PROCEEDINGS, Eleventh Workshop on Geothermal Reservoir Engineering
Stanford University, Stanford, California, January 21-23, 1986
SGP-TR-93

REPRESENTATIVE ELEMENT MODELING OF FRACTURE SYSTEMS BASED ON STOCHASTIC ANALYSIS

T. M. Clemo.

Idaho National Engineering Laboratory
Idaho Falls, Idaho, 83415

ABSTRACT

An important task associated with reservoir simulation is the development of a technique to model a large number of fractures with a single description. Representative elements must be developed before reservoir scale simulations can adequately address the effects of intersecting fracture systems on fluid migration. An effective element model will sharply reduce the cost and complexity of large scale simulations to bring these to manageable levels. Stochastic analysis is a powerful tool which can determine the hydraulic and transport characteristics of intersecting sets of statistically defined fractures. Hydraulic and transport characteristics are required to develop representative elements.

Given an assumption of fully developed laminar flow, the net fracture conductivities and hence flow velocities can be determined from descriptive statistics of fracture spacing, orientation, aperture, and extent. The distribution of physical characteristics about their mean leads to a distribution of the associated conductivities. The variance of hydraulic conductivity induces dispersion into the transport process.

The simplest of fracture systems, a single set of parallel fractures, is treated to demonstrate the usefulness of stochastic analysis. Explicit equations for conductivity of an element are developed and the dispersion characteristics are shown. The analysis reveals the dependence of the representative element properties on the various parameters used to describe the fracture system.

NOTATION

a	depth of fracture into the plane of the element
b	fracture aperture
$\bar{b}$	average (expected) aperture
c(z,t)	solute concentration as a function of z and t
g	gravitational acceleration
k	hydraulic conductivity
ℓ	length of a fracture
λ_ℓ	expected length of l
λ_z	average spacing of fracture centers in the z direction
μ	absolute viscosity
λ_s	average spacing of fracture centers normal to the fracture faces
< >	expectation operator
ln	natural logarithm
p( )	probability density function
	$p_{ln}(x) = \frac{1}{(2\pi\sigma_x^2)^{1/2}} \frac{e^{-[\ln(x/\bar{x})]^2/2\sigma_x^2}}{x}$
	where $\bar{x} = e^{(\langle \ln x \rangle)}$;
	σ_x^2 = variance of ln(x)
	$p_e(x) = 1/\lambda_x \; ^{-x/\lambda_x}$
	where λ_x = mean of x
Q	flow rate
ρ	fluid density
ϕ	pressure
t	time
T	length of the element, in z direction
v	average velocity of fluid in a fracture
W	width of the element, normal to fracture faces
σ^2	variance of ln(b)
z	distance along a fracture

INTRODUCTION

This paper addresses an area of reservoir modeling that has not had significant exposure in the literature. Specifically, the development of representative element models to simulate a small portion of a fracture system. The main thrust of the paper is that, given a statistical description of a fracture system, a large number of fractures can be modeled with a single element. These elements can be combined with other elements and representations of individual fractures to form a reservoir scale simulation of manageable size and cost. A promising technique for deriving the properties of a representative element is the analytical reduction of statistical distributions into expected values. This technique is known as stochastic analysis.

The paper provides a brief background of current research in fractured reservoir modeling, followed by a discussion of what parameters may be needed to characterize a fracture system. The final portion of the paper is an example of using stochastic analysis to model a simple fracture geometry.

BACKGROUND

Modeling of fractured media has been based on two primary approaches, continuum and discrete. These two approaches are briefly discussed below and compared to a third approach which contains elements of both.

The continuum approach is based on a lumped parameter model of the fracture system, where the continuum is composed of representative elements. These elements model the hydraulic and transport behavior of a large number of fractures. For simulations of real fracture systems, the hydraulic and transport properties are determined from a statistical description of the fracture system. A requirement of the elements is that they represent a fracture system that is sufficiently large such that effects of individual fractures can not be distinguished in the response of the model. The scale must be large enough so that the fractured rock can be treated as if it were homogeneous. Consequently, reducing a large number of fractures to a single representation has become an active area of research (Dershowitz 1984).

A homogeneous porous media approximation is the most common method of representing a fracture network as a continuum. A major assumption of the porous media model is that transport dispersion can be modeled as a Gaussian random process using a dispersivity coefficient to determine the variance of transport about the mean movement. Recently a number of studies have called into question the validity of the porous media approximation (Simmons 1982, Dagan 1982, Schwartz et al. 1983).

The discrete approach represents the opposite end of the spectrum. All fractures which are considered relevant are modeled as individual entities. The fractures can be described from knowledge of the individual fractures in the system or be a stochastically generated realization based upon a statistical description of the fracture system. The discrete approach trades the difficulties of determining a representative element for a large amount of information processing required of the model. Presently these requirements limit discrete fracture simulations to reservoirs with few relevant fractures or small portions of a fracture system. Discrete fracture simulations are a promising approach to determining proprieties for representative elements (Dershowitz 1984, Long 1985, Schwartz et al. 1983).

At the Idaho National Engineering Laboratory, a slightly different approach to reservoir scale simulations of fracture systems is in development. The dual permeability approach is a compromise between discrete modeling and continuum modeling. Dual permeability treats the most important fractures in the system discretely and models the rest with representative elements. Furthermore, by incorporating most of the fractures into representative elements, the dual permeability approach can significantly reduce the processing requirement of the simulation. By treating the most important fractures discretely the complexity of the representative element is reduced. The complexity of the elements can be controlled by choosing the level of detail modeled discretely. In many cases the dual permeability approach will allow simulation of reservoirs that are too large for discrete simulation, yet are dominated by a few major fractures or faults making the continuum representation impossible. The fracture system found at the Raft River geothermal field is an example of a system that is highly dominated by a few flow channels and therefore not readily simulated as continuous media. Figure 1 was generated using statistical distributions of fracture spacing based on acoustic televeiwer log data from Raft River (Miller et al., 1984).

Figure 1 is the motivating example for this analysis. In Figure 1 a few large and widely spaced fractures comprise two dominant fracture sets. A third set of small closely spaced fractures completes the flow net. The first observation to be made from Figure 1 is the sparse nature of the large fractures. To model this system with a continuum model would require a much larger representative volume than depicted in the figure. The second observation to note is the large number of small fractures. A discrete simulation of this system would have to ignore most of these fractures. Unfortunately, these lesser fractures may provide a significant hydraulic connection between the larger fractures.

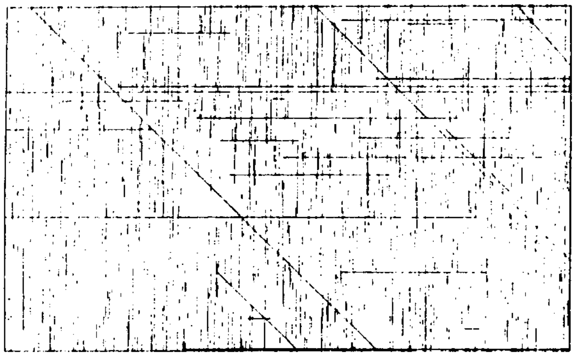

Figure 1: Two-dimensional Representation of a Fracture System. Spacing Acquired from Raft River Data.

The figure provides motivation to develop a two-dimensional representative element for a single set of parallel fractures, the closely spaced set in Figure 1. The representative element must simulate the hydraulic connection provided by this set. It must also simulate the transport dispersion characteristics of the fracture set. One option is to develop type curves based on discrete fracture simulations of various realizations of the fracture set. The geometry of a single parallel set of fractures makes stochastic analysis a viable tool to develop the element models.

Before proceeding with the model development, an aside on complexity is in order. This paper deals with the most simple case which can be studied analytically, a single set of parallel fractures. The next level of complexity involves two fracture sets which intersect. If these sets are such that few intersections occur, then the system is amenable to stochastic analysis. More complex is two fracture sets with frequent intersections. Whether these systems can be solved analytically or must be developed from dual permeability or discrete simulations is not clear. Intersections of multiple sets is probably beyond stochastic analysis. The power of the dual permeability approach will allow these more complex elements to be built up from simpler elements. This paper treats the simplest geometry, keeping a general approach for application to more complex systems.

FRACTURE SYSTEM CHARACTERIZATION

A statistical description of fracture sets comprising the system is required before a representative element model can be developed. This section identifies the parameters that may effect the element properties. A brief review of current statistical models is presented along with the statistical models chosen for this analysis.

Evans (1983) provides an excellent discussion of statistical distributions of fracture parameters. The fracture sets can be described by probability functions of location or spacing, extent or size, fracture aperture, shape, and orientation. Fracture surface characteristics and fracture tortuosity may also be important although these factors are not considered in this development. Fracture shape may be an important factor in developing a representative element. For this treatment however, fractures are assumed to be all of the same depth into the plane of the element.

Evans reports the spacing between fractures found from line samples have been described as lognormal or exponential. Exponential spacing results from a uniform random placement of fractures in space and is assumed for this study. A lognormal distribution would have only a minor impact on the development described below.

The length of fractures is assumed to be distributed as a negative exponential. Lognormal length distribution is also commonly used. The lognormal distribution was investigated but dropped due to the intractability of the resulting equations. Orientation is treated as fixed within the set.

The consequences of different assumptions about aperture characteristics were easily studied. The fracture aperture is assumed variously as a.)constant, b.)lognormal, c.)proportional to length, and d.)lognormally distributed about a mean proportional to length. In the latter assumption the variance of the logarithm of aperture is constant. A lognormal distribution is believed to result from the multiplicative effects of different distributions (Hahn 1967). In this light, given a proportionality of mean aperture and length, a constant variance of the logarithm seems the most appropriate assumption for the aperture.

ELEMENT DEVELOPMENT

As previously mentioned this paper treats a very simple class of representative elements. Specifically the element is a two-dimensional representation of a single set of parallel fractures. The elements are rectangles such that two sides are in the plane of the fracture faces and the other two sides are normal to the fracture faces (Figure 2). The normal sides are treated as constant head boundaries, with the other sides having a no flow condition.

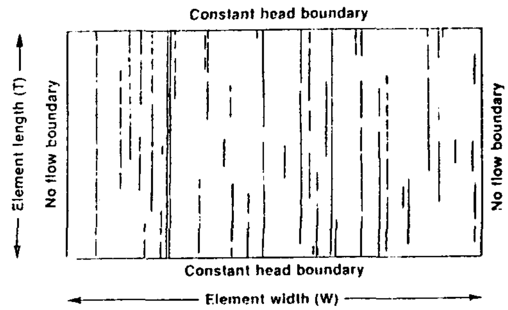

Figure 2: Example Representative Element.

The representative element considered here must simulate pressure response, bulk flow, and transport. Pressure and flow modeling require both a capacitive term to model the pressure response to fluid storage and a conductive term.

Storage is a bulk property determined for the reservoir as a whole. The storage of an individual element is determined by the reservoir storage multiplying the volume represented by the element.

Hydraulic conductivity and fluid transport require more complex analysis than the storage term. The development of these properties and their dependence on the statistical model assumptions are presented in the next two sections.

HYDRAULIC CONDUCTIVITY

Conductivity requires a calculation of the average flow through the element for a unit pressure differential. The parallel fracture system requires only a conductivity between the two fixed head sides. More complicated elements will require a conductivity tenser in two and three dimensions. All fractures that act as a flow path contribute to the conductivity. Limiting the model to laminar flow, the average velocity along a fracture is related to the pressure drop by (Lamb, 1945)

$$v = \frac{\rho g}{12\mu} b^2 \frac{d\phi}{dz} \qquad (1)$$

The flow is then the cross sectional area of the fracture times the average velocity. For constant depth fractures as is normally assumed in two dimensional models, the flow is proportional to the cube of the fracture aperture as;

$$Q = \frac{\rho g}{12\mu} b^3 a \frac{d\phi}{dz} \qquad (2)$$

The flow through a fracture connecting both sides of an element of length T is;

$$Q = \frac{\rho g}{12\mu} b^3 a \frac{\Delta\phi}{T} \qquad (3)$$

The conductivity of the fracture is defined as flow per unit pressure gradient or;

$$k = Q \div \frac{\Delta\phi}{T} = \frac{\rho g}{12\mu} b^3 a \qquad (4)$$

If the aperture is independent of all other factors and is lognormally distributed, then the expected conductivity of a fracture is;

$$\langle k\rangle = \int_0^\infty \frac{g}{12\mu} b^3 a \, P_{ln}(b)\, db \qquad (5)$$

$$\langle k\rangle = \frac{\rho g}{12\mu} \bar{b}^3 d \;\; e^{9\sigma^2/2} \qquad (6)$$

Equation 6 reveals a strong dependence of the average conductivity upon the variance of the logarithm of aperture.

The length distribution of fractures also affects the element conductivity. Only fractures which connect both sides of the element contribute to flow in this parallel system. Consider fractures with centers, Z, distributed uniformly along the z axis of the element. The fracture will connect the two sides only if the length, ℓ, is such that;

$\ell > 2Z$ if $Z > T/2$ or $\ell > 2(T-Z)$ if $Z < T/2$.

This criteria is symmetric about T/2.

For a negative exponential distribution of fracture length, the distribution function of connecting fractures becomes

$$P(\ell) = \frac{1}{\lambda_z} \frac{(\ell-T)}{\lambda_\ell} e^{-\ell/\lambda_\ell} \quad \text{for } \ell>T \qquad (7)$$

$$= 0 \quad \text{for } \ell \le T$$

For fractures with aperture unrelated to length and expected fracture spacing, λ_s, the conductivity of the element is

$$\langle k\rangle = \frac{1}{\lambda_s} \frac{g}{12\mu} \bar{b}^3 e^{9\sigma^2/2} \frac{\lambda_\ell}{\lambda_z} e^{-T/\lambda_\ell} \qquad (8)$$

The expression for conductivity in Equation 8 has the units of a material property (i.e. conductivity per unit area normal to the direction of flow). The conductivity of the element is not a material property. It is a measure of the effective hydraulic conductance of a specific element. Equation 8 reveals that the effective conductivity of the element decreases exponentially with separation T due to the exponential length distribution of fractures.

The above assumption that aperture is independent of fracture size is not realistic. The following development assumes that the aperture is a direct function of the length as;

$$b = \alpha\ell \qquad (9)$$

The expected conductivity of an element is

$$\langle k\rangle = \frac{1}{\lambda_s} \frac{\rho g}{12\mu} \int_T^\infty \alpha^3 \ell^3 \rho(\ell)\, d\ell \qquad (10)$$

$$\langle k\rangle = \frac{1}{\lambda_s}\ \frac{\rho g}{12\mu}\ \alpha^3\ \frac{\lambda_\ell}{z}\ [T^3+6T^2\lambda_\ell+18T\lambda_\ell+24\lambda_\ell]e^{-T/\lambda_\ell} \tag{11}$$

Figure 3 presents a comparison of the sensitivity of conductivity to element length, T, for apertures correlated and uncorrelated to fracture length. Figure 3 makes it clear that more attention needs to be payed to mean fracture length if apertures are assumed to be uncorrelated with length.

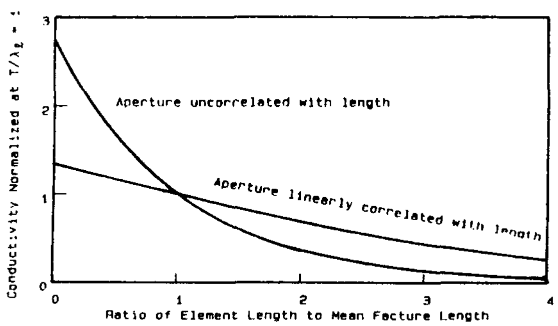

Figure 3: Sensitivity of Conductance to Element Length.

If aperture is related to fracture size by a lognormal distribution where $b = \beta\ell$ and σ^2 = constant

$$P(b:\ell) = \frac{1}{(2\pi\sigma^2)^{1/2}}\ \frac{e^{-[\ln(b/\beta\ell)]^2/2\sigma^2}}{b} \tag{12}$$

now

$$\langle k\rangle = \frac{1}{\lambda_s}\frac{\rho g}{12\mu}\int_T^\infty\int_0^\infty b^3 p(b:\ell)p(\ell)\ db\ d\ell \tag{13}$$

which is identical to the direct proportionality case with $\alpha = \beta e^{3\sigma^2/2}$

The conductivities developed above are the expected values for elements of width W. It is important to consider the effect of the width of the element on the variance of the conductivity. The variance of the conductivity of the fractures increases proportionately to the expected number of fractures, W/λ_S contained by the element. The variance in the number of fractures represented by the element is also W/λ_S for an exponential spacing of fractures (Parzen 1962). These variances of the element conductivity are independent and therefor additive. The conductivity of the elements also increase linearly with W/λ_S which means the relative variance deceases linearly with the element width.

As mentioned earlier, the conductivity determined in this analysis has the units of a material property but is a measure of the expected conductivity of a specific geometry. The representative element properties are strictly modeling tools and should not be construed as a measure of physical properties.

TRANSPORT

Once fluid enters a fracture in this system, it remains in the fracture until it reaches the end of the element. Consider a pulse of solute entering the element at time zero. Following the development presented by Simmons (1982), the distribution of the expected concentration as a function of time and the distance along the element is:

$$c(z,t) = \int_0^\infty \delta(t-z/v)p(v)dv = p\ (z/t) \tag{14}$$

The distribution (dispersion) of the pulse is simply a scaled version of the velocity profile. To model the element, the time distribution of breakthrough is needed. Replacing Z by T

$$c(T,t) = p(T/t) \tag{15}$$

Using Equation 1,

$$T/t = \frac{\rho g}{12\mu}\ b^2\ \frac{\Delta\phi}{T} \tag{16}$$

$$t = \left(\frac{\rho g}{12\mu}\right)^{-1}\left(\frac{\Delta\phi}{T^2}\right)^{-1} b^{-2} \tag{17}$$

The transit time t is therefore directly related to aperture and

$$p(t)dt = p(b)db \tag{18}$$

From 17,

$$db = 1/2\ \left(\frac{\rho g}{12\mu}\right)^{-1/2}\left(\frac{\Delta\phi}{T^2}\right)^{-1/2} t^{-1/2}\ dt \tag{19}$$

Given a lognormal aperture distribution as in the first of the three conductivity calculations,

$$p(t) = \frac{1}{(2\pi\sigma_t^2)^{1/2}}\ \frac{e^{-[\ln(t/\bar{t})]^2/2\sigma_t^2}}{t} \tag{20}$$

Where;

$$\bar{t} = \frac{\rho g}{12\mu}\ \frac{\Delta\phi}{T^2}\ \bar{b}^2 \text{ and } \sigma_t^2 = 4\sigma^2 \tag{21}$$

The transit time for a solute packet can be found using the Monte Carlo technique. The effect of the Poiseuille profiles can be incorporated into a Monte Carlo simulation by dividing the transit time by a generated velocity ratio from the Poiseuille velocity distribution. The distribution function is;

$$P(\gamma) = 1/3\ \left(\frac{\gamma}{1-2\gamma/3}\right) \tag{22}$$

Where γ = the ratio of the velocity at a random position in the fracture to the mean velocity.

Equations 23 and 24 present the distribution function for transport given a linear correlation of aperture and fracture length.

$$P(t) = \frac{1 - \frac{T\alpha}{\kappa t^{1/2}}}{2\alpha^2 \lambda_\ell^2} e^{-\kappa} e^{-t^{1/2}/\alpha\lambda_\ell} \tag{23}$$

Where,

$$\kappa = \left[\frac{\rho g}{12\mu} \frac{\Lambda\psi}{T^2}\right]^{1/2} \tag{24}$$

When aperture is lognormally distributed about a fracture length dependent mean, Equation 25 defines the probability distribution of velocities. Equation 25 does not yield a closed form solution and must be evaluated numerically.

$$P(b) = \int_T^\infty P(b:\ell)P(\ell)\, d\ell \tag{25}$$

CONCLUSIONS

The Dual permeability approach to reservoir scale simulation has some important advantages over discrete fracture simulation or the continuum approach. These advantages are embodied in the ability to adjust the degree of complication treated discretely versus the complexity of the representative elements. Further, some reservoirs may be simulated by only a dual permeability model.

The dual permeability approach will provide a more robust simulation capacity because the technique does not require homogeneity of the elements. It will allow the modeler to treat highly important fractures discretely and yet retain full information of the influence of the minor fractures.

The primary research, needed to be performed to develop dual permeability modeling, is the development of representative elements. This study involved the simplest of fracture systems to be modeled as a representative element. The study provided a complete description of the hydraulics and transport properties of this class of two dimensional elements. These elements can be used in a dual permeability model which simulates transport of solute as discrete particles.

Even these simple elements can provide a significant reduction in simulation cost. More complicated elements can also be created, although some tough problems need to be addressed. The first hurdle is the description of a fracture intersection, both hydraulically and in terms of transport. Fracture interactions will introduce transition probabilities into the transport equation. Once the simple elements have been described, more complicated elements may be amenable to analytic development. If not, a dual permeability model can be used to empirically find the element properties as is now done to develop continuum models.

ACKNOWLEDGEMENT

This work was performed by EG+G Idaho, Inc. for the U. S. Department of Energy under contract No. DE-AC07-76ID01570.

REFERENCES

Dagan G. (1982), Stochastic modeling of groundwater flow by unconditional and conditional probabilities, Water Resources Research, 18(4), 835-848, 1982.

Dershowitz W. S. (1984), Rock Joint Systems, Ph.D. thesis, MIT., 1984.

Evans D. D. (1983), Unsaturated Flow and Transport Through Fractured Rock - Related to High-Level Waste Repositories, NUREG/CR-3206, University of Arizona, 34-43, 1983

Hahn J.P. (1967), Statistical Models in Engineering, Whiley, 1967.

Lamb H. (1945), Hydrodynamics, Dover, New York, 1945.

Long J. C. S. (1985), Verification and Characterization of Continuum Behavior of Fractured Rock at AECL Underground Research Laboratory. BMI/OCRD-17, LBL-14975. University of California. February, 1984.

Miller J. D., D. W. Allman, D. Wulf, (1984), Raft River Fracture Characterization and Reservoir Simulation, SE-PB-84-58, Idaho National Engineering Laboratory, 1984.

Parzen E. (1962), Stochastic Processes, Holden-Day, Pg. 30, 1962,

Schwartz, F. W., L. Smith, A. S. Crowe (1983). A stochastic analysis of macroscopic dispersion in fractured media. Water Resources Research, 19(5), 1253-1265, 1983

Simmons C. S. (1982). A stochastic-convective transport representation of dispersion in one-dimensional porous media systems, Water Resources Research, 18(4), 1193-1214, 1982.