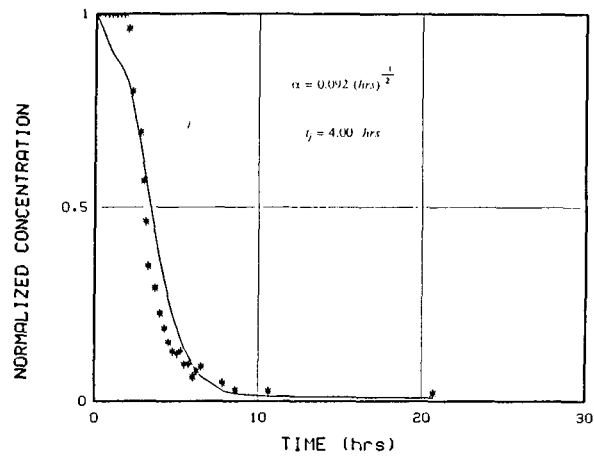


Figure 9: The result of curve fitting the matrix diffusion model to the data from Well #2 - Test a.

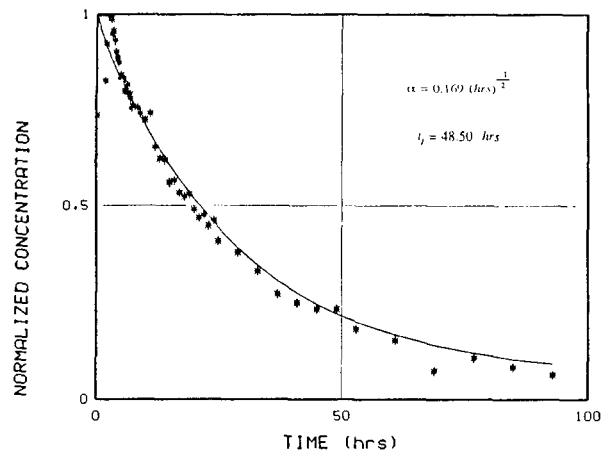


Figure 10: The result of curve fitting the matrix diffusion model to the data from Well #2 - Test c.