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A RESERVOIR ENGINEERING ANALYSIS OF A VAPOR-DOMINATED GEOTHERMAL FIELD

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INTRODUCTION

The purpose of the study is to develop a simplified model to match past performance of a vapor-dominated geothermal reservoir and to predict future production rates and ultimate reserves. The data are fictitious, but are based on real data. A lumped parameter model was developed for the reservoir that is similar to the model developed by Brigham and Neri *(1979, 1980)* for the Gabbro zone, and a deliverability model was developed to predict the life and future producing rate declines of the reservoir. This report presents the development and results of this geothermal reservoir analysis.

DESCRIPTION OF THE RESERVOIR

During the course of production from the reservoir, flow rates and pressures have declined during several periods during which the number of wells has remained approximately constant. This suggests that the reservoir is undergoing depletion. It is reasonable to assume that there exists a boiling water zone deep in the reservoir. The rock matrix between this deep zone and the producing zone consists of relatively tight vertical fractures. The model presented in this report is based on this concept of a deep boiling water zone which supplies steam to a shallower producing horizon. The pressure drawdown measured in the producing zone is a combination of a pressure drop due to depletion of the boiling water and a pressure drop due to frictional flow of the steam as it rises through these vertical fractures.

PRESSURE AND PRODUCTION DATA

The fictitious pressure and production data used for this study are presented in Table 1. The value zero for the number of months corresponds to the beginning of production.

The p/Z data listed in Table 1 are average p/Z values for the entire reservoir. The Z-factor data were calculated assuming isothermal conditions (480°F) exist in the reservoir. The WT data for steam were taken from Keenan and Keyes (1969), and the resulting Z-factors are listed in Table 2. Note that for pressures above *570* psia, the Z-

Table *1*

CUMULATIVE PRODUCTION & AVERAGE p/Z
Units A-C, D, E, and F

Months	Cumulative Production (10^9 lbs.) Gross	Net	p/Z
0	0.0	0.0	707
71	31.0	31.0	706
77	33.8	33.8	705
93	44.4	44.4	704
107	57.0	57.0	698
117	66.8	66.8	696
132	84.3	80.1	695
144	105.3	98.3	686
154	131.9	120.0	672
167	166.3	148.8	660
175	189.4	167.0	643
182	211.1	184.5	626
192	244.7	209.7	598
200	272.0	231.4	585
206	291.7	248.3	579
212	311.9	262.9	574
220	336.4	281.8	568
226	356.7	297.2	561
235	389.6	321.7	546
249	441.4	360.2	533

factors were calculated by extrapolating the values at the lower pressures. They result from the fact that the data have been altered. These synthetic values of Z do not affect the validity of the concepts used.

PREVIOUS HISTORY MATCHING EFFORTS

In their study of the Gabbro Zone, Brigham and Neri(1979, *1980)* combined the standard gas material balance with an empirical power law equation to describe pressure drawdown in the producing zone. The empirical power law equation was derived to model the transient pressure behavior that existed between the top of the reservoir, where the wells are completed, and the constant pressure boiling water interface deep in the reservoir. We will review the development of Chis empirical equation because of its importance to this model.

Table *2*

REAL GAS COMPRESSIBILITY FACTORS
FOR STEAM AT 480°F

Pressure (psia	Z	Pressure (psis	Z
620	0.8038	460	0.8666
600	0.8124	440	0.8736
580	0.8207	420	0.8805
560	0.8289	400	0.8872
540	0.8368	380	0.8938
520	0.8446	360	0.9003
500	0.8521	340	0.9067
480	0.8594		

To derive an equation for the pressure drop from the deep boiling zone through the fractured zone to the producing horizon, we can envision that the flow geometry is approximately linear. This is transient flow, and therefore the magnitude of the pressure drop will depend on the terms in the p_D function for linear flow, and the timing of the pressure transient will depend on the terms in the t_D function. Analytical solutions for such problems have been published by Miller *(1962)* and by Nabor and Barham *(1964).* Nabor and Barham's solutions are summarized in Fig. 1, where their term $F(t_D)$ is the p_D function for linear flow at a constant rate.

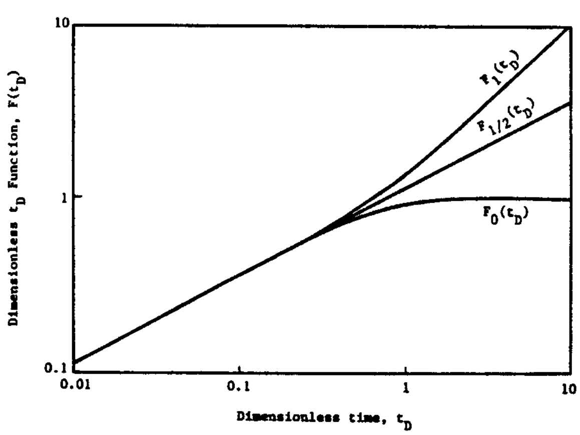

Figure *1.* Dimensionless pressure change and efflux functions, linear aquifers. (After Nabor and Barham)

In Fig. 1, the system that most closely approximates a boiling water interface is the constant pressure outer boundary, represented by the $F_o(t_D)$ curve. This is marked more heavily in Fig. *1.* This curve also presumes an inner boundary condition of constant flow rate. For the actual variable flow rate, it is necessary to use superposition to calculate the transient pressure drop. A discussion of the use of superposition is in Brigham and Neri *(1980).* Only a brief description follows.

Let us study the $F_o(t_D)$ curve in Fig. 1 in detail. A good approximation to this curve is to assume that p_D is proportional to the square root of time until $t_D = 0.785$ and to assume it is a constant equal to 1.00 after $t_D = 0.785$. The maximum error using the approximation is only about 10%. In most real systems, we do not know the parameters in t_D well enough to be able to relate the real time to t_D; however, we can assume a value for the real time that is equivalent to $t_D = 0.785$ and observe how this affects Eq. 1. In this report, we will refer to this time as the "lag time." This phrase was chosen for it is meant to imply the time required to reach effective steady-state flow. If, for example, we were to use incremental times of *10,* 10, *5* and *10* months, and a lag time of *30* months, by superposition the equivalent steady state follow rate would be:

$$q_{eq} = q_1 + \frac{1}{\sqrt{30}}\left[(q_2 - q_1)\sqrt{25} + (q_3 - q_2)\sqrt{15} + (q_4 - q_3)\sqrt{10}\,\right] \quad (1)$$

Equation 1 gives us a basis for a general formulation for calculating the equivalent flow rate as a function of the Lag time. Notice that any time longer ago than the lag time does not affect the equivalent flow rate.

Because the transient propertiea of the reservoir are not known, the lag time is not known. Thus, it is necessary td calculate a least-squares fit assuming various lag times, and then choose the lag time which gives the best fit to the data. The calculated equivalent flow rates were based on the gross steam rate from the reservoir, calculated from the data in Table 1. These flow rates are listed in Table *3* for a lag time of 30 months. Other lag times were also used *(40, 50* months) with similar results; but only the 30 month data are shown here.

When we combine the concept of peservoir depletion in a deep boiling zone with the concept *of* linear flow from that zone to the producing horizon, the reservoir depletion model can be written in the following form:

$$(p/Z)_{top} = (p/Z)_{deep} - \Delta(p/Z)_{flow} \quad (2)$$

where:

$(p/Z)_{top}$ = the p/Z seen at the producing zone; it is less than the value of p/Z within the deep boiling interval due to linear flow from the deep zone to the producing zone.

$(p/Z)_{deep}$ = the value of p/Z at the deep boiling zone; this value drops as the zone depletes.

$\Delta(p/Z)_{flow}$ = the drop in p/Z due to steam flow from the deep zone to the upper producing interval.

The problem now is to define the changes in p/Z as a function of the volume produced and the producing rate. First, let us consider $(p/Z)_{deep}$. The work by Brigham and Morrow (1977) shows that the value of p/Z in a boiling system is nearly linear with cumulative production, at least for the first 1/3 to 1/2 of the total depletion history. Because some of the condensed water is reinjected in this reservoir and clearly shovs signs of evaporation, it seems proper to use only the net cumulative production for this depletion term (Gp_{net}). With this type of model, the equation fs:

$$(p/Z)_{deep} = A - B\ Gp_{net} \tag{3}$$

where:

A = the initial p/Z of the deep reservoir system.

B = the constant which defines the depletion rate of the reservoir; a larger B signifies a smaller reservoir.

The next problem was to determine $\Delta(p/Z)$ due to linear flow. Dee (1983) showed that an equation of the following form would be accurate to within 2.2%.

$$\Delta(p/Z)_{flow} = C \frac{(q_{eq})^{0.987}}{(p/Z)_{top}^{0.257}} \tag{4}$$

We found that units A-C, Unit D and Unit F were acting in different ways. Units D and F were causing a greater pressure drop than Units A-C. Thus Eq. (4) was used separately for each of these units and the history match is shown on the left hand side of Fig. 2. The least squares fit to the data using Eq. 3 and 4 is the following:

$$(p/Z)_{top} = 718.5 - 0.1544\ Gp_{net} - 71.2 \frac{(q_{A-C})^{0.987}}{(p/Z)_{top}^{0.257}}$$

$$- 436.7 \frac{(q_D)^{0.987}}{(p/Z)_{top}^{0.257}} - 717.4 \frac{(q_F)^{0.987}}{(p/Z)_{top}^{0.257}} \tag{5}$$

Let us now turn to prediction of future performance of the reservoir. In order to extrapolate the data, it was necessary to estimate the future reservoir production rates subsequent to 249 months. It was *also* necessary to predict whether new power plants would act like Units A-C, Unit D, or Unit F in their linear flow behavior. Table *4* summarizes these estimates:

Next, it was necessary to calculate net cumulative production and equivalent flow rates for the future based on the data from Tables 3 and 4. A reinjection rate of 25% of the gross production has been assumed for the net cumulative production figures. These projections are listed in Table 5 for a lag time of 30 months. In Table 5, the column labelled "Units A-C" includes Units A, B, C, E, and half of Unit G. The column labelled "Unit D" includes Unit D and Unit H. The column labelled "Unit F" includes Unit F and half of Unit G.

Using the data from Table 5, it is possible to project p/Z decline into the future using

Table 3

GROSS EQUIVALENT FLOW RATES
(Units A-C, E together; Units D and F separate)
Equivalent flow rate, q_{eq}, 10^9 lbs./mo.

	Gross Rate			t_{lag} = 30 mo.		
		Units			Units	
Months	A-C, E	D	F	A-C, E	D	F
71	0.46	0	0	0.46	0	0
77	0.47	0	0	0.46	0	0
93	0.66	0	0	0.60	0	0
117	0.98	0	0	0.92	0	0
132	1.17	0	0	1.11	0	0
144	1.75	0	0	1.53	0	0
154	2.66	0	0	2.19	0	0
167	2.64	0	0	2.53	0	0
175	2.58	0.31	0	2.61	0.16	0
182	2.48	0.62	0	2.55	0.37	0
192	2.61	0.75	0	2.58	0.59	0
200	2.63	0.79	0	2.60	0.71	0
206	2.43	0.74	0.10	2.52	0.74	0.046
212	2.61	0.64	0.13	2.58	0.71	0.077
220	2.38	0.55	0.13	2.47	0.64	0.11
226	2.54	0.68	0.16	2.51	0.66	0.13
235	2.76	0.68	0.22	2.64	0.66	0.18
249	2.74	0.71	0.25	2.72	0.69	0.23

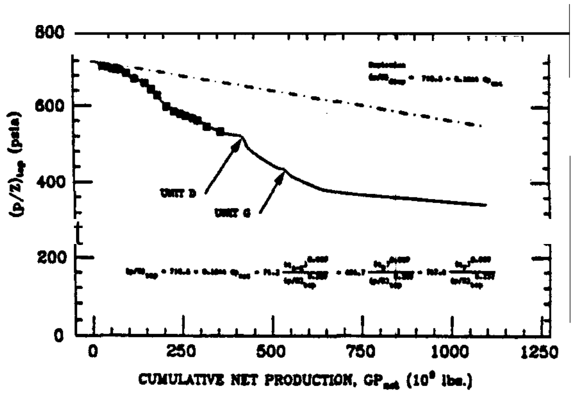

Figure 2. Projected P/Z decline (t_{lag} = 30 months)

Eq. *5.* However, such predictions do not take into account the deliverability of the reservoir.

DELIVERABILITY *AND* FUTURE PRODUCING RATES

In general, for gas flow from a reservoir, it is possible to calculate flow rate based on a version of the Forchheimer equation, known as the universal deliverability equation.

Table 4

SUMMARY OF PROJECTED NEW UNIT BEHAVIOR

Unit Letter	Starting Month	Unit Size	Gross Production Rate from Study Area (10^9 lbs./mo.)	Equivalent Unit Behavior
E	On Line	Large	0.49	A–C
F	On Line	Small	0.21	F
G	305	Small	0.35	A–C, F
H	269	Large	0.70	D

Table 5

FUTURE PRODUCTION AND CALCULATED EQUIVALENT FLOW RATES

Year	Net Cumul. Prod. (10^9 lbs.)	Units A–C; q_{eq} t_{lag} = 30	Unit D; q_{eq} t_{lag} = 30	Unit F; q_{eq} t_{lag} = 30
23.5	435.7	2.80	1.01	0.21
24.0	455.5	2.80	1.14	0.21
24.5	475.4	2.80	1.24	0.21
25.0	495.2	2.80	1.33	0.21
25.5	515.1	2.80	1.40	0.21
26.0	534.9	2.80	1.40	0.21
26.5	556.3	2.88	1.40	0.29
27.0	577.8	2.91	1.40	0.32
27.5	599.2	2.94	1.40	0.35
28.0	620.6	2.96	1.40	0.37
28.5	642.0	2.97	1.40	0.38
29.0	663.4	2.975	1.400	0.385
29.5	684.9	2.975	1.400	0.385
30.0	706.3	2.975	1.400	0.385
30.5	727.7	2.975	1.400	0.385
31.0	749.1	2.975	1.400	0.385

That equation (Dee, 1983) is:

$$\frac{\bar{p}^2 - p_{inlet}^2}{q} = \frac{(\Delta p)^2}{q} = a' + b'q \qquad (6)$$

where:

$\bar{p}$ = the average producing zone pressure (psi)

p_{inlet} = the pressure at the inlet to the power plant (psi)

q = the producing rate (Mlb./mo./well)

a' *6* b = unknown constants

The constant, a', expresses the Darcy resistance to flow in the reservoir. The constant, b', expresses the sum of non–Darcy flow in the reservoir plus flowing friction within the well and surface flow lines.

Graphing $\Delta(p)^2/q$ versus q should produce a straight line whose slope and intercept yield the desired values of the unknown constants, a' *6* b'. Various values of q and $\Delta(p)^2/q$ for specific Unit areas are listed in Table 6, while the corresponding values for all Units combined are listed in Table 7.

Some of the data from Tables 6 and 7 are shown graphically in Pigs. 3 and 4. Fig. 3 shows the result for Unit C. The other units indicated similar results. However, when the units were combined and the average reservoir pressure was used, (Table 7) the data also fit a good straight line (Fig. *4),* even though theory Indicates that heae is no rea–son to expect this to happen. We found that the reservoir production rate could be matched with a maximum error of 6.0% using the following equation:

Table 6

DELIVERABILITY DATA

YEAR	Unit A $\bar{p}$	Unit A p_{inlet}	Unit A q	Unit A p^2/q	Unit B $\bar{p}$	Unit B p_{inlet}	Unit B q	Unit B p^2/q	Unit C $\bar{p}$	Unit C p_{inlet}	Unit C q	Unit C p^2/q	Unit D $\bar{p}$	Unit D p_{inlet}	Unit D q	Unit D p^2/q
12	482	70	1188.9	191.3	-	-	-	-	-	-	-	-	-	-	-	-
13	4S2	70	1047.6	190.3	500	105	2002.0	119.4	548	105	2140.7	135.1	-	-	-	-
14	431	70	948.6	190.7	469	105	1778.8	ll7.S	534	105	1974.2	138.9	-	-	-	-
15	407	70	854.4	188.1	437	105	1655.4	108.7	s01	105	1878.9	127.7	547	105	2620.5	110.0
16	380	70	731.9	190.6	410	105	1497.6	104.9	464	105	1626.7	123.6	501	105	2260.0	106.2
17	354	70	674.7	178.5	381	105	1369.7	97.9	435	105	1485.7	119.9	470	105	1947.5	107.8
18	336	70	600.6	179.8	356	105	1239.8	93.3	415	105	1285.2	125.4	450	MS	1666.6	114.9
19	324	70	584.1	171.3	342	105	1237.4	65.6	403	105	1297.2	116.7	430	105	1528.3	113.8
20	317	70	733.3	130.4	336	105	1139.1	89.4	394	105	1281.2	112.6	429	105	1550.1	111.6
21	-	-	-	-	-	-	-	-	-	-	-	-	414	105	1364.8	ll7.5

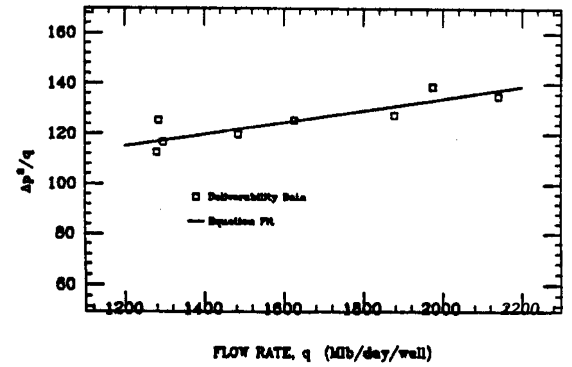

Figure 3. Deliverability analysis (Unit C)

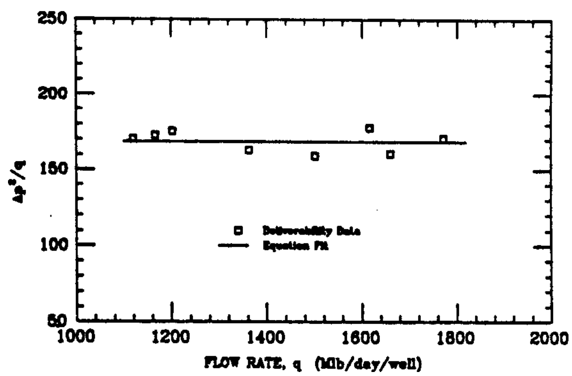

Figure 4. Deliverability analysis (all units combined)

Table 7

DELIVERABILITY DATA
(ALL UNITS COMBINED)

Year	p	p_{inlet}	q	$\Delta p^2/q$
13	560	105	1772.1	170.7
14	546	105	1616.9	177.6
15	527	105	1660.2	160.6
16	500	105	1502.0	159.1
17	483	105	1363.3	163.0
18	471	105	1203.4	175.2
19	461	105	1166.7	172.7
20	449	105	1119.7	170.2

p = (psia).
q = (Mlb/day/well).

$$\bar{p}^2 - p_{inlet}^2 = 5.54\, q \qquad (7)$$

where:
q = flow rate (Mlb./mo./well)

As Fig. 4 indicates, the non-Darcy component, b', was found to be negligible. This does not mean that the non-Darcy term is negligible for this reservoir. This is an artifact of the reservoir pressure averaging process used to fit the equation.

We can now project flow rates and pressures into the future, assuming that plant inlet pressures remain constant at 105 psi. These projections require a trial and error calculation, because both flow rate and pressure are interdependent in Eqs. 5 and 7. Rapid convergence to the answers occurred in 2 to 4 iterations. The trial and error method that we used sets both the pressure and flow rate at the new level of iteration equal to the values at the old level of iteration. Eq. 5 then produced a new value for p/Z, and Eq. 7 produced a new value for q. These new values were then used to continue the iteration in Eq. 5 until convergence was achieved.

Inherent in these projections of flow rate and pressure is the underlying assumption of future drilling. We have assumed three scenarios in our predictions: the future deliverability will equal 2.0, 2.5, and 3.0 times the current deliverability of the reservoir. However, fewer than 2.0 times the current number of wells will be needed to produce twice the current deliverability of the reservoir, because newer wells will be drilled in higher pressure areas and will therefore have better deliverability than older wells.

The flow rates were projected for 32 years through the year 55. An example of these projections is listed in Table 8, where both production rate and pressure are shown. The gross flow rate projections are graphed in Fig. 5.

An important point to notice is that there is not a significant difference between the three different assumptions of future drilling. The three curves in Pig. 5, representing 2.0, 2.5, and 3.0 times the current deliverability of the reservoir are each separated by only two to three years. This emphasizes the fact that drilling new wells can only temporarily relieve the problem of deliverability.

Projections of future p/Z decline are presented on the right hand side of Fig. 2. The solid line is $(p/Z)_{top}$ and the dashed line is $(p/Z)_{deep}$. In Fig. 2 the pressure begins to drop rapidly in the year 23 (Gp_{net} = 415.8 * 10^9 lbs./mo.) after Unit H goes on production. This is because Unit H was assumed to have the more tenuous connection with the

Table 8

PROJECTIONS OF GROSS FLOW RATE (10^9 lbs./mo.) 6 PRESSURE (psia)
(ASSUMES 25 TIMES THE CURRENT DELIVERABILITY OF THE RESERVOIR)

	tlag = 30 months		
Year	Gp_{net} (10^9 lbs.)	q	p/Z
23.0	413.8	3.71	521.5
24.0	455.5	4.41	474.0
25.0	495.2	4.41	449.0
26.0	534.9	4.41	434.6
27.0	577.8	4.76	405.7
28.0	620.6	4.76	388.8
29.0	663.4	4.76	377.2
30.0	706.1	4.71	371.0
31.0	747.9	4.61	367.1
32.0	789.0	4.53	364.2
33.0	829.4	4.46	361.3
34.0	869.2	4 39	358.4
35.0	908.3	4.32	355.5
36.0	946.8	4.25	352.7
37.0	984.7	4.18	349.8
38.0	1022.0	4.11	347.1
39.0	1058.6	4.05	344.4
40.0	1094.7	3.98	341.7
41.0	1130.2	3.92	339.0
42.0*	1165.2	3.86	336.4

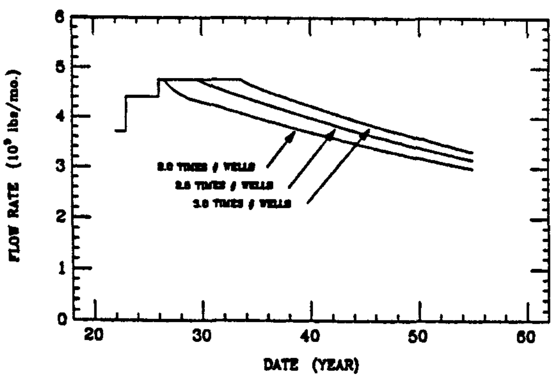

Figure 5. Projected flow rates (t_{lag} = 30 months)

deep boiling zone like Unit D. Another sudden drop in pressure can be seen, beginning in the year 26 (Gp_{net} = 534.9 x 10^9 lbs./mo.) due to production in the Unit G area. After these rapid drops, the pressure tends to level off again and almost flattens completely at about 340 psia. This flattened portion of the $(p/Z)_{top}$ curve corresponds to the period of flow rate decline seen in Fig. 5.

CONCLUSIONS

The reservoir pressure and production data used herein indicate that depletion is occurring in this reservoir. A reasonable assumption of the flaw behavior is that there exists a zone of boiling water deep in the reservoir, which supplies steam to the producing horizon where the wells are completed. The pressure drop seen at this producing zone is a combination of depletion of the boiling water and frictional flaw effects. The frictional flow drawdown is a transient pressure drop due to ffrictlonal losses as the steam rises through relatively tight vertical fractures.

Using the above concepts, we have successfully developed a lumped parameter model describing pressure drawdown in the reservoir. Depletion of the boiling water zone is assumed to fit linearly with p/Z. The transient linear vertical flow is calculated using a lag time concept to change transient flow into equivalent steady state flow. The lag time is unknown, but a lag time of 30 months has produced a reasonable fit. Various areas within the system have experienced different drawdown behavior, and therefore, the flow rates from these areas were separated from the total flow rate and were then incorporated into separate flow and pressure drop parameters.

The deliverability problem described by these example data is a reservoir problem, and a sustained flow rate can only be maintained until approximately the 30th year. However, subsequent to that time, the flaw rate decline will be gradual, in the neighborhood of two percent per year. This is quite similar to the behavior of several geothermal reservoirs.

Many people feel there is considerable "perched" and adsorbed liquid water in inaccessible areas within producing horizons of geothermal steam reservoirs. As the pressure drops, this "perched" water could boil and the resulting steam would then flow toward the highly permeable channels connected to the wells. Presumably, the flow connection between the perched water and the permeable channels is tenuous. In other words, we are describing a two-porosity system. An important point is that the reservoir model developed herein fits this physical picture equally well. The resulting equations would be identical.

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