# NONISOTHERMAL INJECTIVITY INDEX CAN INFER WELL PRODUCTIVITY AND RESERVOIR TRANSMISSIVITY

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## ABSTRACT

In geothermal wells injection tests are
commonly used to obtain well and reservoir
data. These tests are typically conducted in
a series of step rates followed or preceded
by a complete shutin. Usually the tempera-
ture of the injected fluid is different from
that of the reservoir fluid. Because of the
strong temperature dependence of fluid
viscosity and to a lesser extent, fluid
density, nonisothermally related pressure
responses must be considered. The noniso-
thermal injectivity index obtained from these
tests depends on the mobility ratio of the
cold region to the hot reservoir and the ex-
tent of the cold spot. This paper proposes a
method of estimation of the apparent viscosity
which accounts for these effects and relates
the nonisothermal injectivity index to the
isothermal injectivity index.

## INTRODUCTION

In geothermal wells injection tests are
commonly used to obtain well and reservoir
data. The temperature of the injected fluid
is invariably different from the temperature
of the in situ reservoir fluid. Since fluid
mobility, $k/\mu$, plays the major role in
pressure transient behavior, the strong
temperature dependence of fluid viscosity and
to a lesser extent, fluid density, has a large
impact on the test. The dynamic viscosity of
water changes by an order of magnitude between
20 °C and 300 °C, the major changes ocurring
between 20 °C and 100 °C. However, the de-
crease in fluid density from 20 °C to 300 °C
is approximately 30%. To interpret such tests
the nonisothermally related pressure transient
must be considered.

Several authors have discussed the interpret-
ation of pressure transients during cold water
injection into hot reservoirs. In particular,
Bodvarsson and Tsang (1980) and Mangold et.
al. (1981) considered the behavior of noniso-
thermal pressure transients in geothermal

reservoirs and illustrated the effect of the
temperature dependent fluid properties,
viscosity and density. Tsang and Tsang (1978)
developed a semi-analytic solution for
pressure transients during cold water injection
tests. O'Sullivan and Pruess (1980) and Garg
(1980) discussed the analysis of injection and
falloff tests in two-phase geothermal
reservoirs. These studies demonstrated that
the pressure transients during injection tests
can be used to determine the transmissivity of
the reservoir. In recent studies by Benson
and Bodvarsson (1982) and Benson (1982) methods
for calculating the skin factor and reservoir
properties from nonisothermal injection tests
were developed and the conditions in which the
pressure transients behaved like a composite
reservoir or moving front dominated problem
for both single- and step-rate tests discussed.
In the present study their work is extended by
relating the nonisothermal injectivity to the
reservoir transmissivity.

## FORMULATION OF THE PROBLEM

Injection of cold water is frequently used as
a testing procedure for geothermal wells, as
for example in completion tests at the end of
drilling. Injection tests are typically con-
ducted in a series of step rates followed or
preceded by a complete shutin. Since the
duration of each step is seldom very long,
usually few hours, stabilized time is usually
not reached. The observed pressure is there-
fore transient and the injectivity so obtained
only an approximation.

Now consider the system illustrated in Figure 1.
It is that of a single well at the axis of a
radial system of two concentric regions which
can have different properties. Defining non-
dimensional time, radius and pressure as:

$$t_{DR} = \frac{k_1 t}{\phi_1 \mu_1 c_1 R^2} \qquad \ldots (1)$$

$$r_D = \frac{r}{R} \qquad \ldots (2)$$

References and illustrations at end of paper.

$$P_{D1} = \frac{2\pi k_1 h\rho_1}{q\mu_1} (P_i - P_1) \; ; \; 0 < r_D < 1 \quad (3)$$

$$P_{D2} = \frac{2\pi k_2 h\rho_2}{q\mu_2} (P_i - P_2) \; ; \; 1 < r_D \quad (4)$$

Then the flow equations in the two regions are:

$$\frac{\partial^2 P_{D1}}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial P_{D1}}{\partial r_D} = \frac{\partial P_{D1}}{\partial t_{DR}} \quad \ldots (5)$$

and

$$\frac{\partial^2 P_{D2}}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial P_{D2}}{\partial r_D} = \frac{1}{\gamma}\frac{\partial P_{D2}}{\partial t_{DR}} \quad \ldots (6)$$

where

$$\gamma = \frac{\{k/\phi\mu c\}_2}{\{k/\phi\mu c\}_1} \quad \ldots (7)$$

Initial conditions are $P_1$ and $P_2$ are $P_i$ everywhere so:

$$P_{D1}(r_D,0) = P_{D2}(r_D,0) = 0 \quad \ldots (8)$$

There are three boundary conditions to consider; the inner boundary, the discontinuity and the outer boundary. At the inner boundary the condition is:

$$\lim_{r_D \to 0} r_D \frac{\partial P_{D1}}{\partial r_D} = -1 \quad \ldots (9)$$

At the discontinuity, the flux and pressure must be continuous so:

$$P_{D1} = P_{D2} \quad \ldots (10)$$

$$\frac{\partial P_{D1}}{\partial r_D} = \delta \frac{\partial P_{D2}}{\partial r_D} \quad \ldots (11)$$

where

$$\delta = \frac{\{k\rho/\mu\}_2}{\{k\rho/\mu\}_1} \quad \ldots (12)$$

Here, only the infinite outer boundary case is considered. Then the outer boundary condition is:

$$\lim_{r_D \to \infty} P_{D2}(r_D, t_{DR}) = 0 \quad \ldots (13)$$

## RESULTS

Following the procedure of Horne et. al. (1980) the long time solution for $P_{D1}$ can be written as:

$$P_{D1} = \frac{1}{2}\{- Ei(-\frac{r_D^2}{4t_{DR}}) + Ei(-\frac{1}{4t_{DR}}) - \frac{1}{\delta} Ei(-\frac{1}{4\gamma t_{DR}})\} \quad \ldots (14)$$

For a large time this may be approximated as:

$$P_{D1} = \frac{1}{2}\{ \ln(r_D^2) + \frac{1}{\delta}\ln(\frac{1.781}{4\gamma t_{DR}})\} \quad (15)$$

In real space Eq. 15 becomes:

$$P_1 = P_i + \frac{q\mu_1}{4\pi k_1 h\rho_1}\{ \ln(\frac{R}{r})^2 + (\frac{k_1\rho_1\mu_2}{k_2\rho_2\mu_1}) \ln(\frac{2.246 k_2 t}{\phi_2\mu_2 c_2 R^2})\}(16)$$

For an isothermal homogeneous geothermal system the pressure at the well during injection can be written as:

$$P_{wf} = P_i + Bq + Cq^2 \quad \ldots (17)$$

where C is a constant governed by the pore structure of the porous medium, the radius and the condition of the well. The quadratic flow rate term accounts for the non-Darcy's flow effects. Assuming these effects to be negligible for the purpose of this study, the injectivity index can be expressed as:

$$II = \frac{q}{P_{wf} - P_i} = \frac{1}{B} \quad \ldots (18)$$

Because of the relatively short duration of each injection step in the test, the pressure at the well is considered to be transient. Then B can be approximated as:

$$B = \frac{\mu}{4\pi k h\rho} \ln(\frac{2.246 kt}{\phi\mu c_t r_w^2}) \quad \ldots (19)$$

For homogeneous rock properties and single phase condition, Eqs. 16 and 17 along with Eq. 19 yield:

$$\frac{\mu_a}{\rho_a} \ln(\frac{2.246 kt}{\phi\mu_a c_t r_w^2}) = \frac{\mu_1}{\rho_1}\{ \ln(\frac{R}{r_w})^2 + (\frac{\rho_1\mu_2}{\rho_2\mu_1}) \ln(\frac{2.246 kt}{\phi\mu_2 c_t R^2})\} \ldots (20)$$

Eq. 20 is a root solving problem to find the apparent viscosity which will represent the nonisothermal effect in a conventional manner. Figures 2 - 4 show how this apparent viscosity is related to the position of the thermal front

and the duration of the injection steps for the properties listed in table I. Thus the nonisothermal injectivity index is for engineering purposes approximately related to the isothermal injectivity index as:

$$II_{ISOTH} \simeq ( \frac{\mu_a \rho_2}{\rho_a \mu_2} ) \, II_{NONISOTH} \qquad (21)$$

DISCUSSION

Figure 5 has been constructed using the data in table I and the apparent viscosity relative to the cooled radius. It shows the transmissivity of the hot reservoir as a function of the injectivity depending on the size of the cooled radius. Figure 5 could equally well have been constructed from a rearrangement of Eq. 16. Figure 5 shows that when the nonisothermal injectivity and the size of the cooled zone are known ( see Benson, 1982 ), the transmissivity of the reservoir can be determined. If the transmissivity and the nonisothermal injectivity are known, the size of the cold spot can be estimated. It is also possible to project the well productivity as its temperature recovers from the injection and the cold zone diminishes. Here it is assumed that the reservoir transmissivity does not change between injection and production. However, as Grant (1982) has pointed out, the injection transmissivity is in many instances observed to be several times greater than that for production. This might be due to an increase in permeability with injection either because of hydraulic fracturing or thermal contraction of rock and opening of fissures.

It has been pointed out (Benson and Bodvarsson 1982, Benson 1982) that the presence of a cooled zone around the well acts as an apparent positive skin. Hence, the presence of a positive skin will shift the curves on Figure 5 up and to the left while the presence of a negative skin will shift them down and to the right. Of particullar interest is the effect of fractures, since transmissivity or injectivity is usually dominated by natural fractures in geothermal systems. The presence of fractures will contribute to a negative skin, but the extent of a cooled zone is much greater at the fracture feed points than in the rock matrix. How these effects counteract each other is not fully known, but numerical studies indicate that the heat transfer from rock matrix to fissures significantly changes the radius of the zone occupied by the cold injection fluid (Bodvarsson and Tsang 1982).

EXAMPLES

A simulated nonisothermal step rate injection test was run on the numerical simulator, PT (Pressure - Temperature) (Bodvarsson 1981).

The test consisted of three six-hour steps with injection rates of 0.10, 0.15 and 0.20 kg/s/m of 20 °C fluid into a 300 °C reservoir followed by a complete shutin. The reservoir properties used for the simulation are given in table I. The reservoir is a liquid saturated homogeneous porous medium. No cold spot exists around the well prior to injection. The observed injectivity at the end of each step is shown in Figure 6, which is similar to Figure 5. Since the transmissivity of the hot reservoir is constant during the test, one can see that the decrease in injectivity is a result of an increasing cold spot around the well. The size of the cold spot as determined by the transmissivity and the injectivity for each step is in good agreement with the location of the thermal front according to the simulator which is shown in Figure 6. Assuming that the isothermal injectivity is identical to the well productivity, the production index can be estimated by extrapolation, using Figure 6, as the well warms up.

The transmissivity and the injectivity index for a few of the wells in the Krafla geothermal field are plotted in Figure 7. The Krafla geothermal reservoir is fracture dominated so an average skin factor of s = -5 is assumed for the field. Following the methodology developed by Benson and Bodvarsson 1982, and Benson 1982, the size of the cold spot around the wells due to drilling and injection testing is estimated to be 2 to 3 m on the average. However, since the reservoir is fracture dominated, the distance to the thermal front can be considerably greater in the fractures or around the feed points in the wells. On the other hand the distance to the thermal front in the nearly impermeable rock matrix, between the feed points in the wells, can be of the order of a few cm and is controled by the thermal conduction of the rock matrix. The size of the cold spot as obtained from Figure 7 when the data is plotted according to the methodology of this paper is therefore some weighted average of these two distances. It seams therefore that the existence of a cold spot around the feed points in a well in a hot fractured reservoir has very litle effect on the relationship between transmissivity and injectivity. Other factors such as the skin effect are problably dominating the effect of the cold invasion zone in fracture dominated reservoirs.

CONCLUSIONS

Through theoretical considerations an apparent viscosity has been established which relates the nonisothermal injectivity index, obtained from nonisothermal step rate tests, to the isothermal injectivity index. Thus it is possible to use the nonisothermal injectivity index to estimate the reservoir transmissivity and infer the well productivity.

## NOMENCLATURE

$c_t$ = total system compressibility, $Pa^{-1}$

$C_r$ = specific heat capacity, $J/kg\,°C$

$h$ = thickness, m

$II$ = injectivity index, kg/sPa

$k$ = permeability, $m^2$

$P$ = pressure, Pa

$q$ ◻ mass flow rate, kg/s

$r$ = radius, m

$R$ = radius of cold spot, m

$t$ = time, s

$T$ = temperature, $°C$

$\mu$ ◻ dynamic viscosity, Pa s

$\rho$ = density, $kg/m^3$

$\phi$ = porosity

$\lambda$ = thermal conductivity, $W/m\,°C$

Subscripts

a = apparent

c = cold

D = dimensionless

i = initial

inj = injection

r = rock

w = well

wf = well flowing

1 = inner (cold) region

2 = outer (hot) region

## ACKNOWLEDGMENTS

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## Table I. Reservoir Properties

| Property | Value |
|---|---|
| Permeability, $k_r$ | $1.0 \cdot 10^{-14}$ $m^2$ |
| Thickness, h | 1.0 m |
| Porosity, $\phi$ | 0.20 |
| Specific heat capacity, $C_r$ | 1000.0 $J/kg\,^\circ C$ |
| Density, $\rho_r$ | 2200.0 $kg/m^3$ |
| Thermal conductivity, $\lambda$ | 2.0 $W/m\,^\circ C$ |
| Total system compressibility, $c_t$ | $1.0 \cdot 10^{-9}$ $Pa^{-1}$ |
| Wellbore radius, $r_w$ | 0.10 m |



CASE: 20°C INTO 100°C

Figure 2. Apparent viscosity as a function of the duration of the injection steps and the size of the cold spot for injection of 20 °C water into 100 °C reservoir.
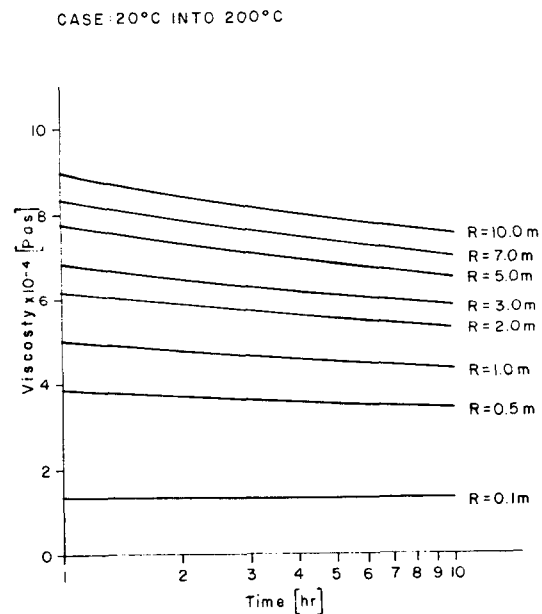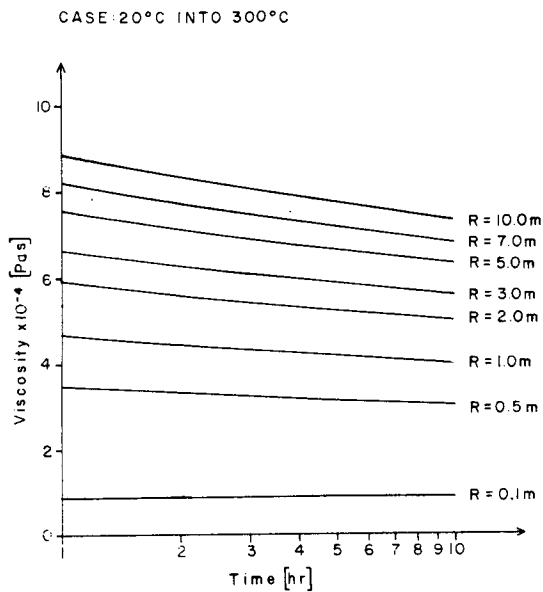


Figure 1. Schematic picture of the reservoir system before and after nonisothermal injection



CASE: 20°C INTO 200°C

Figure 3. Apparent viscosity as a function of the duration of the injection steps and the size of the cold spot for injection of 20 °C water into 200 °C reservoir.

CASE:20°C INTO 300°C



Figure 4. Apparent viscosity as a function
of the duration of the injection
steps and the size of the cold
spot for injection of 20 °C water
into 300 °C reservoir.



Figure 6. The relationship between
transmissivity and injectivity
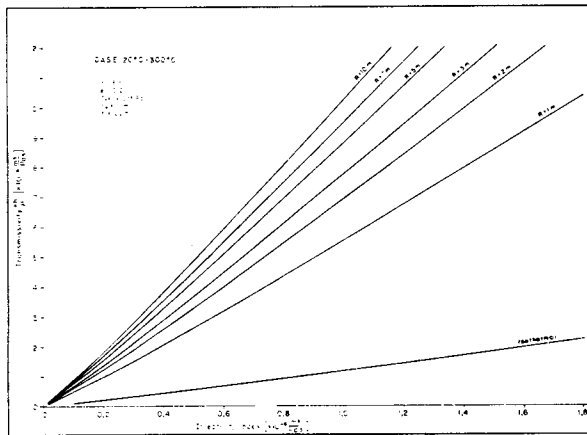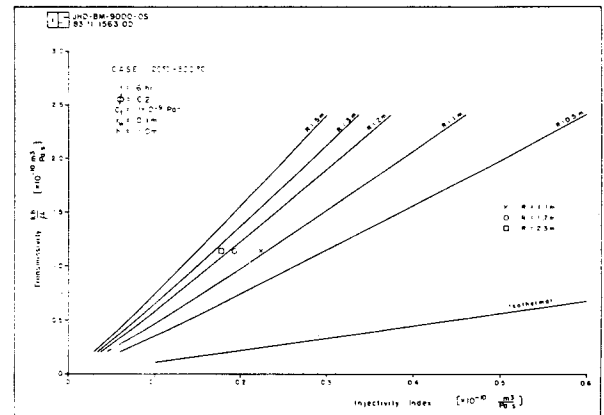for the simulated example.



Figure 5. Transmissivity as a function of
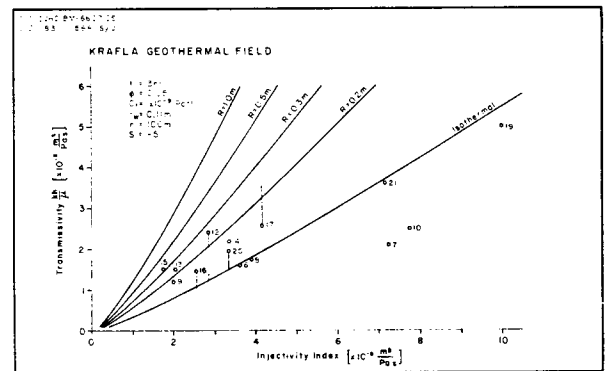the injectivity and the size of
the cold spot.



Figure 7. The relationship between
transmissivity and injectivity
for a few of the wells in the
Krafla geothermal field.