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DECLINE CURVE ANALYSIS FOR INFINITE DOUBLE-POROSITY SYSTEMS WITHOUT WELLBORE SKIN

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ABSTRACT

This paper presents a transient pressure analysis method for analyzing the rate decline of a constant pressure well producing in an infinite double-porosity reservoir, without wellbore skin. This analysis method may be used to interpret well test rate data, and to compute the rate behavior of an infinitely acting reservoir that is being produced at constant pressure.

The development of the pseudo steady state bg-log type curve is presented along with a hypothetical example of its use. This type curve allows the estimation of the two controlling parameters in double-porosity systems: A and ω. The first perameter, A describes the Interporosity flow, and the second parameter, ω describes the relative fracture storatlvlty. This paper considers the estimation of these two parameters. The estimations of permeabillties and storativities have been described in the past, hence, are not considered.

In a double-porosity system, with pseudo steady state Interporosity flow, the initial infinite acting rate decline, representing only the fracture system, is folbwed by a constant rate flow period. The length of this constant rate flow period is controlled by the parameter ω. The beginning of this period is controlled by the interporosity flow parameter, A. Following this constant rate period, the rate resumes an Infinite homogeneous decline, representing the total system, fractures and matrix. The parameters A and ω may be estimated from a log-log match of rate data to the type curve.

A comparison between rate responses of two translent flowing matrices and the pseudo steady state matrix Is presented. Transient Interporosity flow allows the matrix to Increase the well flowrate in the early and transition portions of the flow. The final decline, representing the total system, is identical to the decline with a pseudo steady state matrix.

INTRODUCTION

Decline curve analyses are used to interpret rate-tlme data, estimate reservoir properties, and then, compute the future behavior of the system. The decline in the rate of production with time, be it a single well or an entire reservoir, when produced under a constant pressure drawdown, has been described In the literature [1-6]. *Fetkovich*[4] introduced the concept of "transient decline" and "depletion decline", by merging *Arps* depletion decline with the transient solution for a finite wellbore in a cbsed outer boundary radial reservoir. The transient period of the rate response occurs before the closed outer boundary effects are significant, and the reservoir appears to infinite in lateral extent.

As naturaliy-fractured reservoirs were considered, it became evident that fluid flows both In the fractures and in the matrix blocks. *Barenblatt and Zeltov*[7] presented a double-porosity model where the diffusion equation applied to the fractured medium and the fluid stored in the matrix flowed into the fractures at a pseudo steady state condition. *Warren and Root*[8] introduced two parameters characterizing the flow In a two porosity system: the interporosity flow parameter, λ, and the relative fracture storage parameter, ω.

Mavor and Cinco-Ley[9], *Da Prat et al*[10], and *Raghavan and Ohaeri*[11] introduced the concept of decline curve analysis in double-porosity systems. They considered the rate-time behavior of a constant pressure well in an infinite or closed outer boundary radial system.

The response of a double-porosity reservoir depends on the type of interporosity flow. Transient Interporosity flow was considered for various matrix shapes: slabs[12,13], spheres[13], and cylinders[14]. *Raghavan and Ohaeri*[11] considered decline curve analysis for both the *Warren and Root*[8] pseudo steady state model and the transient Interporosity flow model. Also, *Moench*[15,16] considered a generalized solution to the interporosity flow, introducing the concept of fracture skin. *Moench*[15,16] showed that pseudo steady state interporosity flow occurs when a large fracture skin is present, and the transient Interporosity flow as previously considered occurs when fracture skin is not present.

This paper presents a log-log type curve matching technique for estimating λ and ω from rate-time data taken during a constant-pressure well test in a double-porosity reservoir with pseudo steady state matrix flow. Comparisons between the pseudo steady state and the transient interporosity flow models are presented.

MATHEMATICAL MODEL

The mathematical description of the behavior of double-porosity systems has been described in the past[7-20]. In developing the model, It is assumed that the reservoir is Infinitely large, the fluid is slightly compressible with a constant compressibility, gravitational forces are negligible, the porosity and permeabilities of the fractures and the matrix are not functions of pressure , and that the fluid enters the well through the fractures.

Deruyck et al[19] observed that the pseudo steady state and the transient interporosity flow models may be handled in a similar mathematical way. They presented the fracture pressure equation[19,20]:

$$\frac{k_f}{\mu}\nabla^2 p_f = \left(\varphi\mu c_t\right)_f \frac{\partial p_f}{\partial t} - q^* \qquad (1)$$

where the terms are defined in the nomenclature. The Laplace transformation of the dimensionless form of equation 1, making use of the Initial condition $p_f = p_m = p_i$,yields[9,10]:

$$\frac{d^2\bar{p}_{fD}}{dr_D^2} + \frac{1}{r_D}\frac{d\bar{p}_{fD}}{dr_D} - sf(s)\bar{p}_{fD} = 0 \qquad (2)$$

for which the Laplace dimensionless rate solution for a constant pressure inner boundary is:

$$\bar{q}_D = \frac{\sqrt{sf(s)}K_1(\sqrt{sf(s)})}{sK_0(\sqrt{sf(s)})} \qquad (3)$$

$\bar{p}_{fD}(r_D,s)$ and $\bar{q}_D$ are the Laplace transformations of $p_{fD}(r_D,t_D)$, and q_D respectively. The dimensionless groups are defined as:

$$p_{fD} = \frac{p_i - p_f}{p_i - p_{wf}} \qquad (4)$$

$$q_D = \frac{qB\mu}{k_f h(p_i - p_{wf})} \qquad (5)$$

$$t_D = \frac{k_f t}{\left[(\varphi Vc_t)_f + (\varphi Vc_t)_m\right]\mu r_w^2} \qquad (6)$$

$$r_D = \frac{r}{r_w} \qquad (7)$$

The variable $f(s)$ depends on the assumed interporosity flow model. For the pseudo steady state model:

$$f(s) = \frac{\omega(1-\omega)s + \lambda}{(1-\omega)s + \lambda} \qquad (8)$$

For the transient interporosity flow model, with slab-shaped matrix[19,20]:

$$f(s) = \omega + \frac{\lambda}{3s} a \tanh(a) \qquad (9)$$

where: $a = \sqrt{\frac{3(1-\omega)s}{\lambda}}$

For the transient interporosity flow model, with slab-shaped matrix and with matrix skin[15,16]:

$$f(s) = \omega + \frac{\lambda}{3s}\left[\frac{a \tanh(a)}{1 + S_F\, a \tanh(a)}\right] \qquad (10)$$

For the transient interporosity flow model, with spherically-shaped matrix[18,19]:

$$f(s) = \omega + \frac{\lambda}{5s} b \coth(b) \qquad (11)$$

where: $b = \sqrt{\frac{15(1-\omega)s}{\lambda}}$

For the translent interporosity flow model, with spherically-shaped matrix and with matrix skin[15,16]:

$$f(s) = \omega + \frac{\lambda}{5s}\left[\frac{b \coth(b) - 1}{1 + S_F[\, b \coth(b) - 1\,]}\right] \qquad (12)$$

The parameters ω and λ are defined as:

$$\omega = \frac{(\Phi Vc)_f}{(\Phi Vc)_f + (\Phi Vc)_m} \qquad (13)$$

$$\lambda = \alpha \frac{k_m}{k_f} r_w^2 \qquad (14)$$

and the other parameters are defined in the nomenclature.

For the pseudo steady state interporosity flow, the value of $f(s)$ takes on three distinct approximations. At early times, $t \to 0$, $s \to \infty$, $f \to \omega$, and Eq. 3 inverts to[10]:

$$q_D = \frac{\sqrt{\pi}}{\pi}\left(\frac{t_D}{\omega}\right)^{-\frac{1}{2}} \qquad (15)$$

At intermediate times, λ controls the flow, $3 \to \lambda/s$, and Eq. 3 Inverts to:

$$q_D = \frac{\sqrt{\lambda}K_1(\sqrt{\lambda})}{K_0(\sqrt{\lambda})} \qquad (16)$$

At late times, $t \to \infty$, $s \to 0$, $f \to 1$ and Eq. 3 inverts to[10,11]:

$$q_D = \frac{1}{\frac{1}{2}\left(\ln t_D + 0.800\right)} \qquad (17)$$

At intermediate times for the transient interporosity flow with a matrix skin of $S_F > 0.33$, $f(s) \to \lambda/3S_F$, and Eq. 10 inverts to:

$$q_D = \frac{\sqrt{\frac{\lambda}{3S_F}}K_1\sqrt{\frac{\lambda}{3S_F}}}{K_0\sqrt{\frac{\lambda}{3S_F}}} \qquad (18)$$

The Laplace dimensionless rate solution is numerically inverted using the algorithm developed by *Stehfest*[21].

TYPE CURVE

In this section, the log-log type curve for the pseudo steady state interporosity flow is presented, followed by a comparison between the pseudo steady state and the transient interporosity flow models. In contrast to a single porosity homogeneous system, there are two parameters controlling the flow in the system, ω and A.

The effects of ω and λ on the rate response are now considered. Figure 1 presents two cases with different ω and the same λ. The case where $\omega = 1$ is a single-porosity system, and A has no effect on the rate response, yielding the lower curve. When $\omega < 1$, the rate response is different. At a certain time, the rate becomes asymptotic to a constant, and only after a period of time resumes its decline. The early decline represents the flow only in the fractures. This decline starts with a slope of $-1/2$, as described by Eq. 16. The transition flow period, represents an increasing amount of flow from the matrix into the fractures. The constant rate during the transition flow period is given by Eq. 16. The second decline occurs when the pressure in the matrix and in the fracture at a given spatial point are practically identical. This is the total system decline, for which the rate is give by Eq. 17[10,11].

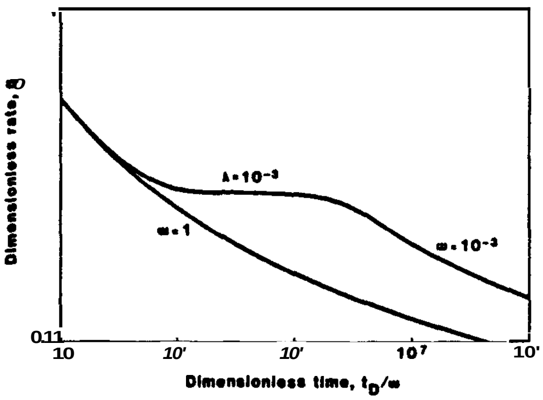

FIGURE 1: A typical response of an Infinite double-porosity system, PSS (ω=0.001, λ=0.001, and S=0).

The constant rate flow period described by Eq. 16 is only a function of A and the time shift of the second decline is only a function of ω, hence, A and ω domains may be mapped independently. Figure 2 presents a log-log type curve for an infinite two porosity system without wellbore skin. A rate response of a constant pressure well would start on the infinite acting single porosity curve for $\omega = 1$. Then, after a transition period, the rate will become constant following a constant λ line. At later times, the rate response will follow the decline of a different ω line. By matching the initial decline and the constant rate period, the value of A can be estimated. By matching all the three flow periods, the values of A and ω can be estimated. The use of this log-log type curve is demonstrated in the next section.

Transient interporosity flow produces a different rate response than the pseudo steady state one. Two kinds of matrix geometries are considered: slabs and spheres. Figure 3 presents the rate response for an infinite double- porosity system wlth transient interporosity fbw. The lowermost curve is for a single-porosity system where $\omega = 1$. The two curves for ω = 0.001 and A = 0.001, representing slabs and spheres are similar, with the Curve for the spherically-shaped matrix above the curve for slabs. These two curves merge into a single curve at the end of the transition flow period and in the second decline period.

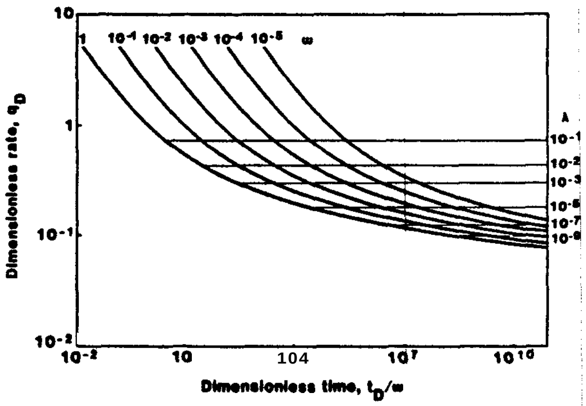

FIGURE 2: Log-log type curve for an Infinite double-porosity system, PSS flow model (S=0).

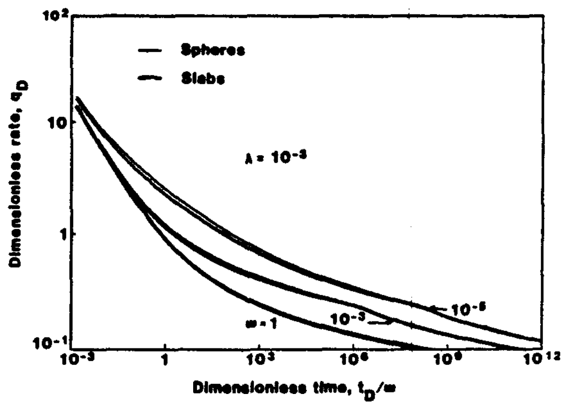

FIGURE 3: Response of an Infinite double-porosity system with transient matrix flow (S=0 and S_F=0).

Figure 4 compares the rate response of the pseudo steady state and transient interporosity flow models for a fixed value of λ. The early time transient interporosity rate response is above that of the pseudo steady state one. The final declines for the two interporosity flow models are identical.

The transient flow response curves separate from the homogeneous case at early times and do not follow the same transition response as well. The larger the value of ω, the more the deviation between the curves. Figure 6 compares the rate response of the pseudo steady state and the transient interporosity flow models, for a fixed value of ω. As the value of A increases, the difference between these two interporosity flow models reduces.

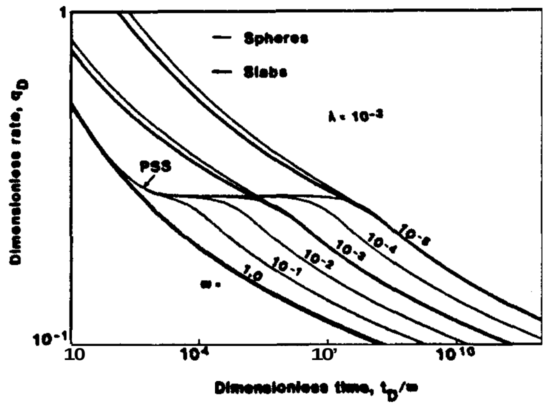

FIGURE 4: Comparison between the PSS and the transient Interporosity flow models for a fixed A and various values of ω.

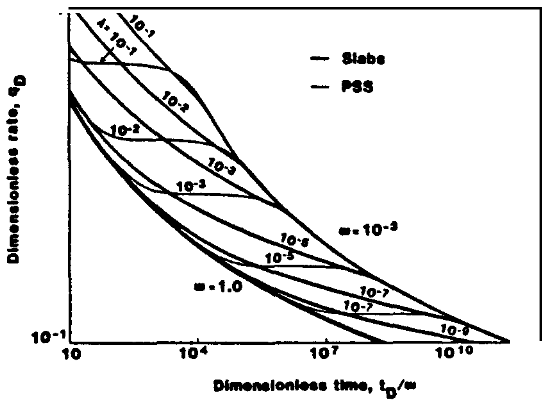

FIGURE *6:* Comparison between the PSS and the transient interporosity flow models for a fixed ω and various values of A

For the transient Interporosity flow models, without fracture skin, the early time period and the beginning of the transition flow period depend on the values of ω and A This dependency complicates the bg-log type curve matching procedures, which are not considered in this paper.

Transient and pseudo steady state interporosity flows are described by a transient interporosity flow with fracture **skin**[15]. In Figure 6, a set of rate responses with fracture skin are compared to pseudo steady state and transient responses. When S_F = 0, Eq. 10 is identical to Eq. 9, hence, the transient flow is a special fracture skin case with zero skin. For S_F > **0.33,** the matrix acts like a pseudo steady state matrix, and the constant rate, given by Eq. 18, occurs for a period that depends only on *w.* The lines of constant A on Figure 2 are identical to lines of constant $\lambda/3S_F$ for a transient matrix with fracture skin. A high fracture skin prevents flow from the matrix to the fractures, hence, the rate response is for a single porosity system, with fracture characteristics.

TYPE CURVE MATCHING EXAMPLE

In this section, a hypothetical example of a rate history from an infinite two porosity system without wellbore skin is analyzed. Only the estimation methods of λ and ω are presented. The analysis for the fracture permeability and the total storage were presented by *Da Prat et al*[9], and ***Raghavan and Ohaeri***[10] are not considered here.

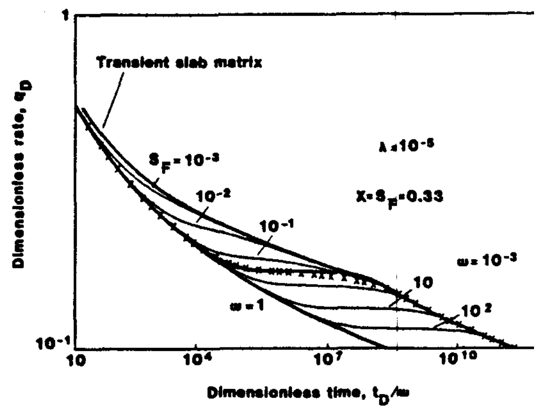

FIGURE 6: Comparison of the PSS model and the transient interporosity flow model with fracture skin.

This example illustrates the use of the type curve presented in Figure 6. The infinite acting early time data are matched on the log-log(type curve to the first curve on the left (See Figure 7). This curve is for ω = 1. The late time data deviate above the first infinite acting curve and the rate becomes constant. The value of A is estimated by matching the constant rate portion of the data to a constant A line. For this example, A = 0.00001. If this test were bnger, the rate would resume the second decline along another ω curve, allowing the estimation of ω. For this example, only the upper limit of ω is estimated as 0.01. Once a bg-log match is made, other parameters, such as fracture permeability, can be calculated from the rate and time match point.

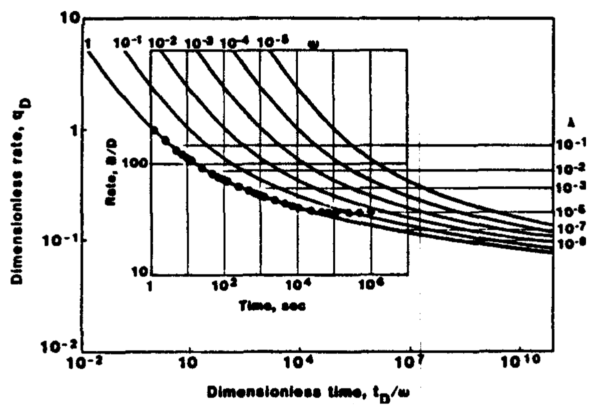

FIGURE 7: Log-log type curve matching example for an infinite double-porosity system.

DISCUSSION

The log-log type curve presented In this paper has two applications. The first application is for analyzing **well** test rate data for estimating the values of λ and w. The second application is to compute the decline rate of a well producing at a constant pressure in an infinite double-porosity reservoir.

Assuming the matrix produces at a pseudo steady state condition permits an independent determination of λ and ω. The value of A is determined by the deviation of the rate response from the homogeneous response, and the value of the rate during the constant rate flow period. In order to determine ω, the matrix pressure must decline together with the fractures, acting as a single porosity system. Hence, the length of the constant rate flow period determines the value of ω.

The rate response of a double-porosity system with transient interporosity flow does not yield a decllne pericd where only the fractures are produced. The transiticn flow period starts early in the decline process, and does not end in a constant rate flow period. This transition flow period Is affected by both λ and ω, hence complicates the log-log type curve analysis.

The rate response of an infinite double-porosity system with transient interporosity flow is modeled as a single case of a transient interporosity flow with a negligible fracture skin. With a significant fracture akin, the rate response is similar to the rate response when the matrix produces at pseudo steady state, yleldlng a constant rate flow period. the length of this period is inversely proportional to ω, and the value of this constant rate is a function of A and fracture skin.

NOMENCLATURE

B = formation volume factor
K_0 = modified Bessel function, third kind, zero order
K_1 = modified Bessel function, third kind, first order
V = ratio of volume of one porous system to bulk volume
S_F = dimensionless fracture skin
c = compressibility
h = formation thickness
k = permeabillty
p = pressure
p_D = dimansionless pressure
$\bar{p}_D$ = Laplace transform of p_D
q = volumetric rate
q_D = dimensionless rate
$\bar{q}_D$ = Laplace transform of q_D
q^* = matrix flow rate
r_D = dimensionless radlus
r_w = wellbore radius
s = Laplace variable
t = tlme
t_D = dimensionless time
λ = interporosity flow coefficient
μ = viscosity
φ = porosity
ω = dimensionless fracture storage

Subscripts

D = dimensionless
f = fracture
m = matrix
t = total
w = Well

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