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SLUG TESTING IN WELLS WITH FINITE-THICKNESS SKIN

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ABSTRACT

We present an analysis of the slug test in a well surrounded by an annulus of altered material, which is treated as a skin of finite thickness. By assuming the skin has a thickness, the storage capacity of the altered material is included in the analysis. The problem is solved in the Laplace domain. The solution is found in terms of well-bore storage and the thickness, hydraulic conductivity, and specific storage of the skin. Type curves are generated by numerical inversion of the Laplace transform solution. We find that standard methods of analysis, involving a skin of infinitesimal thickness, are adequate for open-well or drill-stem tests. However, for pressurized tests the response may differ markedly from standard slug-test solutions.

INTRODUCTION

The slug test is known to be a valuable tool for evaluating the hydraulic properties of water-bearing formations. It has the advantage that it is simple and inexpensive to run and can usually be performed in a short period of time. It has the disadvantage, however, that only a small region around the well bore is investigated. Consequently it is extremely sensitive to the hydraulic properties of the material in the immediate vicinity of the well. In many instances the hydraulic properties in this region have been altered by such factors as invasion of the pores or fractures of the formation by drilling mud, the buildup of scale due to chemical precipitation, or well-Stimulation endeavors. Historically, this annulus of altered material has been treated as a skin of infinitesimal thickness. Ramey and Agarwal (1972) presented an analytical solution for a slug test in a well with a skin of infinitesimal thickness and Ramey et al. (1975) showed that the solution could be approximated by a single set of type curves.

In this paper, which is a revised and abbreviated version of a paper by Moench and Hsieh (1985a), we present an analysis of a slug test in a well with a skin of finite thickness. The solution was found in the Laplace domain and type curves were obtained by numerical inversion. Using realistic values of aquifer and skin hydraulic properties, we show the effects of a finite-thickness skin on slug-test responses for both open-well, or drill-stem, tests and pressurized tests.

MATHEMATICAL MODELS

Skin of Infinitesimal Thickness

Except for the presence of Skin the assumptions of Ramey and Agarwal (1972) are the same as those of Cooper et al. (1967). These assumptions are that the well fully penetrates a confined, homogeneous and isotropic aquifer of constant thickness and of infinite lateral extent. It is also assumed that the hydraulic properties of the skin and aquifer remain constant for the duration of the test.

In the notation of this paper, the solution in the Laplace domain can be written as

$$\bar{h}_w = \frac{K_0(q) + \sigma q K_1(q)}{pK_0(q) + q(p\sigma + 1/\gamma)K_1(q)} \tag{1}$$

where σ is the skin factor, which is a constant of proportionality relating the flux of water through the skin to the difference in hydraulic head across the skin; p $(=q^2)$ is the Laplace transform parameter and is inversely related to the dimensionless time; and γ is the dimensionless well-bore storage. The mathematical notation is collected and defined at the end of the paper.

Skin of Finite Thickness

Figure 1 is a schematic diagram of a slug test conducted in an open well surrounded by a finitethickness skin. Except for the presence of the finite-thickness skin, the assumptions are the same as those of Cooper et al. (1967), outlined above. The boundary value problem that describes the slug test depicted in Figure 1 is set up formally by Moench and Hsieh (1985a). The mathematical problem can be solved in a straightforward manner by the Laplace transform method. The Laplace transform solution for the dimensionless head in the well is

$$\bar{h}_w = \bar{F}(p, \gamma, r_s, \alpha, \delta) \tag{2}$$

where $$\bar{F} = \frac{\alpha\gamma[\Delta_1 K_0(q\beta) - \Delta_2 I_0(q\beta)]}{C_1\Delta_1 - C_2\Delta_2}$$

$$\Delta_1 = \alpha I_0(q\beta r_s)K_1(qr_s) + \beta I_1(q\beta r_s)K_0(qr_s)$$

$$\Delta_2 = \alpha K_0(q\beta r_s)K_1(qr_s) - \beta K_1(q\beta r_s)K_0(qr_s)$$

$$C_1 = \alpha\gamma p K_0(q\beta) + \beta q K_1(q\beta)$$

$$C_2 = \alpha\gamma p I_0(q\beta) - \beta q I_1(q\beta)$$

$$\beta = (\alpha\delta)^{1/2}$$

$$q = p^{1/2}$$

Equation (2) cannot be easily inverted analytically. In this paper type curves are obtained for equations (1) and (2) by numerical inversion using the Stehfest (1970) algorithm. The accuracy of the numerical procedure was checked for the case of $\alpha = 1$ and $\delta = 1$ by comparison with tables published by Cooper et al. (1967).

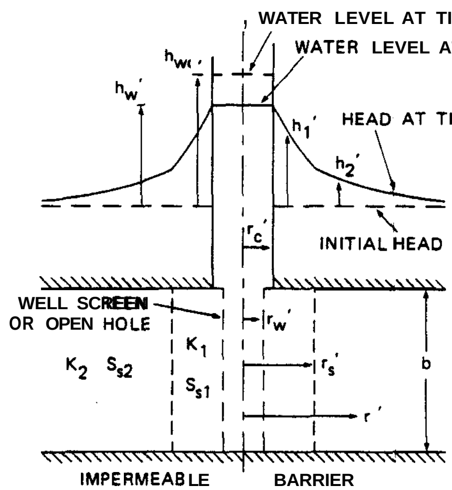

Figure 1. Schematic diagram of a slug test conducted in an open well surrounded by a finite-thickness skin.

RESULTS

It is evident from the number of dimensionless parameters involved that numerous sets of type curves can be generated from equation (2). To illustrate the effect of the finitethickness skin on Yell response, we present some typical type curves for both open-well tests and pressurized tests. Open-well tests are distinguished from pressurized tests by the magnitude of the well-bore storage. For open-well tests the well-bore storage is dependent upon the cross-sectional area of the riser pipe: for pressurized tests it is dependent upon the fluid compressibility and compliance of the well-test equipment. Because the well-bore storage of a pressurized test may be four to five orders of magnitude smaller than that of an open-well test, the dimensionless well-bore storage parameter (γ)

may be four to five orders of magnitude smaller for pressurized tests than for open-well tests. For open-well tests, γ generally ranges from 10^2 to 10^6 and for pressurized tests, γ generally ranges from 10^{-2} to 10^2.

Open-well tests

In this section we compare type curves for a slug test in a well with a skin of finite thickness with corresponding type curves for a skin of infinitesimal thickness. Moench and Hsieh (1985a and 1985b) show that the solution for a finite-thickness skin with negligible skin storativity is equivalent to the solution for a skin of infinitesimal thickness provided α, t, and γ in the latter solution are replaced by

$$\sigma^* = \alpha \ln(r_s) \qquad (3)$$

$$t^* = t/r_s^2 \qquad (4)$$

and

$$\gamma^* = \gamma/r_s^2 \qquad (5)$$

Ramey et al. (1975) showed that the infinitesimally thin skin solution can be represented as a function of two dimensionless groups: t/γ and $\gamma e^{2\sigma}$. Thus, if skin storativity is negligible, the finite-thickness skin solution should also be a function of two dimensionless groups: t^*/γ^* ($=t/\gamma$) and $\gamma^* e^{2\sigma^*}$.

Figure (2) shows type curves generated from the finite-thickness skin solution using realistic skin and aquifer parameters. A value of $\delta=1$ is used because it is considered unlikely that the porosity and rock compressiblity in the altered region will differ significantly from that of the aquifer material. Values of $\gamma^* e^{2\sigma^*}$ are shown in the inset. Using the definitions (3)-(5), we also generated type curves from the infinitesimally thin skin solution, equation (1). When compared with one another, the two sets of type curves are found to be identical. For these parameters the same relationship was found to hold for all values of γ equal to or larger than 10^2. This suggests that for open-well slug tests, the skin storativity may be neglected, and the skin thickness may be considered as infinitesimally thin.

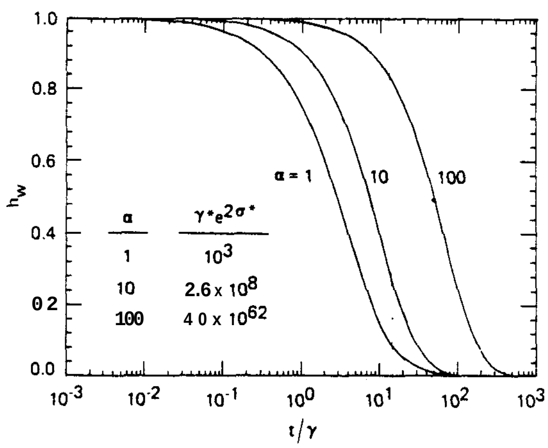

Figure 2. Type curves for an open-well slug test for $\gamma=10^3$, $r_s=2$, $\delta=1$, and the indicated values of α. Inset shows the corresponding values of $\gamma^* e^{2\sigma^*}$.

For open-well tests, in order to show a deviation from the solution for a skin of infinitesimal thickness, it is necessary to use a very large value of skin storativity. Figure 3 shows a comparison of type curves similar to those in Figure 2 but with a value of skin storativity that is unreasonably large. At early time. the finitethickness skin solution shows a marked departure from the type curves for a skin of infinitesimal thickness. With an increase in r_s it was found that the departure is even greater. Thus if δ and/or r_s are very large, the equivalence between the solutions may not hold.

From the definition of the dimensionless quanitiies t and γ, we find that

$$t/\gamma = 2\pi K_2 L t'/C \qquad (6)$$

Thus with known C, a match of slug test data to an appropriate type curve should yield an estimate of the transmissivity (K_2L) of the test interval. For large values of α, however, there is nonuniqueness in the match and an accurate estimate of transmissivity cannot be obtained.

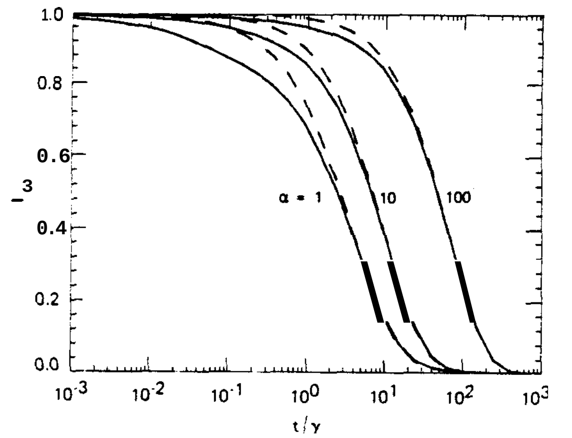

Figure 3. Type curves for an open-well slug-test for $\gamma=10^3$, $r_s=2$, $\delta=100$ and the indicated values of α.

Pressurized Tests

An increase of complexity is encountered in analyzing a pressurized slug test conducted in a well surrounded by a finite-thickness skin. Because well-bore storage is small, the effect of the well-bore skin is enhanced. In general, the storativity of the skin cannot be neglected. As a result, the head in the well may respond to skin properties at early time and to aquifer properties at a later time. This response is markedly different from the standard type curves, and data analysis by curve matching can be extremely difficult.

Because the response to pressurized tests may be as much as five orders of magnitude more rapid than the response to open-well tests, it is advantageous to seek the short-time asymptotic approximation to equation (2). This is given by Moench and Hsieh (1985a) in real space as

$$h_w = \exp(a^2t)\ \mathrm{erfc}[(a^2t)^{1/2}] \qquad (7)$$

where $a^2 = \delta/(\alpha\gamma^2)$

In equation (7) h_w is a function of a single dimensionless group, a^2t. Substituting the definitions of the dimensionless quantities, we find

$$a^2t = 4\pi^2(r_w')^2K_1S_{s1}L^2t'/C^2 \qquad (8)$$

We expect that for very small γ, the asymptotic solution (7) will accurately approximate the head response of a pressurized slug test. Thus, if skin is present and γ is sufficiently small, a match of slug-test data with the solution (7) will yield only the product K_1S_{s1}, and no information about the aquifer hydraulic properties will be obtained. In the absence of skin ($\delta = 1$ and $a = 1$) equation (7) becomes identical to the short-time solution of Bredehoeft and Papadopulos (1980).

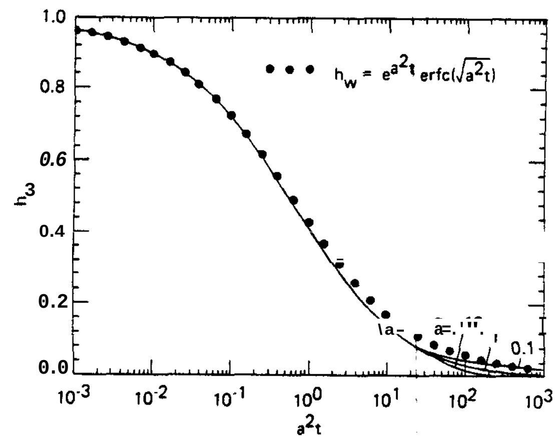

Figure 4. Type curves for a pressurized slug test for $\gamma=10^{-1}$, $r_s=2$, $\delta=1$, and the indicated values of a. The asymptotic solution for short time is also shown for comparison.

Figure 4 shows type curves for pressurized slug tests with $\gamma = 10^{-1}$. The short-time solution given by (7) is also shown for comparison. Note that the short-time solution accurately describes the type curves over a large period of the head response. If γ is decreased to 10^{-2} or less, the type curves will be indistinguishable from the short-time solution.

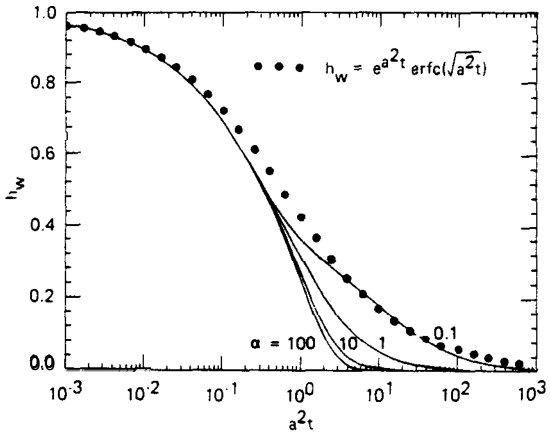

Figure 5. Type curves for a pressurized slug test for $\gamma=1$, $r_s=2$, $\delta=1$, and the indicated values of a. The asymptotic solution for short time is also shown for comparison.

Figure 5 shows that the aquifer properties begin to affect the shapes of the type curves as γ is increased. For a tenfold increase in γ, the type curves show dramatic differences from those of Figure 4. Initially, the type curves follow the short-time solution, indicating that the head response in the well is primarily controlled by the hydraulic properties in the immediate vicinity of the well. As time increases, the type curves deviate from the short-time solution as well as from one another. For a > 1, the type curves fall below the $a = 1$ (no skin) curve, indicating a faster head response due to the higher hydraulic conductivity in the aquifer material away from the well. For a < 1, the type curve deviates above the $a = 1$ curve, indicating a slow down in head response due to the lower hydraulic conductivity in the aquifer. If γ is increased by another ten or a hundred fold, the storative effect of the skin at early time will be entirely obliterated, and we approach the case of an open-well test.

Figure 6 shows the effect of changing the skin radius, r_s, while keeping the value of γ at 10^{-1}. These curves show similar behavior as those in Figure 5. The explanation for this behavior is simple. As r_s is decreased, the effect of the skin storativlty is also diminshed, and the type curves are controlled more and more by the hydraulic properties of the aquifer.

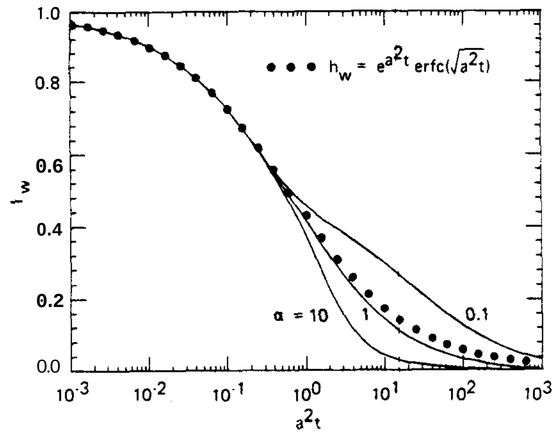

Figure 6. Type curves for a pressurized slug test for $\gamma=10^{-1}$, $r_s=1.1$, $\delta=1$, and the indicated values of a. The asymptotic solution for short time is also shown for comparison.

DISCUSSION

It appears that for realistic values of skin and aquifer parameters the conventional theory of Ramey and Agarwal (1972) and Ramey et al. (1975) for open-well test8 is adequate. The reason for this is that the relatively large volume of water involved in open-well tests tends to overwhelm the storative effects of the finite-thickness skin. Thus, data analysis may yield reasonable estimates of the transmissivity of the test interval. For pressurized tests, on the other hand, the well response may show significant departure from convehtional theory. This is due to the fact that the very small volume of water involved in the slug test can be easily dissipated by the storative effects of the finite-thickness skin. Consequently, pressurized test data may yield only the product of hydraulic cohductivity and specific storage of the skin.

Because of the large number of parameters involved it may be difficult to separate effects of the skin from those of the aquifer. There is a trade-off between the time available to perform the test and the volume of material investigated. We recommend that open-well tests be performed whenever time permits. We also recommend, particularly when performing pressurized tests, that the flexibility in varying γ be exploited and that two or more tests be carried out on a given test interval using different values of γ. In this manner it may be possible to unravel effects of skin parameters from aquifer parameters.

NOTATION

b	aquifer thickness, [L]
C	well-bore storage coefficient, $[L^2]$ $C = \pi(r_c')^2$ for open well $C = V_w \rho_w g C_{sys}$ for pressurized well
C_{sys}	compressibility of the combined fluid-well system, $[LT^2M^{-1}]$
g	acceleration of gravity, $[LT^{-2}]$
h	dimensionless head in skin or aquifer
h'	hydraulic head in skin or aquifer, [L]
h_w	dimensionless head in well
h_w'	hydraulic head in well, [L]
h_{wo}'	initial hydraulic head in well, [L]
$I_0(x)$	modified Bessel function of the first kind and order zero
$I_1(x)$	modified Bessel function of the first kind and order one
$K_0(x)$	modified Bessel function of the second kind and order zero
$K_1(x)$	modified Bessel function of the second kind and order one
K	hydraulic conductivity, $[LT^{-1}]$
L	thickness of test interval, [L]
p	Laplace transform variable
r	dimensionless radial distance from center of well
r'	radial distance from center of well, [L]
r_c'	radius of riser pipe, [L]
r_w'	well radius, [L]
r_s'	distance from center of well to outer boundary of skin, [L]
S_s	specific storage, $[L^{-1}]$
t	dimensionless time
t'	time, [T]
V_w	volume of fluid in a pressurized well, $[L^3]$
ρ_w	density of fluid in the well, $[ML^{-3}]$
x	argument of the Bessel functions
σ	skin factor for skin of infinitesimal thickness

Subscripts:

1	skin
2	aquifer

Dimensionless groupings:

$$q = p^{1/2}$$
$$\beta = (\delta\alpha)^{1/2}$$
$$r_s = r_s'/r_w'$$
$$\alpha = K_2/K_1$$
$$\delta = S_{s1}/S_{s2}$$
$$\gamma = \frac{C}{2\pi(r_w')^2 S_{s2} L}$$
$$t = \frac{K_2 t'}{(r_w')^2 S_{s2}}$$

REFERENCES

Bredehoeft, J.D., and S.S. Papadopulos, 1980, A method for determining the hydraulic properties of tight formations, Water Resour. ~~Res., 16~~(1), 233-238.

Cooper, H.H., J.D. Bredehoeft, and I.S. Papadopulos, 1967, Response of a finite-diameter well to an instantaneous charge of water, Water Resour. Res.. 3(1), 263-269.

Moench, A. F.. and P.A. Hsieh, 1985a, Analysis of slug test data in a well with finite-thickness skin. Proceedings of the IAH 17th International Congress on the Hydrogeology of Rocks of Low Permeability, Jan. 7-12, 1985, Tucson, AZ.

Moench, A. F. and P.A. Hsieh, 1985b, Comment on "Evaluation of Slug Tests in Wells Containing a Finite-Thickness Skin" by C. R. Faust and J. W. Mercer, Water **Resour.** Res (in **press).**

Ramey, H.J., Jr., and R.C. Agarwal, 1972, Annulus unloading rates as influenced by wellbore storage and skin effect, Trans. Soc. ~~Pet. Eng. AIME, 253~~, 453-462.

Ramey, H.J., Jr., R.C. Agarwal, and I. Martin, 1975, Analysis of 'slug test' or DST flow period data, J. Can. Petrol. Technol., 37-47.

Stehfest, H., 1970, Numerical inversion of Laplace transforms, Commun. ACM, 13(1), 47-49.