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THERMODYNAMIC BEHAVIOUR OF SIMPLIFIED GEOTHERMAL RESERVOIRS

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ABSTRACT

Starting fmn the basic laws of conservation of mass and energy, the differential equations that represent the thermodynamic behaviour of a simplified geothermal reservoir are derived. Its application is limited to a reservoir of high permeability as it usually occurs in the central zone of a geothermal field. A very practical mthod to solve numerically the equations is presented, based on the direct use of the steam tables. The method, based in one general equation, is extended and ilustrated with a numerical example to the case of segregated mass extraction, variable influx and heat exchange between rock and fluid. As it is explained, the method can be easily coupled to several influx models already developed somewhere else. The proposed model can become an important tool to solve practical problems, where like in Los Azufres México, the geothermal field can be divided in an inner part where flashing occurs and an exterior field where storage of water plays the main role.

SYMBOLS AND UNITS

- C_r Specific heat of the rock (Kj/Kg°C)
- h specific enthalpy (Kj/Kg)
- $\dot{m}$ Mass flaw rate extracted (Kg/s)
- M Mass of fluid in the reservoir (Kg)
- M_r Mass of the rock matrix (Kg)
- P Pressure (bar)
- q Mass flow rate influx (Kg/s)
- t Time (s)
- T Temperatura (°C)
- u specific internal energy (Kj/Kg)
- v Specific volume (m^3/Kg)
- V Velocity (m/s)
- $\forall$ Volume of the reservoir (m^3)
- x Steam quality
- ϕ Effective porosity
- ρ Density (Kg/m^3)

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- g Gas
- f Fluid
- fg Gas-Fluid
- w water
- e Extraction
- r Recharge
- ()' at next step

INTRODUCTION

In the study of geothermal reservoirs, it has been a very successful approach to divide the field in a near field, or reservbir itself, where most of the thermodynamic changes take place (such as flashing, heat transfer) and a far field that is mst responsible for the recharge of fluid to the reservoir. In general, in the far field only water flows, although, some heat transfer might takes place.

Some authors (1) have divided very drastically this two zones. In the central one it is assumed that permeability is high enough to consider it as a reservoir where there exist constant properties in the horizontal plane. In the vertical mst assume an homogeneous fluid even in the saturation region. For the far field several influx models are used (2) (3) some of them coupled to the pressure term of the inner reservoir.

To make the procedure more practical, mst of the time the equations are simplified making analogies to petroleum engineering technology or in some cases using polynomials to represent several pieces of the curve of thermodynamic behaviour of water and steam (4). Most of these equations are derived by fitting curves to the points taken from the steam tables.

In this work it is only presented the case of a reservoir in two phase, where water and steam are present in saturation form. The intention is to develop, step by step, the thermodynamic equations that control the process in this inner part of the reservoir where the permeability is quite high. The presentation starts with the mst simple case, that is:

a) Constant mass flow rate extraction from a reservoir where the two phase fluid is homogenously distributed in all directions. No heat transfer from the rock to the fluid, nor recharge is considered.

And then goes to the more complicated but mre realistic case:

b) As in a) but now the rock-fluid system is assumed to be in equilibrium by accepting heat transfer fran the rock to the fluid during extraction.

an important step that makes the model more practical is:

c) In the model, one my allow segregation of the two phases making possible to extract fran the top dry steam at saturation conditions, representing shallow wells as it happens in Los Azufres [5] or only fran the bottom, pure water also at saturation conditions.

To make the model more realistic, recharge of hot water is considered. This term is usually very difficult to determine in practice in volcanic reservoirs, and on the other hand, has a very important influence on the results of pressure depletion:

d) Recharge or influx is accepted at constant flow rate or at any other variable flow rate as it happens in pressure dependent models. The temperature of this water can also change with time.

The thermodynamic behaviour is presented as a set of partial differential. equations. To make then practical to the user, the approach @re presented consists in changing the independent variables to the most used in ———mics when dealing with saturated mixture, ~.e., pressure (p) and steam quality (x). To solve the problem by finite differences, the thermodynamic properties are obtained fran the steam tables. This can be done by directly reading off the values fran the table or (as it was done here) by forming a storage in the computer with the values of the tables around the values one will use. This might look as a very time and memory consumming process. In fact it is not, only several lines with pressure, temperature, internal energy and specific volume on each are necessary. Intermediate values are obtained by linear interpolation as one usually does it with the steam tables. Since the pressure is expressed in integers, it is easy to use it as an index in the (T), (u) and (v) term in the computer.

Having explained in the text the deduction of the equations, the physical assumptions for every case, the numerical aproximation to solve the differential equations and the way to use the steam tables as a tabulated storage in the computer, the method is then applied to a cubic reservoir having 1 Km in each side.

Finally several extraction policies, well depth, recharge flow rate and influx water temperature are analyzed and presented in graphic fran.

ZERO DIMENSION MODEL

Consider a constant volume of reservoir (V), as shown in Fig. 1, with effective porosity (ϕ) containing a mass of fluid (M). If mass is extracted at a rate ($\mathring{m}$), by conservation of mass one has

$$\mathring{m} = - \frac{dM}{dt} \quad (1)$$

since

$$M = \frac{V\phi}{v} \quad (2)$$

$$\frac{dM}{dt} = - \frac{M}{v}\frac{dv}{dt} \quad (3)$$

Replacing (2) and (3) in (1), one obtains

$$\mathring{m} = \frac{M}{v}\frac{dv}{dt} \quad (4)$$

applying now the law of conservation of energy, that in this case becomes the first law of thermodynamics since work and energy are involved, one obtains.

$$\mathring{m}\left(h + \frac{V^2}{2}\right)\Big|_{in} - \mathring{m}\left(h + \frac{V^2}{2}\right)\Big|_{out} = \frac{dMu}{dt}$$

The left hand side is the net ballance of energy flawing in and flowing out. The enthalpy therm takes care of the internal energy of the flowing fluid and the flow work necesary to push it in or pull it out. In the right hand side, the process was considered adiabatic (no heat is flowing through the boundaries of the volume of Fig. 1) and no shaft work is being done inside the reservoir. The rate at wich stored energy changes inside, is the time derivative of the mass of fluid and its specific internal energy, since changes in kinetic and potential energy inside the reservoir are negligible.

a) No heat transfer and no influx

For the case of the reservoir to Fig. 1, without recharge (zero influx), one obtains

$$\mathring{m} \quad \frac{dMu}{dt} \quad (5)$$

Since by definition h = u + pv, replacing and expanding the previous equation, it becomes

$$- \mathring{m}h = (h-pv)\frac{dM}{dt} + M\frac{dh}{dt} - Mp\frac{dv}{dt} - Mv\frac{dp}{dt} \quad (6)$$

replacing $\frac{dM}{dt}$ fran (1) and $Mp\frac{dv}{dt}$ from (4) the whole equation (6) reduces to

$$\boxed{dh = v\,dp} \quad (7)$$

The main point in arriving to an expression like equation (7) is to have the pressure (p) as the independent variable in order to use the steam-tables directly, entering with (p).

To solve equation (7) numerically, by means of finite differences one can use

$$h' = h + v\Delta p \quad (8)$$

b) Considering heat transfer from the rock to the fluid.

If mass is extracted fran the confined reservoir, with no recharge, the pressure (p) will decrease, and since we are considering the case of fluid at saturation condition, the temperature of the fluid (T) will also go down. In this case heat is transfered frcm the rock to the fluid.

The condition we impose is that rock and fluid remain in equilibrium during the process, that is, both are allways at the same temperature.

Equation (5) becomes in this case

$$-\mathring{m}\, h = \frac{dMu}{dt} + \frac{dMr\, ur}{dt} \qquad (9)$$

The last term takes care of the change in stored energy (ur) in the rock matrix (Mr), and can be expressed as

$$Mr \frac{dur}{dt} = MrCr \frac{dT}{dt} \qquad (10)$$

Where Cr is the specific heat of the rock. Since the temperature (T) is related directly to pressure (p) at saturation conditions

$$Mr \frac{dur}{dt} = MrCr \frac{\partial T}{\partial p}\Big|_{sat} \frac{dp}{dt} \qquad (11)$$

Recognizing that equation (9) and (5) are the same, except for the addition of a stored heat tenn like eq. (11), one can go directly to equation (6) and obtain for this case.

$$-\mathring{m}\, h = (h-pv)\frac{dM}{dt} + M\frac{dh}{dt} - Mp\frac{dv}{dt} - Mv\frac{dp}{dt} + MrCr \frac{\partial T}{\partial p}\Big|_{sat} \frac{dp}{dt} \qquad (12)$$

and following the same procedure as before, arrive to

$$dh = \left(1 + \frac{Mr}{M} Cr \frac{\partial T}{\partial p}\Big|_{sat}\right) v\, dp \qquad (13)$$

Wich is identical to equation (7) except for the stored heat term.

c) Considering recharge, or influx.

The conservation of mass will be in this case

$$q - \mathring{m} = \frac{dM}{dt} \qquad (14)$$

Where (q) is the influx mass flow rate.

The first law of thermodynamic will be

$$q\, hr - \mathring{m}\, h = \frac{dMu}{dt} \qquad (15)$$

after going through a similar procedure to the one used to go from equation (1) and (5) to equation (7), one obtains

$$dh = v\, dp + \left[\frac{hr}{M} q\right] dt \qquad (16)$$

In this equation (dt) appears complicating the numerical solution. One can change variables but will allways end with (p) and some other independent variable, essentially needed to define the saturated fluid.

In other words, this case depend$ on two independent variables, and can not be solved as easily as cases a) and b). So, in the next section a different approach will be tried.

CASE IN ZERO DIMENSION

In the previous section, three equations where deduced for three particular ca$es. In this one, a general approach is presented in order to be able to solve the following cases by using just one general equation:

a) Heat transfer fran the rock to the fluid.
b) Recharge to the reservoir, or influx, by allowing a mass flow rate (q) at an enthalpy (hr). Both of them (q) and (hr), can be a function of the pre$sure difference inside the reservoir and the far field.
c) Mass extraction frcm the re$ervoir can be constant or variable.
d) The enthalpy (he) of the extracted fluid can be the one corresponding to an homogeneous fluid (Fig. 2), or in the case where segregation of steam and water at saturation conditions occur$ (Fig. 3) the one corresponding to steam when extracting frcm the upper with shallow wells, or, to water when extracting fram the bottom with deep wells.
e) With an appropriate selection of the mass extraction, one can prcduce a constant electric power at the surface of the field, at a given separation pressure.
f) A mobility term can be included to withdraw a proportion of steam to water different than homogeneous fluid.

After presenting the equations, the independent variables are changed into saturation pressure (p) and steam quatility (x), by using the chain rule for differentiation with more than one variable. This is because those are the two independent variables when using steam tables in their usual format.

The main assumptions made in this model are that inside this central core of the reservoir, the high permeability is enough to permit the use of a zero dimension model, and that the fluid is in saturation condition. Of course if the fluid is subcooled water the model can be adapted with minor changes but this is out of the scope of this work. Also the reservoir does not receive heat fran the surroundings other than the one coming in with the influx water.

a) Conservation of mass

$$q - \mathring{m} = \frac{dM}{dt} \qquad (17)$$

Since

$$M = \frac{V\,\phi}{v}$$

$$q - \overset{\circ}{m} = -M \frac{1}{v} \frac{dv}{dt} \qquad (18)$$

and

$$dt = -\frac{M}{q - \overset{\circ}{m}} \frac{1}{v} \, dv \qquad (19)$$

b) First law of thermodynamics

$$q \, h_r - \overset{\circ}{m} \, h_e = \frac{dMu}{dt} + M_r C_r \frac{dT}{dt} \qquad (20)$$

rearranging the previous equation in a very similar way as the one used to obtain equation (6), (7) and (11) one obtains

$$\alpha \, dv = du + \beta \, dp \qquad (21)$$

being

$$\alpha = \frac{1}{v} \left[\frac{qhr - \overset{\circ}{m}he}{q - \overset{\circ}{m}} + u \right] \qquad (22)$$

and

$$\beta = \frac{1-\phi}{\phi} \, v \, \rho r \left. \frac{\partial T}{\partial p} \right|_{sat} \qquad (23)$$

by making use of the *chain* rule, (dv) and (du) can be transformed in

$$du = \left(\frac{\partial uf}{\partial p} + x \frac{\partial ufg}{\partial p}\right) dp + u_{fg} dx$$

$$dv = \left(\frac{\partial vf}{\partial p} + x \frac{\partial vfg}{\partial p}\right) dp + v_{fg} dx$$

Finally, equation (21) becomes

$$dx = -\frac{\alpha\left[\frac{\partial vf}{\partial p} + x \frac{\partial vfg}{\partial p}\right] + \left[\frac{\partial uf}{\partial p} + x \frac{\partial ufg}{\partial p}\right] + \beta}{u_{fg} + \alpha \, v_{fg}} \, dp \qquad (24)$$

where α and β are defined in equations (22) and (23). This is the most general equation for the inner reservoir with recharge and heat transfer.

METHOD OF SOLUTION

Starting from a given state of the *reservoir* where one knows all the properties of the fluid, the pressure is decreased Δp (usually one bar, or any other integer, in order to find all the corresponding values in the codified steam tables, directly without interpolation).

A new steam quality is found by using equation (24) and

$$x' = x + \Delta x$$

The other state variables and parameters are found as shown below, in the same *order*

$$p' = p + \Delta p$$

$$v' = v'_f + x' \, v'_{fg}$$

$$u' = u'_f + x' \, u'_{fg}$$

$$h' = u' + p'v'$$

$$M' = V \, \phi / v'$$

$$\Delta t = (M - M')/(\overset{\circ}{m} - q)$$

$$t' = t + \Delta t$$

Knowing now all the values at $p + \Delta p$, the value of $\overset{\circ}{m}$ and q can be redifined or recalculated if necessary, depending on the problem. Then, all the procedure is repeated starting from equation (24). It is important to realize that α and β defined in equations (22) and (23), must be recalculated at every step in (p), since they are not constants.

The proposed method has a big advantage because it uses the steam tables as already tabulated, in a very simple and short way. But more important is that by making the pressure (p) the independent variable, one can always use the same Δp increment regardless of the mass flow rate extracted or recharged. The time increment will be a result that can be said that it adjusts itself to the process, being small at the beginning and large when the quasi stady state is reached.

Example

The best way to show the advantages of using the described procedure is to present a numerical example. Then, to use it to examine the effect of applying different explotation policies to the hypotetical field.

Let the following be the characteristics of a reservoir.

$V = 1 \times 1 \times 1$ Km = 109 m^3

$\phi = 0.2$

$\rho_r = 2550$ (Kg/m^3)

Cr = 1.4 (Kj/Kg°C)

The initial conditions are:

p = 86 bar

T = saturation

x = 0.5

Operating regime

$\overset{\circ}{m}$ = the necessary to produce 10 MW at the surface (i.e. it changes every Δp)

he = in some cases h or hf or hg.

q = 0.2, 0.5 or 0.8 times $\overset{\circ}{m}$

hr = 640 and 84 (Kj/Kg)

Integration Step Δp = 1 bar.

DISCUSSION OF RESULTS

The results of the previous problem are shown in Figs. 4 to 9. All of then corresponds to a mass extraction (ṁ) sufficient enough to produce 10 MW at 10 bars of pressure separation at the surface.

In Fig. 4, with no recharge, it is clear that the reservoir expires much faster when the fluid is extracted fran the lower part (sawted water).

The effect of heat stored is important. Fig. 5 represent the importance of the recharge to the reservoir, with an influx mass rate 80% of the mass extraction, the reservoir can last 6 times longer than 0% recharge. 'he same effect is shown in Fig. 6 for an extraction fran the lower part of the reservoir (i.e., water).

So far the example has been worked starting with a reservoir having half steam half water (x = 0.5), in Fig. 7 it is shown the effect of initial steam quality extracting from the bottom. Notice that there exists a "worst" condition when the shortest life of the reservoir is obtained (extracting fran the lower part).

The effect of the temperature of the water recharging the reservoir is not important as indicated in Fig. 8.

Finally Fig. 9 represents the evolution of the entalphy. Notice that when extracting pure steam from the upper part of the reservoir the entalphy of the extracting fluid slowly increases and that of the reservoir (average) drastically increases. But when the extraction is done fran the lower part (i.e., pure water) the entalphy of the extracting fluid slowly decreases and that of the reservoir drastically increases. This is important when one models a reservoir using homogeneous (constant) properties in every element of the grid and try to match the average entalphy of the model to the one measured in the field on wells that extract saturated water from the lower part. The matching in this case doesnt make sense.

CONCLUSIONS

A general method has been developed to calculate the behaviour of a geothermal reservoir under the assumption of very high permeability and homogeneous distribution of properties (i.e., zero dimension).

The equations were rearranged and presented in such a form that the pressure (p) is always the independent variable, and the steam quality (x) when necessary. This allows to perform the numerical integrations with Δp increments, improving in this way the stability and convergence of the method.

To represent the equation of state of the fluid, the steam tables are directly used performing linear interpolation as one usually does it in hand calculations.

The method was applied to an hyptetical reservoir using numerical values that approaches a real geothermal field.

By changing the quality of the fluid extracted, recharge flaw rate and temperature, initial fluid enthalpy at the reservoir and rock heat transfer, several curves were presented showing the decaying of the reservoir.

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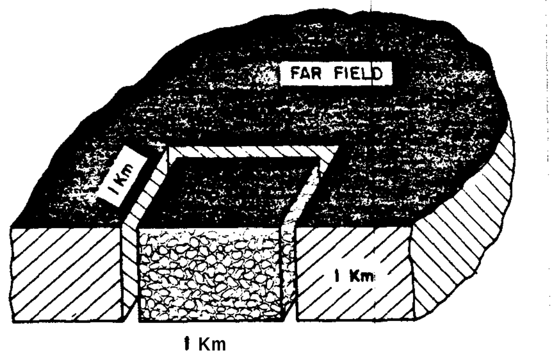

FIG. I INNER PART OF A RESERVOIR AND FAR FIELD.

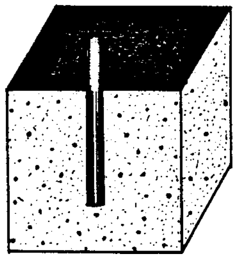

FIG. 2 HOMOGENEOUS REPRESENTATION AND EXTRACTION IN A RESERVOIR.

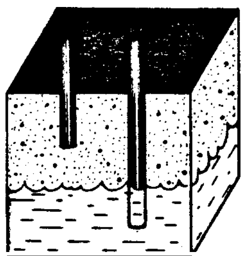

FIG. 3 SEGREGATED RESERVOIR WITH AN UPPER WELL EXTRACTING STEAM AND A DEEP WELL EXTRACTING SATURATED WATER .

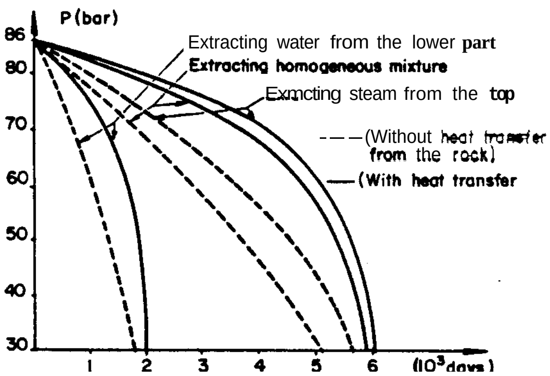

FIG. 4 PRESSURE DRAW DOWN VS TIME ($\dot{m}$ TO PRODUCE 10 MW, INITIAL CONDITION X=0.5, ZERO RECHARGE EXTRACTION FROM THE TOP, BOTTOM AND HOMOGENEOUS.

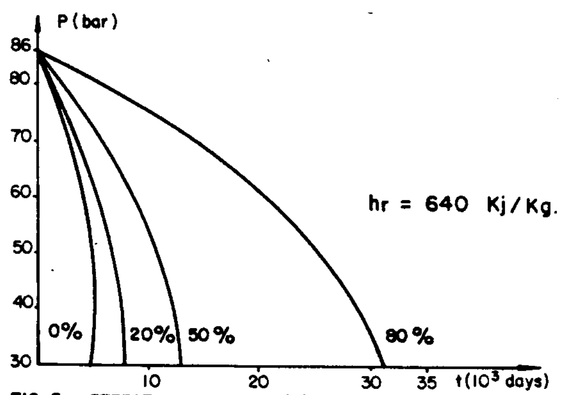

FIG. 5 EFFECT OF RECHARGE (q) EXPRESSED AS A PERCENT OF THE MASS EXTRACTION ($\dot{m}$), EXTRACTING PURE STEAM FROM THE TOP TO GENERATE 10 MW.

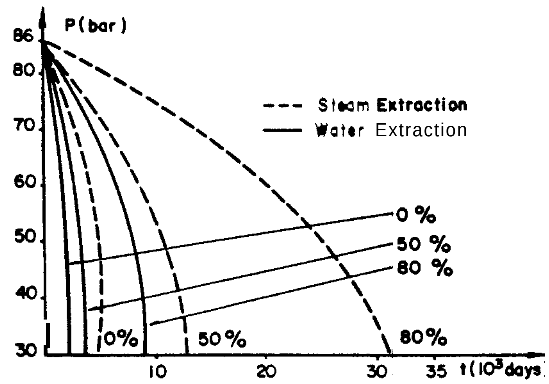

FIG. 6 EFFECT OF RECHARGE INFLUX (q) EXPRESSED AS A PERCENT OF MASS EXTRACTION ($\dot{m}$), EXTRACTING PURE WATER FROM THE LOWER PART TO GENERATE 10 MW. (PART OF FIG.5 REPRODUCED FOR REFERENCE).

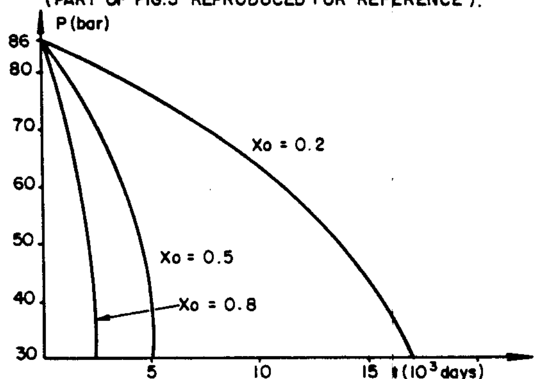

FIG. 7 PRESSURE DRAW DOWN WHEN GENERATING 10 MW, - RECHARGE INFLUX (q) IS 20% OF $\dot{m}$, EXTRACTING FROM LOWER PART, STARTING AT DIFFERENT INITIAL STEAM QUALITY (X) IN THE RESERVOIR.

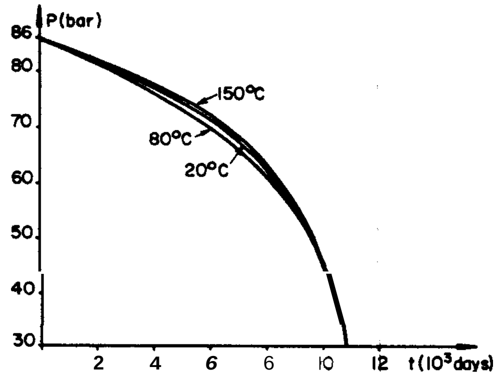

FIG. 8 PRESSURE DRAW DOWN WHEN GENERATING 10 MW, RECHARGE IS 50% OF EXTRACTION ($\dot{m}$) AT DIFFERENT TEMPERATURES OF THE RECHARGED WATER.

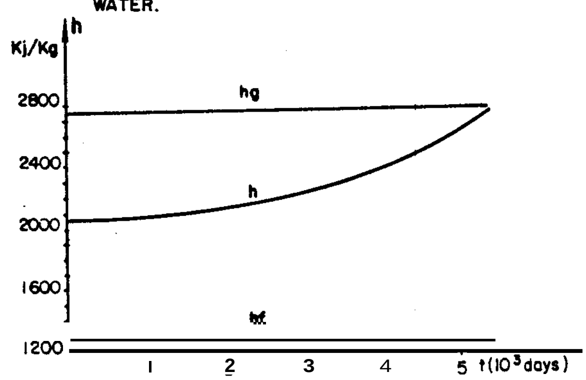

FIG. 9 VARIATION OF THE ENTHALPY OF THE STEAM, WATER AND MIXTURE DURING EXTRACTION PRODUCING 10 MW.