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A Fundamental Approach to the Simulation of Flow and Dispersion in Fractured Media

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ABSTRACT

Fracture systems may be generalized in terms of number and orientation of sets of parallel fractures and the distribution of length, width, thickness and separation. Borehole measurements may be used to particularize these parameters for a specific site.

Global flow and dispersion in an aquifer occur in the interconnected fractures and may be related to specific fracture elements. A fluid dynamics code named SALE has been used to solve the Navier-Stokes equations for laminar flow in these elemental geometries. A marker particle calculation has been added to characterize longitudinal dispersion due to the velocity profile across the fracture and lateral dispersion due to flow disturbances at junctions.

Local flow and dispersion in the matrix occur in the finer fracture structure and are evaluated using porous media approaches.

These results or models are integrated in a 2D isothermal reservoir simulator named FRACSL. Discrete fractures are superimposed on the edges or diagonals of rectangular grid elements. Water may flow from node to node through the matrix or through the fracture. The heads are found by iterating for the distribution which conserves the appropriate local mass. Marker particles are used to monitor the tracer dispersion due to motion in the fractures, in the matrix and between the two.

Results are given showing flow and dispersion in an orthogonal junction and in a sample fractured reservoir.

INTRODUCTION

Geothermal, waste management and secondary and tertiary petroleum recovery activities have provided the stimulus for recent advances in the characterization of fractured reservoirs. Much of this work has been directed towards the evolution of specific fractured media approaches which depart significantly from those used for porous media. An essential characteristic of these approaches is the determination of fracture system geometric and flow characteristics.

A reservoir fracture system may be described in terms of two subsets: A number of interconnected fractures which constitute the global flow system and a system of progressively smaller fractures which begin at the global flow system and terminate in the matrix microcrack structure. The secondary system, while nominally quiescent, constitutes the bulk of the matrix porosity and surface area.

Fractured reservoir characterization for petroleum recovery is described by Nelson[1]. A basis for characterizing the geometry of fracture systems is presented by Nelson[2] and by Stearns and Friedman[3] and a significant body of data is available in the literature, particularly from the research at the Stripa mine in Sweden and from the Canadian waste management programs.

Numerical simulations of flow in fractured media depart from the use of heterogeneous and anisotropic porous media characteristics and model the flow in unique and discrete fractures or in repeated fracture patterns. Some flow simulations[4,5] superimpose discrete fractures onto the edges of porous media grid cells. This approach results in a single set of grid point pressures and distinct matrix and fracture flow paths between those grid points.

These flow simulations are essential to the study of solute transport, both as an end product and as another tool for characterizing fractured reservoirs. Typical solute transport models, however, revert to porous media theory. A common approach is to implement different dispersivities in the fracture and in the matrix.

While studies[6] have shown the significance of geometry and head gradient orientation on dispersion or mixing in individual fractures and fracture junctions, these characteristics have not been reflected in reservoir level numerical simulation.

This paper describes initial numerical studies aimed at characterizing the flow and dispersion characteristics of the global flow system in terms of those of its elements and integrating those characteristics with those of the matrix in a complete reservoir simulation. One objective of this work is to develop a model whose parameters can be adjusted to describe a particular reservoir by matching measured hydrologic and tracer dispersion responses. This model will ultimately be used for assessing the thermal potential of a reservoir. Specific models will be developed to correlate data measured in 1982 and 1983 at Raft River, Idaho and East Mesa, California as part of the U.S. Department of Energy Hydrothermal Injection Research Program. Other phases of this program are reported elsewhere in these proceedings.

APPROACH

The complete numerical simulation effort, a portion of which is presented in this paper, includes the following tasks:

1) Integrate formation characterization data from the literature into a base which can be used together with site-specific data such as lithology, core samples and acoustic televiewer, spinner, temperature and conductivity logs to generate representative fracture patterns for the site being modeled.

2) Generate flow and dispersion characteristics for the elements of the global flow system as a function of geometry and flow conditions.

3) Integrate the element characteristics into a reservoir simulation.

4) Develop specific models of the Raft River and East Mesa reservoirs to correlate measured flow and tracer dispersion.

This work is an outgrowth of and is complemented by extensive laboratory scale physical modeling.[6,7]

The numerical simulation accomplished to date consists of initial development and demonstration of the fracture element and reservoir simulations. Completion of the current simulations and the addition of heat transfer and rock-water interactions is included in future planning.

ELEMENT SIMULATION

The specific motivation for this phase of the program is experimental work showing that complete mixing does not occur at fracture junctions.[6] This, together with consideration of the dispersion resulting from the velocity profile across a fracture, suggests that reservoir dispersion may be a strong function of identifiable fracture element characteristics.

The elements of interest, in the order traversed in injection, are the well-head piping, the wellbore, the junction of the wellbore and the initial fractures, single fractures, fracture junctions and the junctions of flowing and "dead-ended" fractures. Subsequent smaller fractures down to the micro-crack level are, for the present, treated using porous media models.

The essential capability required in the mathematical model is a flow simulation. Once the steady state velocity field has been established the dispersion characteristics are found by tracing imaginary marker particles through the flow element, thereby determining the lateral position at the exit as a function of that at the inlet. The resulting pressure drop and particle inlet-outlet displacement curve are then transferrable to the reservoir simulation. This process is a deterministic study of advective dispersion only. Diffusion, where significant, will be added in the reservoir simulation.

While the majority of the elements considered are essentially two dimensional, the wellbore -initial-fracture junction and the single fracture itself require three dimensional treatment.

The code selected for these simulations is SALE[8] (Simplified Arbitrary Lagrangian-Eulerian). This code is used extensively to solve a variety of fluid dynamics problems by numerical solution of the Navier-Stokes equations in two dimensions. This code is extremely well documented and was already installed (in two-dimensions) with plotting capability at the Idaho National Engineering Laboratory (INEL). A three dimensional version[9] is available but has not yet been installed.

SALE provides an implicit treatment of the pressure calculation for flow at the incompressible limit, thereby avoiding Courant-like time step restrictions While the grid may be moved with the fluid in the Lagrangian manner or arbitrarily rezoned, in the subject application it is held fixed in the Eulerian manner. A non-rectangular grid may be coded by the user. A problem may include the following boundary conditions: 1) a free Lagrangian surface, 2) a free-slip (symmetry or non-viscous) vertical or horizontal wall, 3) a curved or tilted free-slip wall, 4) a no-slip wall, 5) a continuative outflow boundary, 6) a specified inflow or outflow boundary and 7) a specified pressure boundary. Initial simulations of steady flow between parallel plates and accelerated flow in a pipe agreed well with theory.

A fracture junction model was built into the SALE code along with a marker particle calculation. Figure 1 shows the velocity vectors calculated for the orthogonal junction of two parallel sided fractures of

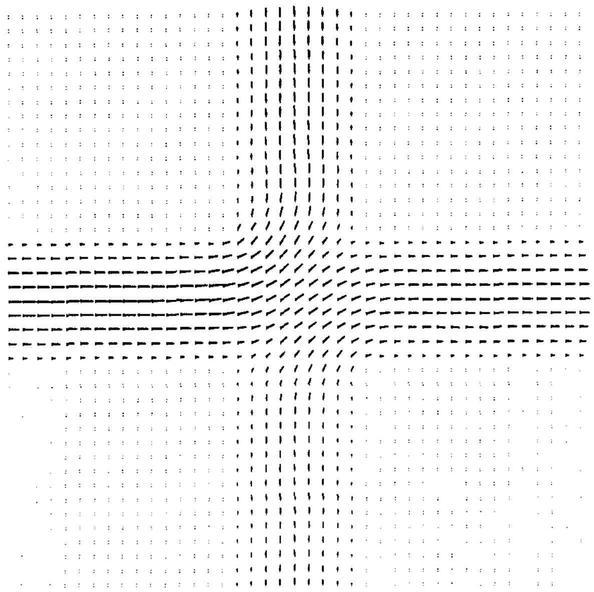

Figure 1. Velocity vectors for orthogonal junction of two fractures. Vertical inlet velocity at 50% of horizontal inlet velocity.

equal width. The case shown is a 40 cell by 40 cell grid with inlet (bottom and left) and exit (top and right) leg lengths double their widths. The left inlet flow is at a mean velocity of 312.4 ft/day, the bottom inlet flow is lower by a factor of two and the width of each fracture is .0625 inch. Poiseuille velocity profiles[10] are imposed at each of the inlets and both exits are at zero pressure. While the exit flow straightens within a length equal to one width, a length twice the width was found necessary to reestablish a symmetrical Poiseuille velocity profile.

The complete set of input parameters for the fracture junction include angle of intersection, relative width, and velocities in three of the legs. Three velocities are sufficient since the fourth is found from continuity. Since the pressure drops were found to be linear with velocity, the spectrum of velocities is defined by specifying two velocity ratios, e.g., leg B to leg A and leg C to leg A. Initial results are limited to orthogonal junctions and equal widths and were made for 3 different ratios of bottom and left inlet velocities with exit velocities corresponding to equal exit pressures.

Calculated dispersion characteristics are given as displacement functions. A displacement function relates the lateral position of a marker particle at the inlet to its position at the outlet and, in the case of an asymmetric junction, specifies which outlet. Lateral position in the fracture is an essential consideration since the advective movement varies so strongly from the wall to the centerline. The existence of multiple outlets leads to the plot format shown in Figure 2. The inlet lateral position is shown on a continuous abscissa for the two inlets and the exit lateral position is shown on a continuous ordinate for the two exits. The sketches at the top of the figure illustrate the graphical process of rotating the left inlet down to the x axis and rotating the top exit up to the y axis. The result is a continuous function, as shown for vertical inlet velocities of 10%, 50% and 100% of the horizontal inlet velocity. The three curves demonstrate the intrusion of the stronger horizontal stream into the horizontal exit for small vertical inlet velocities.

Steady state was achieved for each of these cases in .5 seconds of problem time. This calculation, at time steps of 5 ms, and the subsequent tracing of marker particles through the system was accomplished in 49 CP

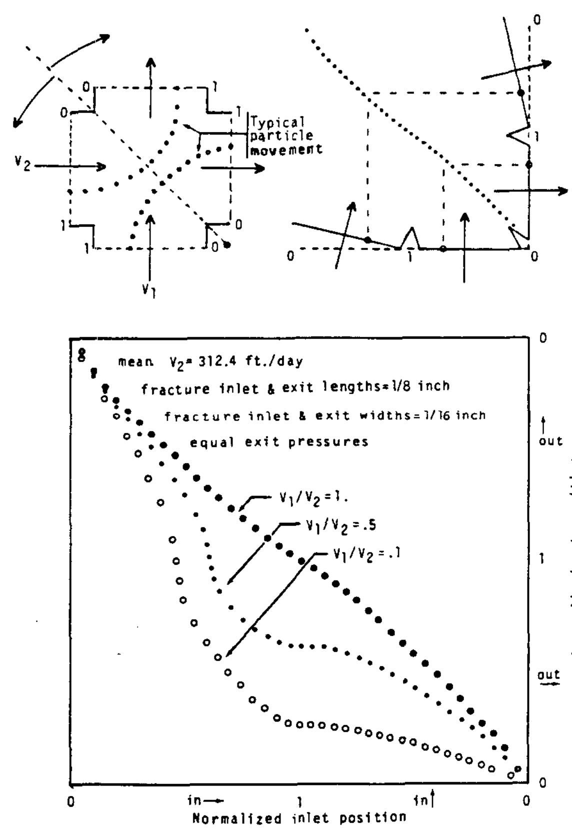

Figure 2. Displacement function for orthogonal junction of two fractures.

seconds on a CYBER 176 computer. While trivial levels of numerical (non-physical) stabilization were sufficient to maintain stability and avoid distortion due to alternate node coupling and donor cell differencing, a small spatial pressure oscillation was noted. Resolution of this problem and verification by comparison to physical model results are in progress.

RESERVOIR SIMULATION

Suitable codes described in the literature with the discrete fracture capability essential to this approach were unavailable for external use. A new code was therefore assembled from the basic flow relationships and simplified models of marker particle displacement processes. The model calculates flow between points on a rectangular grid through the matrix or through a fracture which may connect adjacent grid points horizontally, vertically or diagonally. The grid is two-dimensional, the boundaries are impermeable and specified external inflow or outflow may be applied at any grid point. The heads at the various grid points are found by dividing the grid into quarter-cells and assigning each quarter cell to the nearest grid point. Continuity is then applied to the matrix flow and fracture flow entering or leaving the volume assigned to each grid point.

The simulation was implemented within the ACSL[11] (Advanced Continuous Simulation Language) problem solver code. ACSL is a proprietary, author supported code which provides the input/output, function generation, integration, etc. software to solve a dynamic problem described by the user in terms of algebraic and first-order ordinary differential equations. It also provides a direct calculation of a steady state by a Newton-Raphson iteration which drives the derivatives towards zero. The resulting code is named FRACSL.

Sample flow calculations were performed on the fractured reservoir shown in Figure 3. The reservoir is 900 ft long by 360 ft high by 1 ft thick and incorporates a 9 cell by 9 cell grid, an open wellbore at the left edge and an extensive fracture system as shown by the heavy vertical or diagonal lines. Hydraulic parameters are given in Table 1. In this simplified model the junction resistance is not specifically selected as a function of junction characteristics. The ACSL direct steady state mode was used to calculate the head and flow distribution for a 25 gpm injection flow and a 25 gpm withdrawal flow. In each case, the head derivatives, after subtracting the steady state drawdown rate, were forced to zero. Table 1 and Figure 4 show the results for the injection case.

The following discussion summarizes the marker particle models incorporated in the code.

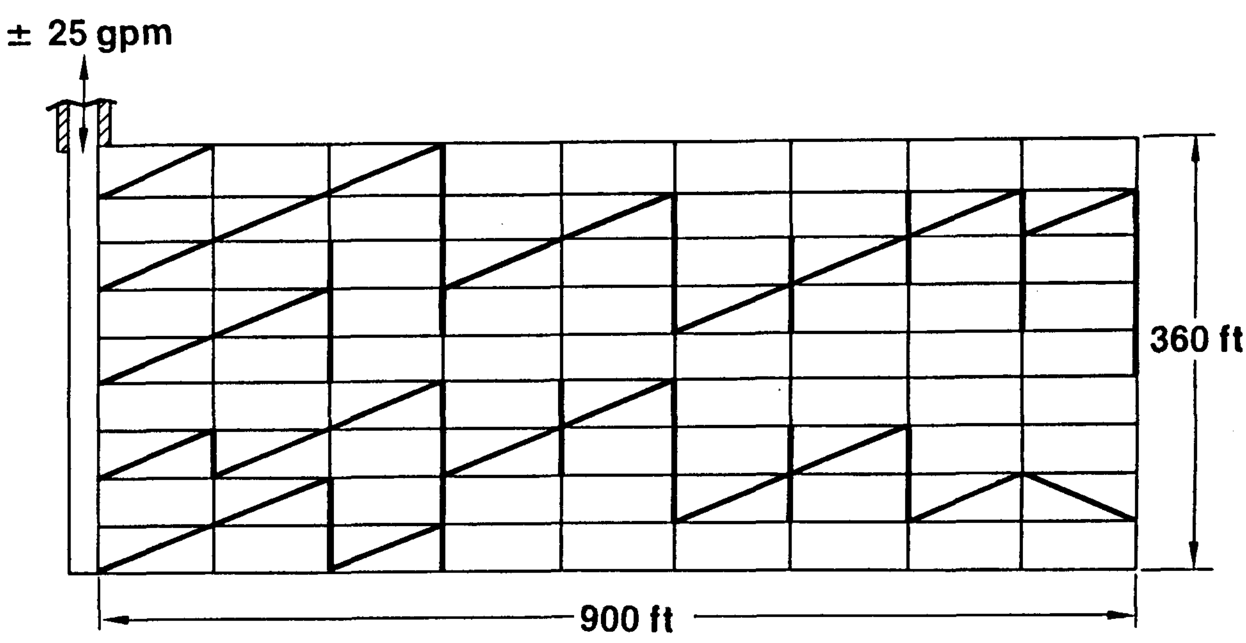

Figure 3. Sample Fractured Reservoir

Table 1. Sample Reservoir Flow Simulation

Water Temperature	70°F
Well Flow Rate	± 25 gpm
Fracture Aperture	.01 ft
Fracture Conductivity	2.25E+6 ft/day
Matrix Conductivity	1.0 ft/day
Head at Well	100 ft
Minimum Head in Fracture (Injection)	35.2 ft
Minimum Head in Matrix (Injection)	11.3 ft
Fracture Velocity Range	1560-178500 ft/day
Matrix Velocity Range (x or y component)	.00069-.672 ft/day

A tracer slug injection is modeled by injecting ten groups of 25 particles into the fracture system at suitable time intervals. Each group of 25 is distributed across the width of the 5 fractures which intersect the well. Each particle is assigned a mass proportional to its velocity, thereby approximating a constant concentration input. Dispersion in the wellbore and the junctions of the wellbore and the initial fractures will be added later.

A particle in a fracture is moved with a velocity which corresponds to the local value on a Poiseuille velocity profile fitted to the mean fracture velocity. Diffusion is added by drawing a sample from an appropriately scaled normal distribution. Three dimensional dispersive effects will be determined using SALE-3D and incorporated in the reservoir model.

A particle traversing a junction completes the time increment in the exit fracture. The lateral position in the exit fracture is determined from the SALE generated displacement function for the appropriate inlet position, geometry and flow ratios.

Particles move from the fractures into the matrix based on the mass balances for the individual grid points and the matrix quarter cells and half fractures which terminate at that grid point. The volume of water moving from the fracture into the matrix in this regime is equal to the excess of fracture inflow over fracture outflow, multiplied by the time interval. This volume is distributed to the various half fractures at the grid point according to their length-times-volumetric-flow products. The assumption is made that the water enters the matrix in a "dead-ended" fracture which creates sufficient dispersion in the through-flow that the volume entering the matrix may come, with equal likelihood, from any depth in the flowing fracture. The validity of this simplification will be determined from fracture characterization and SALE dispersion studies.

The model is implemented numerically by dividing the volume to be transferred to the matrix into 10 equal cells and selecting those cells at random over the entire width and depth of

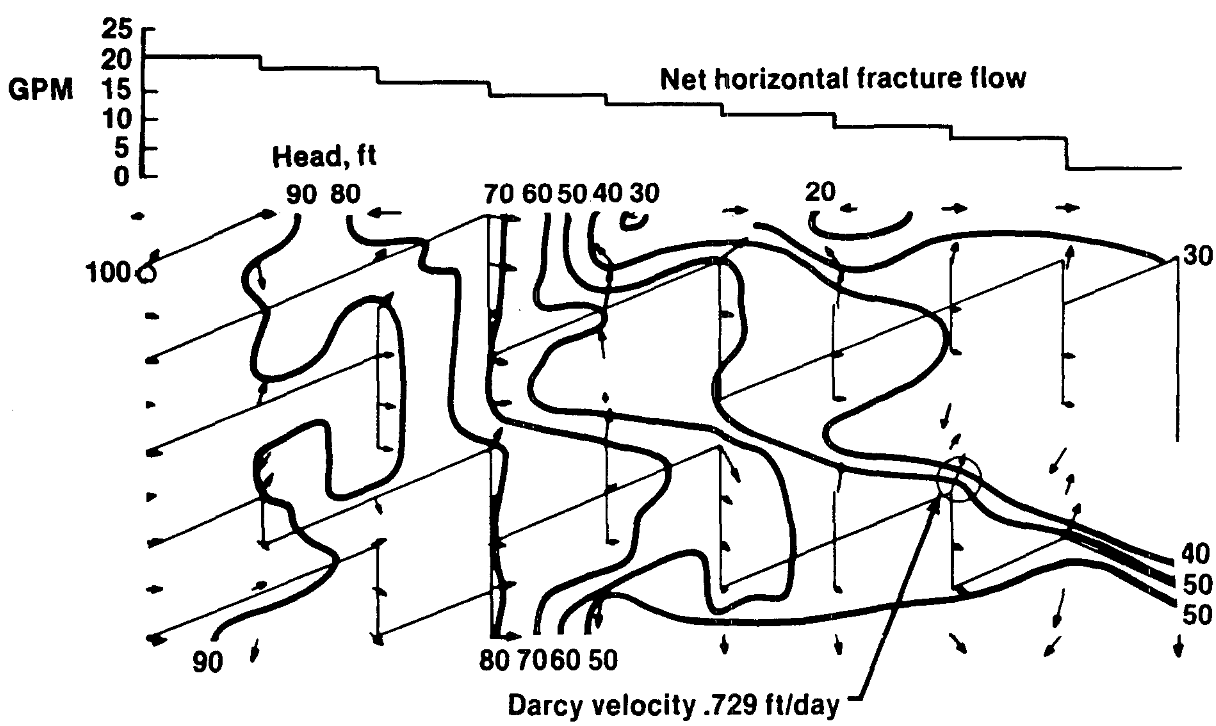

Figure 4. Fracture Flow, Head Contours and Matrix Velocities for 25 gpm Injection

the half-fracture. Any markers in these ten cells are removed from the fracture and assigned an equal position in the matrix.

Marker particle motion in the matrix is computed as the sum of advection, porous media based dispersion and molecular diffusion. The advective displacement is found by interpolating for the velocity vectors at the beginning of the time increment and multiplying by the time increment. Dispersive displacements in the direction of the advection and normal to it are found from separate normal distributions with standard deviations equal to the square root of twice the product of the appropriate dispersivity, in feet, times the advective displacement[12]. Molecular diffusion is computed in a similar manner.

The same models apply during the withdrawal phase except that the fracture-to-matrix transfers are reversed and the particles are counted as they leave the initial fractures to reenter the well. The matrix-to-fracture transfer is again based on the mass balance for the appropriate grid point and associated matrix and half fractures. The volume of water moving from matrix to fracture during a time increment is equal to the excess of fracture exit flow over fracture inlet flow, times the time increment. That volume is assigned to the individual half fractures on the basis of their flow-times-length products. The total volume of water associated with the grid point is equal to the area of matrix times the unit thickness times the matrix porosity. The transferred water is taken from a strip of appropriate width along the fracture. Any marker in that strip at the end of the time step or which crosses it during the time step is moved normally into the fracture and assigned a lateral position drawn from a random distribution.

Results have been generated for a simplified model without molecular diffusion and with particle lateral position at the exit of a junction equal to that at the inlet. The ten groups of 25 particles were released into the initial fractures at equal intervals during a .01 day injection period with flow conditions as shown in Figure 4. Table 2 gives the initial and end-of-injection positions for the first 5 particles released into the lowest fracture in the reservoir for a case without matrix dispersion.

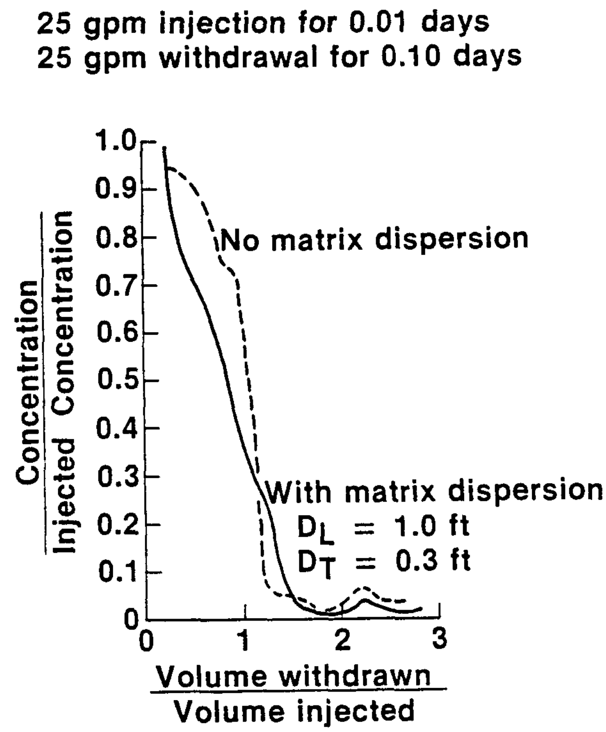

Figure 5. Recovery Curves

As shown in the table, three of the particles had transferred to the matrix by the end of injection.

Flow was then reversed for a period of 0.10 days and the particle mass returning to the well was monitored. The ratio of recovered-to-injected concentration was found by differentiating the total tracer return and divi-

Table 2. Marker Particle Positions (No Matrix Dispersion)

Particle	Initial Position in Fracture - Distance from Fracture Bottom	End-of-Injection Position in Reservoir: Longitudinal Distance from Left Edge	Vertical Distance fm. Bottom	Host - Fracture or Matrix	Distance from Fracture
1	10%	52.2%	41.1%	Fracture	---
2	30%	22.2%	6.7%	Matrix	.046' right
3	50%	3.4%	3.4%	Matrix	.015' below
4	70%	82.1%	15.5%	Fracture	---
5	90%	33.9%	22.8%	Matrix	.001' below

ding by the injection rate. Figure 5 shows the concentration ratio as a function of the withdrawn-to-injected volume ratio. A purely advective case would give a function which steps from 1.0 for volume ratios less than 1.0 to 0. for ratios greater than 1.0. Results for the injection-withdrawal sequence without matrix dispersion are shown by the dotted line. Substantial spreading of the sharp front has occurred and the tail of the curve extends to zero concentration at a volume ratio of 6.4. The solid curve shows the effect of the addition of matrix dispersion with longitudinal dispersivity of 1.0 ft and transverse dispersivity of 0.3 ft. The front is spread further and the tail is extended with 7.0% of the tracer still in the reservoir after 10 injection volumes have been withdrawn.

SUMMARY

An approach to numerical simulation of fractured reservoirs has been developed which includes characterization of fracture system geometry, numerical simulation of flow and dispersion in fracture elements, numerical simulation of flow and dispersion in fractured reservoirs and correlation with laboratory and field test data.

A general purpose fluid flow code has been modified to simulate flow and the dispersion of marker particles in orthogonal fracture junctions. Results are given for three different flow conditions.

A reservoir simulation code has been assembled. Flow is calculated in the matrix and in discrete fractures arranged along the edges or diagonals of the rectangular grid blocks. The dispersion of marker particles through the reservoir is simulated. Sample results are given.

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