ONE COMPONENT TWO PHASE FLOW IN HORIZONTAL AND VERTICAL DUCTS:
SOME BASIC CONSIDERATIONS

Paul F. Maeder, David A. Dickinson, Dimitris E. Nikitopoulos

Brown University
Providence, Rhode Island 02912

ABSTRACT

For a description and analysis of the flow we consider the conservation equations of the two phases separately, but in thermal and mechanical equilibrium, coupled by the interface shear forces (two fluid model, drift flux model). Coupling may be weak or strong, depending on Froude and Mach numbers of the flow. The fluid is highly compressible, not because the individual phases move at such speeds that their individual density changes are significant but because evaporation (phase change) results in large density changes of the system at moderate pressure or temperature changes once flashing occurs.

The slip between the phases is caused by unequal wall shear stress, acceleration of the fluid or gravitational forces and is hindered by the interface interaction. If we denote by γ the ratio of the liquid density to the vapor density and by σ the ratio of the vapor speed to the liquid speed we find that in horizontal flows $\sigma = \gamma^{1/2}$ yields the maximum slip (neglecting acceleration effects) that can be reached with no interface force acting (assuming equal friction coefficients for both phases at the wall).

If one investigates the conditions of thermodynamic flow similarity between different substances in two phase flow, one finds that the latent heat of vaporization is the principal controlling parameter. Thus, a 5 cm diameter test section in two phase R-114, at room temperature, corresponds to a 30 cm diameter duct in water-steam at boiling conditions at high temperatures such as encountered in geothermal and other power production systems.

1. INTRODUCTION

Two Phase Flow, as a branch of fluid mechanics, has commanded a great deal of attention for the last forty years. Most recently, research has been mainly concentrated on specific problems involved with the production of electricity from geothermal energy and the safety of nuclear power plants, in addition to the chemical engineering process problems which provided the original motivation. The number of approaches taken to the problem, however, is nearly as large as the number of investigators, and it seems as though a consensus has not been reached on some of the most basic questions.

Two basic approaches have been taken. The first encompassed nearly all of the earliest work and is characterized by phenomenological considerations or statistical methods. The development of flow-pattern diagrams [1] and the direct correlation of data [2] for pressure change and void fraction are examples. Notably, most work involved two-substance flow without phase change, such as air and water.

The second approach has been to seek deeper understanding of the physics of the flow in order to successfully and more accurately predict behavior over a fuller range of two phase conditions [7,8,9]. The importance of compressibility in determining the behavior of a single-substance flow is more readily recognized in the most recent work. An up-to-date collection and discussion is given in Ref. [10] and [11].

An investigation of the physical phenomena of two phase flow must center on the basic conservation equations of mass (continuity) momentum, and energy. These may be written in time-averaged form for each of the phases. The interface interaction must be properly accounted for, including the presence or absence of phase change. Hence, six basic equations can be written. Additional information is required for the nature of the interface coupling. If pressure and temperature equilibrium is assumed, combination of the conservation equations yields three independent equations, one each for momentum, energy and dissipation or mechanical energy (in addition to the two continuity equations).

The physical insight that can be gained from these basic considerations is the goal of the present paper.

2. THE BASIC PROBLEM

First, we will assume the case of a horizontal

pipe in which two separate fluids are flowing. We will assume that no phase change takes place. Each fluid has acting on it the pressure difference and the shear stresses as indicated in Fig. 1.
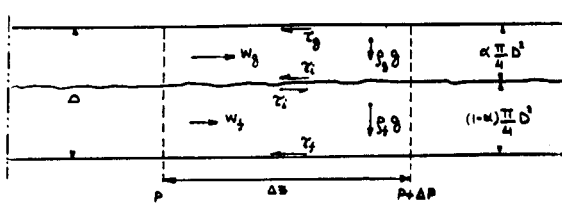

Figure 1.

The shear stress at the interface between the two fluids is shown schematically only. A simple calculation leads to the conclusion that, in this case, the pressure changes are so slight that both fluids can be considered as incompressible. We assume, as is conventional in fluid mechanics, that the pressure is constant throughout a cross-section. We further assume essentially one-dimensional flow and that the fraction of the wall circumference being wetted by one substance is proportional to the cross-section it occupies. Equilibrium between the forces acting on the two fluids then leads to

$$\alpha \frac{dP}{dz} = -\tau_{wg} \frac{\alpha}{R_h} - \tau_i \frac{N_i}{R_h} C$$

$$(1-\alpha) \frac{dP}{dz} = -\tau_{wf} \frac{(1-\alpha)}{R_h} + \tau_i \frac{N_i}{R_h C} .$$

(1a,b)

We see that for the extreme case of negligible interface shear force, the wall shear stress for each component must be the same. Since this is given by the product of a friction factor and the dynamic pressures of the fluids, the latter must be in the inverse ratio of the friction factors. For flows of primary interest the Reynolds number will be large enough so the flow is turbulent and the friction factors as given by the Moody diagram will differ only slightly. In this case, i.e. steam-water flow with no phase change or air-water flow, the densities of the two fluids will be very different and therefore the velocities must differ greatly. Hence, slip between fluids ensues.

If we denote the density ratio of the two fluids by γ, we find that in the case of no interface stress, the slip ratio σ, i.e., the ratio of velocity in one fluid to that of the other, will be equal to the square root of the specific volume ratio. As the interface shear stress increases, this slip ratio will diminish and becomes essentially one if large interface stress is present. Thus, slip changes from $\sigma = \gamma^{1/2}$ for a strongly

stratified flow, to $\sigma = 1$ for complete mixing. In the horizontal pipe, the amount of mixing that occurs will be related to the Froude number since the gravity is the agent which tends to stratify the flow.

Martinelli [2] has proposed to use as an independent variable, accounting for the friction effect, the ratio of the frictional pressure drop each fluid would experience if it were flowing separately in the absence of the other. If we do so (keeping in mind that the continuity equation establishes a relationship between the mass fraction ratio, the slip and the holdup) we obtain Fig. 2 which shows holdup (1 minus vapor void fraction) as a function of the Martinelli variable with slip ratio as a parameter. Some of the proposed correlations given in the literature have been shown [3,4,6] as well as some data from our facility [5].
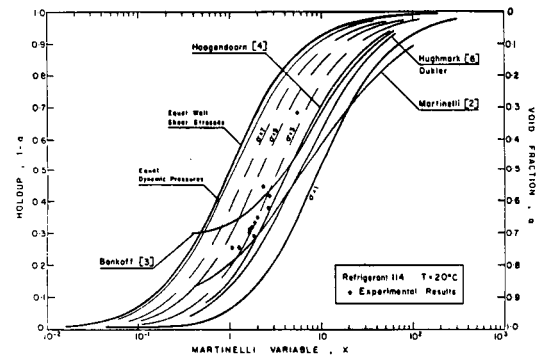

Figure 2.

A significantly different situation exists if the pipe or duct is oriented vertically. Figure 3 illustrates this case.
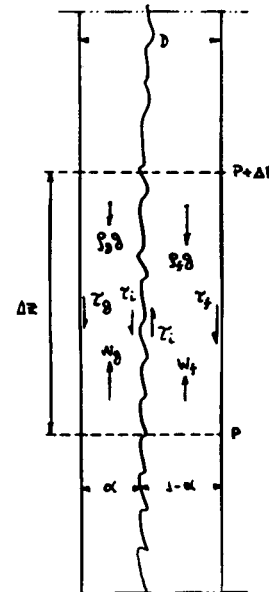

Figure 3.

The gravitational force acting on the two fluids will then be the primary cause of slip. Again, making the assumption that there is no interface shear stress the following relationship for slip ratio, friction factor and Froude number results:

$$\sigma = \gamma^{1/2}\left(1 + \frac{1}{fFr_f^2}\right)^{1/2} . \qquad (2)$$

Slip of the indicated magnitude is quite large. Violent mixing will occur leading to such flow patterns as slug flow, annular flow, etc. It should be noted and is significant that in the horizontal pipe, slip is caused by unequal wall shear stresses while in the vertical pipe slip is primarily caused by gravity acting on fluids of different densities. Since their densities will not change markedly neither fluid will be accelerated significantly. A fully developed flow whose character will not change along the pipe will result. Successive sections of pipe will be similar to each other.

3. COMPRESSIBILITY AND SIMILARITY

If the fluids are two phases of the same substance, the decrease in pressure due to gravitational forces and friction will cause liquid to evaporate. The accompanying large increase in specific volume (decrease in density) will cause acceleration of the flow in a constant area duct. The latter will be more pronounced if there is little or no slip between the phases. In order to accommodate the increased kinetic energy of the flow due to this acceleration, the pressure will decrease further. Finally, an increase in specific volume may produce such a large increase in kinetic energy that it can no longer be accommodated in an adiabatic system. The flow then chokes. We see that the flow is highly compressible even though the vapor phase is still moving at velocities far below its own velocity of sound.
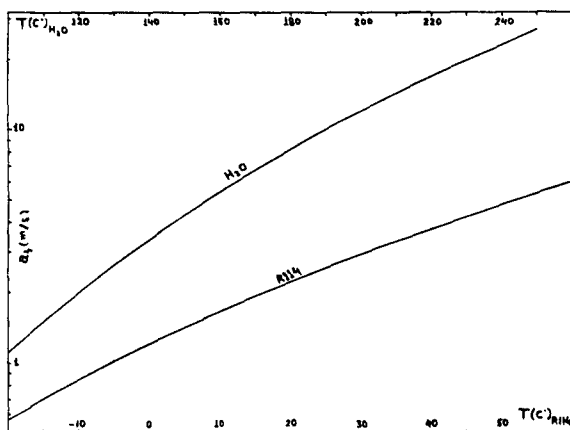
Figure 4.

The choking velocity with no slip is a property of the substance. Figure 4 shows the magnitude of this velocity of "sound" for saturated liquid as a function of temperature, for water and freon R-114 (the substance we are using in our laboratory).

Thus, we can define a Mach number at any point in our flow as the ratio of the flow velocity to the choking velocity at that point. This compressible fluid (two phases of a single substance) will progress from a point where flashing occurs at the flash Mach number, through a series of states until it finally chokes as shown in Fig. 5.
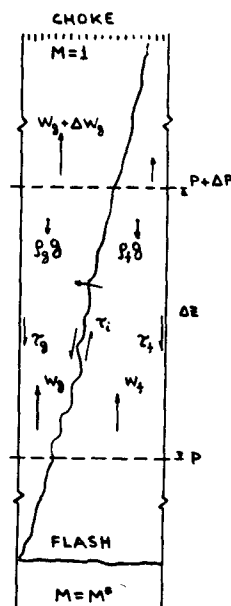
Figure 5.

This leads us to two basic questions: (1) Can we establish any similarity between such a flow of one substance and the flow of another substance? and (2) If we consider one substance only, what are the similarity parameters that are required to be met between the flow of this substance at one initial state and the flow of the same substance at another initial state?

Thermodynamic behavior of substances is complicated and we do not expect that exact similarity laws exist. However, we can create a simplified model of a fluid by assuming the latent heat to be constant, the specific heat and density of the liquid to be constant and the vapor to behave as a perfect gas with an apparent gas constant. For this we can establish a set of scaling parameters which will yield similarity of flows between substances or similarity within the flow regimes of one substance [12].

There is a single dimensionless similarity parameter between substances N_D, the "Dickinson number":

$$N_D = \frac{c_f h_{fg}}{R P_1 v_f} e^{\xi_1},\qquad (3)$$

where ξ_1 is the dimensionless temperature,

$$\xi_1 = -\frac{h_{fg}}{R T_1}\qquad (4)$$

at a reference point.

The analysis also yields the proper scaling parameters for all variables, such as

$$\sqrt{\frac{c_f h_{fg}}{R}}\qquad (5)$$

for velocity and

$$\frac{c_f h_{fg}}{gR}\qquad (6)$$

for length. On the other hand for flow similarity the essential parameters are the Mach, Froude and Reynolds numbers, known from fluid mechanics.

Figure 6 shows the similarity parameter N_D for water substance and R-114 plotted against temperature.
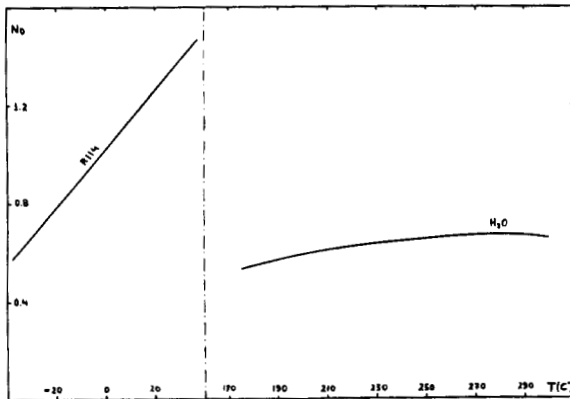
Figure 6.

We see that thermodynamic similarity exists between water substance at 200°C and R-114 at -20°C for instance.
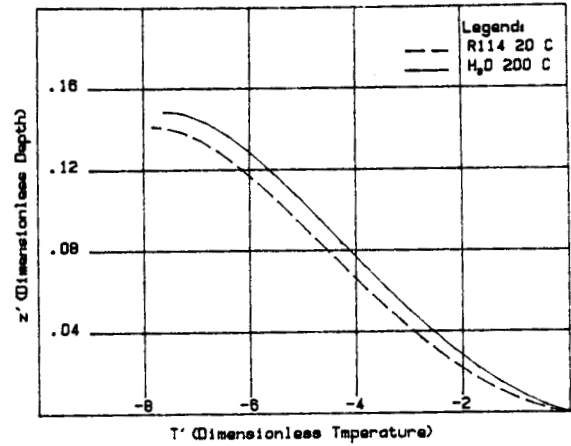
Figure 7.

Figure 7 shows the dimensionless length along a vertical pipe for water substance at 200°C and R-114 at 20°C plotted against the non-dimensional temperature.

The agreement between substances appears to be satisfactory. For purposes of illustration friction effects were neglected (i.e., isentropic flow was assumed). The physical depths are 319 m and 41.5 m for water and R-114, respectively.

4. EQUATIONS FOR THE TWO-FLUID SYSTEM

To complete the presentation in this paper we list the combined conservation equations for the two-fluid model with slip [5].

The conservation of mass equations:

$$\psi x_f = \rho_f \alpha_f w_f; \quad \psi x_g = \rho_g \alpha_g w_g \qquad (7)$$

$$\text{or} \quad \frac{1-x}{x} = \frac{\gamma}{\sigma} \frac{1-\alpha}{\alpha}.$$

The conservation of momentum:

$$\frac{d}{d\tilde{z}}\{\psi w_{cm}\} = \qquad (8)$$

$$-\frac{dP}{d\tilde{z}} + \bar{\tau}_{gw} + \bar{\tau}_{fw} - \rho_m g R_h$$

with $w_{cm} = \sum_p x_p w_p$, $\rho_m = \sum_p \rho_p \alpha_p$ and

$$\bar{\tau}_{pw} = \tau_{pw} \frac{N_p}{C}.$$

The conservation of energy:

$$h + \frac{w_{rms}^2}{2} + g\tilde{z} R_h = h_0 \qquad (9)$$

with $w_{rms}^2 = \sum_p x_p w_p^2$, subscript p indicating the phase (f or g).

The dissipation equation:

$$\psi T \frac{ds}{d\tilde{z}} = \overline{\tau}_{wg} w_g + \overline{\tau}_i (w_g - w_f) + \overline{\tau}_{wf} w_f, \quad (10)$$

where $\overline{\tau}_i = \tau_i \frac{N_i}{C}$.

This last equation is independent of the rest, having been derived from individual phase conservation laws and the Gibbs relation.

Slip σ is contained in all these equations as a parameter while the interface shear stress appears in the dissipation equation. Slip and interface shear are related via the physical phenomena mentioned in the beginning of this paper. Such relationships are usually referred to as closures.

Very little is known about reasonable closures derived, as they should be, from the basic physical interactions between the fluids.

The work done by Taylor [13] and work related to the falling of droplets may serve as a guide to develop such relations. As a primitive approach to the problem one can introduce a pseudo friction factor f so that dissipation is proportional to the kinetic energy of the system $(w_{rms}^2/2)$. This relation can be solved together with equations (7), (9) and (10) for fixed values of f and σ.
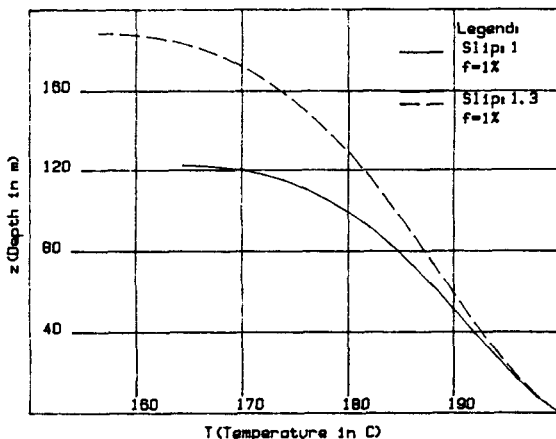

Figure 8.

Figure 8 shows the change in temperature versus length of the two phase flow (from flash to choke) upward in a vertical pipe, for f = 1% and two different values of σ.

We note the large effect on the length of the pipe from flash to choking produced by a

relatively small increase of the slip ratio from 1 to 1.3. Void fraction measurements as reported in Fig. 2 indicate that slip ratios as high as 3 may be present even in horizontal ducts.

On the other hand, as the flow gets closer to choking the mixing of the phases becomes more violent, reducing slip, and the effect shown in Fig. 8 may be exaggerated.

5. CONCLUSIONS

It is clear from the foregoing that the qualitative behavior of the evaporating two phase system with its compressibility effects and choking is well understood. Also, well in hand is the formulation of the time-averaged two-fluid conservation equations. The present difficulties arise with the basic understanding and quantitative assessment of the interface interactions and forces inhibiting the slip between phases. The stronger these interactions are, the smaller will be the slip and the shorter will be the length of the duct from flashing to choking.

In the case of the geothermal reservoir or well in two phase flow, choking greatly reduces productivity. In the case of fluid transmission systems or other applications on the surface this is often recognized and efforts are made to separate vapor and liquid phases and transport them separately, greatly delaying or eliminating choking.

A better understanding, and a development of methods for calculation, of the interactions present if both phases flow together in one duct may lead to similarly effective methods in the Geothermal system.

LIST OF SYMBOLS

Latin letters

w	velocity
P	pressure
z	length
D	diameter of pipe
g	acceleration of gravity
R	gas constant
T	temperature
x	flowing dryness fraction

Latin letters

h	enthalpy
ho	stagnation enthalpy
f	friction factor
s	entropy
C	interior pipe circumference
N_i	length of interface
R_h	hydraulic radius of pipe
N_p	length of pipe circumference wetted by phase p

$Fr = \dfrac{w}{\sqrt{gD}}$ Froude number

$c_f = (\dfrac{\partial h}{\partial T})_{sat}$ specific heat of liquid on saturation line

h_{fg} latent heat

N_D Dickinson number

w_{cm} velocity of the center of mass

w_{rms} rms velocity

$\tilde{z} = z/R_h$

Greek letters

$\sigma = \dfrac{w_g}{w_f}$ slip ratio

$\gamma = \dfrac{\rho_f}{\rho_g}$ density ratio

ρ	density
α	void fraction
τ_w	wall shear stress
ξ	dimensionless temperature
ψ	flowrate per unit area
τ_i	interface shear stress

Subscript g refers to vapor quantities

Subscript f refers to liquid quantities

Subscript p refers to f or g

Note: Change in subscript order on τ changes sign.

REFERENCES

[1] Baker, O. (1954), "Simultaneous Flow of Oil and Gas", Oil Gas J., 53, 185

[2] Lockhart, R.W. and Martinelli, R.C. (1949), "Proposed Correlation of Data for Isothermal Two Phase Two-Component Flow in Pipes", Chem. Eng. Prog., 45, 39.

[3] Bankoff, S.G. (1960), "A Variable Density Single-Fluid Model for Two Phase Flow with Particular Reference to Steam-Water Flow", Trans. ASME, J. Heat Trans., 82, 265.

[4] Hoogendoorn, C.J. (1959), "Gas-Liquid Flow in Horizontal Pipes", Chem. Eng. Sci., 9, 205.

[5] Dickinson, D.A. (1984), "An Investigation of Single-Substance, Horizontal Two Phase Flow", Ph.D. Thesis (in progress), Brown University, Providence, R.I..

[6] Hughmark, G.A. (1962), "Holdup in Gas-Liquid Flow", Chem. Eng. Prog., 58, 62.

[7] Dukler, A.E., Wicks, M., and Cleveland, R.G. (1964), "Frictional Pressure Drop in Two Phase Flow: B: An Approach Through Similarity Analysis", A.I.Ch.E. J., 10, 44.

[8] Zuber, N. and Findlay, J.A. (1965), "Average Volumetric Concentration in Two-Phase Systems", Trans. ASME, J. Heat Trans., 27, 453.

[9] Levy, S. (1963), "Prediction of Two-Phase Pressure Drop and Density Distribution from Mixing Length Theory", Trans. ASME, J. Heat Trans., 85, 137.

[10] Thermohydraulics of Two-Phase Systems for Industrial Design and Nuclear Engineering. J. M. Delhaye, M. Giot and M.L. Riethmueller, eds. Hemisphere, 1981.

[11] Advances in Two Phase Flow and Heat Transfer. S. Kakac and M. Ishii, eds. 2 vols., Martinus Nijhoff, 1983.

[12] Maeder, P.F.. Report in progress.

[13] Davies, R.M. and Taylor, G.I. (1950), "The Mechanics of Large Bubbles Rising Through Extended Liquids and Through Liquids in Tubes". Proc. Roy. Soc., A200, 375.