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# MATRIX DIFFUSION AND ITS EFFECT ON THE MODELING OF TRACER RETURNS FROM THE FRACTURED GEOTHERMAL RESERVOIR AT WAIRAKEI, NEW ZEALAND.

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## ABSTRACT

Tracer tests performed at the geothermal reservoir at Wairakei, New Zealand have been analyzed, using a mathematical and physical model in which tracer flows through individual fractures with diffusion into the surrounding porous matrix. Model calculations matched well with the observed tracer return profiles. From the model, first tracer arrival times and the number of individual fractures (the principal conduits of fluid flow in the reservoir) joining the injector-producer wells can be determined. If the porosity, adsorption distribution coefficient, bulk density and effective diffusion coefficent are known, fracture widths may be estimated. Hydrodynamic dispersion down the length of the fracture is a physical component not taken into account in this model. Future studies may be warranted in order to determine the necessity of including this factor. In addition to the tracer profile matching by the matrix diffusion model, comparisons with a simpler fracture flow model by Fossum and Horne (1982) were made. The inclusion of the matrix diffusion effects was seen to significantly improve the fit to the observed data.

## INTRODUCTION

In many geothermal development schemes, produced geothermal waters are reinjected for the purpose of disposal and pressure maintenance. The known effects of reinjecting water are: improved or degraded thermal recovery (depending on underground flow paths and velocities); permeability changes in the reservoir; pressure maintenance of reservoir fluid; and possible re-routing of natural underground water pathways. Horne (1982) presents a summary of such experience on a worldwide basis.

Since both detrimental and beneficial effects have been observed, reservoir tests to determine the effects of a proposed reinjection system are desireable. Also, various reservoir parameters and the mechanics of fluid flow in the reservoir need be investigated. Interwell tracer tests have made significant contributions to the understanding of fluid flow in natural underground reservoirs. Radioactive and chemical tracers have been used for many years in groundwater hydrology to study the movement of water through porous media, but until recently little has been reported on their use in fractured geothermal systems.

In addition to the test itself, there needs to be some method to analyze the data obtained. To date, tracer returns from geothermal reservoirs have been analyzed in only a semi-quantitative sense to determine transit times, flow velocities and pathways.

In 1982, Fossum and Horne presented an analysis of tracer data from field results at Wairakei, New Zealand, including a model describing linear flow through a fracture with hydrodynamic dispersion. This physical and mathematical model unfortunately proved to be only partially adequate in modeling fluid flow, and does not fit well to many of the more recently obtained test results from the fractured Wairakei geothermal reservoir.

In searching for and testing of a physical model that would better mathematically fit the tracer return data, it has been found here that a 'double-porosity' model is more satisfactory. The 'double-porosity' model formulated in this work includes diffusion of tracer into the porous matrix in addition to flow through the fractures in the reservoir.

## DISCUSSION

The tracer tests which produced the data used in this study were performed by the Institute of Nuclear Sciences, Department of Scientific and Industrial Research, New Zealand. Iodine-131 was used as the tracer. Its half-life is eight days. This eight-day half-life limited the field tests to four to five weeks, by which time a combination of decay corrections and variation of background signals produced unacceptably large errors. This error becomes quite noticeable at late time for some of the tracer return data. For a detailed description of tracer injection methods, well monitoring and counting equipment used at Wairakei see McCabe, Barry and Manning (1983).

The data was corrected for decay and background responses. All negative values have been deleted. Missing data is due to instrument or field problems. Not all of the monitored wells gave sufficient tracer returns and therefore were not analyzed.

Until recently, most mathematical models were based upon a porous media physical model. These porous media type models are useful, but since most geothermal reservoirs are highly fractured they are not entirely applicable, for they assume some type of uniform sweep through the reservoir. Horne and Rodriguez (1983) presented a mathematical model based on the physics of dispersion during fluid flow through fractures, thus forming a basis for the derivation of a transfer function to be used in the interpretation of field observations. Fossum and Horne (1982) utilized this model to analyze some of the tracer return profiles from the Wairakei geothermal field. A double flowpath model was found to give a better data match than a single flowpath model, though interwell flow over long distances was interpreted to occur in only a very few open fissures. However, other tracer test data more recently obtained from Wairakei has proven to be poorly fitted by this simple model.

A possible explanation for this poor fit was indicated by laboratory studies performed by Breitenbach (1982). Significant retention of the tracer in reservoir rocks was observed. The processes producing tracer retention could include adsorption, diffusion, dissolution and ion exchange.

From experimentation, Neretnieks (1980) determined that diffusion into the rock matrix can enhance tracer retardation by many orders of magnitude compared to retardation by surface reaction in fissures only, and that the magnitude of the retardation depends very much on the fissure widths and spacings.

Grisak and Pickens (1980) presented a study concerning the effect of matrix diffusion on solute transport through fractured media. Transport is considered in a manner conceptually similar to 'double-porosity' or 'intra-aggregate' transport models. A finite element model was developed to simulate nonreactive and reactive solute transport by advection, mechanical dispersion, and diffusion in a unidirectional flow field. The numerical model and the laboratory tracer test data provided insight into the processes controlling solute transport in fractured media.

From studies of the migration of radionuclides in the bedrock surrounding nuclear waste repositories, Neretnieks, Eriksen, and Tahtinen (1982) developed a mathematical and physical model describing tracer movement in a single fissure of granitic rock. This model takes into account instantaneous sorption on the surface of the fissure, and loss of tracer from the fluid flowing in the fissure due to diffusion into the porous matrix. It is this model that is used to help gain insight and a physical understanding of the fluid flow implied by the tracer tests performed at the geothermal reservoir field at Wairakei, New Zealand.

Although fractures are the principal paths of groundwater flow and solute transport, the matrix adjacent to the fractures plays an important part in the overall solute transport process. The effect of matrix diffusion is to provide solute storage, with the rate of change of storage within the matrix related to Fick's second law of diffusion.

The net effect of matrix diffusion is to retard the arrival of the solute at any point along the fracture. If the source of the solute is discontinued, the effect will be to flush the fracture and reverse the concentration gradient, causing solute to move from the matrix into the fracture.

Two equations describing the physical situation of one-dimensional advective flow through a fracture with simultaneous tracer adsorption and diffusion into the surrounding porous matrix are as follows:

$$R \frac{\partial C_f}{\partial t} - \frac{2D_e}{\delta} \left.\frac{\partial C_p}{\partial y}\right|_{y=0} + U_f \frac{\partial C_f}{\partial x} = 0 \qquad (1)$$

$$D_a \frac{\partial^2 C_p}{\partial y^2} = \frac{\partial C_p}{\partial t} \qquad (2)$$

A linear equilibrium relationship between the dissolved and sorbed phases of the solute has been assumed. Linear adsorption assumes that once the tracer and rock are brought sufficiently close together, adsorption will be an instantaneous process.

Two different diffusion coefficients, $D_a$ and $D_e$, are presented in Equations 1 and 2. The apparent and effective diffusion coefficients are related as follows:

$$D_a = \frac{D_e}{K_d \rho_b} \tag{3}$$

The effective diffusion coefficient $D_e$ is dependent on temperature, porosity, molecular diffusivity, and the geometry of the rock. $K_d \rho_b$ is a volumetric sorption equilibrium constant and is related to porosity $\phi$, the solid rock density $\rho_s$ and the adsorption distribution coefficient k by the equation,

$$K_d \rho_b = \phi + (1 - \phi) k \rho_s \tag{4}$$

Notice that if the solids are inert, i.e., k=0, the porous rock matrix still has a volumetric sorption equilibrium constant equal to its porosity $\phi$. Rearrangement of Equation 4 gives,

$$R = \frac{K_d \rho_b}{\phi} = 1 + \frac{k \rho_b}{\phi} \tag{5}$$

where R is refered to as the retardation factor.

The retardation factor defines the mean velocity of the moving liquid relative to the mean velocity at which the tracer itself moves through the rock. This factor accounts for the slowing down of a tracer moving with the fluid due to the interaction with the solid. If there is no interaction between the tracer and the solid phase, k becomes zero and R reduces to one.

The initial and boundary conditions are a finite rectangular pulse of tracer with duration $\Delta t$ introduced at the inlet of the fracture at time t=0, and the fracture and rock are originally free of tracer.

The solution to Equations 1 and 2 subject to the given initial and boundary conditions is,

$C_f = 0$ for $t < t_w R$

and for $t > t_w R$,

$$C = \frac{E \alpha_1 \alpha_2}{\sqrt{\pi} (\alpha_2 t - 1)^{1.5}} \exp\left(- \frac{\alpha_1^2}{(\alpha_2 t - 1)}\right) \tag{6}$$

where,

$$\alpha_1 = \left(D_e \phi t_w\right)^{0.5} / \delta \quad \text{and} \quad \alpha_2 = \frac{1}{t_w R}$$

The linear parameter E normalizes the flow fraction to one. This normalization is needed because precise information on the initial concentration injected into the fracture system connected with the producing well is not available. This does not affect the shape of the calculated tracer profile, but merely the size.

Tester, Bivens, and Potter (1982) proposed the use of an objective function F over N measured data points in order to analyze for optimum values $\alpha_1$ and $\alpha_2$ in the transfer function $C(t;\alpha_1,\alpha_2)$ for a given tracer return profile. When F, given by

$$F = \sum_{i=1}^{N} \left(C(t;\alpha_1,\alpha_2) - C_i\right)^2 \tag{7}$$

is minimized, optimum values of $\alpha_1$ and $\alpha_2$ result. A multifracture model assuming one-dimensional flow in separate fractures and which gives the predicted tracer concentration response is given by

$$C = \sum_{j=1}^{M} \varepsilon_j C_j (t;\alpha_{1j},\alpha_{2j}) \tag{8}$$

where $\varepsilon_j$ is the fraction of flow in fracture path j. The relative flow fractions in the fracture system communicating with the production well and the injection well is given by

$$\sum_{j=1}^{M} \frac{\varepsilon_j}{E} = 1 \qquad (9)$$

This multifracture model is used to determine whether the tracer returns to a producing well is a result of flow through one or more fractures. Once the above objective function is minimized, the resulting optimized parameters are used to give information about the fracture system and flow mechanisms in the geothermal reservoir.

Optimization of the parameters in the transfer function $C(t;\alpha_1,\alpha_2)$ is accomplished using a nonlinear least-squares method of curve fitting based on a paper by Golub and Pereya (1973). A computer program calls for the input of the tracer return data, the number of parameters being used, and estimates of the nonlinear parameters. Subroutine VARPRO (written by Stanford University Department of Computer Science) and its accompaning subroutines are called to optimize the objective function. The main program then calls for the plotting of the tracer return data along with the computed best fit tracer return profile, and the optimal values of both the nonlinear and linear parameters of the given transfer function are printed.

## RESULTS

The tracer return data for the various wells were fitted to the mathematical model using the computer program discussed previously. Figures 2, 4, and 6 show the fitted data profiles. The squares represent the data and the solid line is the calculated curve fit. For comparison purposes, Figures 1, 3, and 5 show corresponding curve fits using the model presented by Fossum and Horne (1982). Remember that their model includes only advection and dispersion along one or more non-connecting or channeled fractures. The model presented in this report includes adsorption, advection, and diffusion into the surrounding porous matrix. This inclusion of diffusion gives considerable improvement in the curve fit of the tracer return profiles. Furthermore, in many of the wells only a single fracture model is required to smoothly fit the data, whereas multi-fracture modeling was required in the cases presented by Fossum and Horne (1982). This is more pleasing since most curves can be fitted if several linear combinations of the single path equation are used, irrespective of the physical applicability.

Values for the flow fractions and nonlinear parameters $\alpha$ and $\beta$ for the different calculated tracer return curve fits are given in Table 1.

For a few of the tracer return data, double fracture modeling was possible but did not substantially improve the single fracture curve fits. Where improvement was possible, however, these fits were used in place of the single curve fits.

Not all the tracer returns are well fitted. One such tracer return is from well WK121. The fit (Fig. 6) is better than that obtained with the non-diffusive model (Fig. 5), however it still does not reproduce the entire observed behavior. Reasons for poor fits may be that (1) hydrodynamic dispersion down the length of the fracture needs to be included to better model the fluid and tracer flow, (2) the instantaneous linear adsorption assumption is not valid, or (3) temperature effects on k, and $D_e$ are of importance.

Well WK121 is an interesting case in that a good fit was obtained when modeled as a double fracture case. However, a negative flow fraction is calculated. This anomaly could have a physical or mathematical significance, but most likely is an artifact of the curve fitting technique itself, in that more than one approach to convergence may be possible.

The goal of tracer return analysis is to infer information concerning the flow velocities, fracture widths, flow pathways, and reservoir rock and fluid properties such as diffusion and adsorption coefficients. To do this with the Wairakei data at hand requires some knowledge of the parameters $\rho_b$, $D_e$, k, and $\phi$.

If a nonsorbing tracer is used then k=0, R=1, and $K_d\rho_b=\phi$. In this nonsorbing case some knowledge of the porosity $\phi$ and effective diffusion coefficient $D_e$ is required to calculate fracture width values for the corresponding curve fit. In Table 1, fracture width values are given based on the nonsorbed tracer assumption and the effective diffusion coefficient value of $4.32 \times 10^{-6}$ m$^2$/day ($5 \times 10^{-11}$ m$^2$/s). The value for $D_e$ is a medium value obtained from a range of values given by Neretnieks (1980) for nonsorbing tracers in granites. This value is not necessarily the proper value to be used in this case, but it does allow one to speculate on possible fracture widths. Also, since the matrix porosity of the Wairakei reservoir is not definitively known, porosity values of 1% and 5% were used in the the calculations.

In Table 1, flow velocities have been calculated based on the injector-producer distances and calculated first tracer arrival times. An assumption of the tracer not being sorbed to the reservoir rock (R=1) is also made in these calculations. As the injector-producer distances are not necessarily representative of pathlengths in the reservoir, these calculated velocities are minimum values.

## CONCLUSIONS

1. Tracer diffusion into the matrix of the Wairakei geothermal reservoir is an important factor in the mechanism of tracer flow. Estimated reservoir parameters such as fracture widths, fluid velocities and dispersion characteristics are difficult to accurately interpret in a fractured reservoir without accounting for matrix diffusion. The diffusion of tracers into the rock matrix and their sorption onto the surfaces of the rock are the main mechanisms retarding migration through fractures.

2. In using the fracture model presented by Fossum and Horne (1982) to analyze the Wairakei data, a double flowpath model gave a more accurate data match than a single component model. However, in using the matrix diffusion model presented in this report, single fracture flowpath modeling was sufficient in many of the cases.

3. Without further investigation of representative values for the effective diffusion coefficient $D_e$, bulk rock density $\rho_b$, porosity $\phi$, and the adsorption distribution coefficient k, quantitative values for the various reservoir and fluid flow properties cannot be accurately calculated for the Wairakei reservoir.

4. Further study into the modeling of tracer flow through fractured media which takes into account hydrodynamic dispersion down the length of the fracture in addition to diffusion into the porous matrix may be warranted.

## NOMENCLATURE

| | |
|---|---|
| $\alpha$ | nonlinear parameters |
| $C_f$ | concentration of tracer in fracture |
| $C_p$ | concentration of tracer in porous matrix |
| $D_a$ | apparent diffusion coefficient |
| $D_e$ | effective diffusion coefficient |
| $\delta$ | fracture width |
| E | linear scaling factor |
| $\epsilon$ | fraction of flow |
| F | objective function |
| k | adsorption distribution coefficient |
| $K_d\rho_b$ | volumetric sorption equilibrium constant |
| M | number of proposed fracture paths |
| N | number of data points |
| $\rho_b$ | bulk density of the medium |
| $\rho_s$ | solid rock density |
| R | retardation factor |
| $t_w$ | water residence time |
| $\phi$ | porosity |
| $U_f$ | fluid velocity in the fracture |
| x,y | Cartesian directions |

## ACKNOWLEDGEMENT

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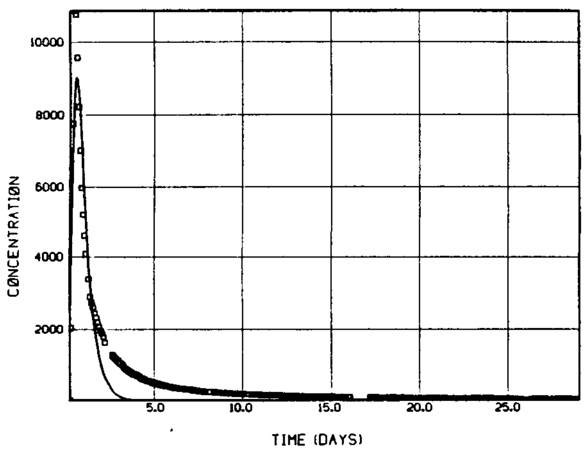


Figure 1: Wairakei (3/79) - CWK24 from WK107 Fossum model: single fracture fit

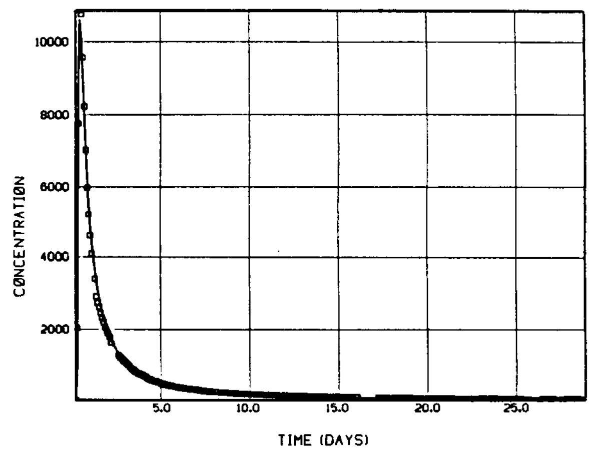


Figure 2: Wairakei (3/79) - CWK24 from WK107 Matrix diffusion model: single fracture fit

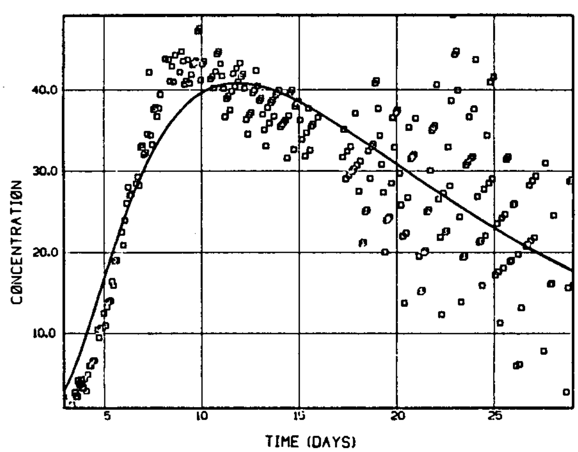


Figure 3: Wairakei (3/79) - CWK70 from WK107 Fossum model: single fracture fit

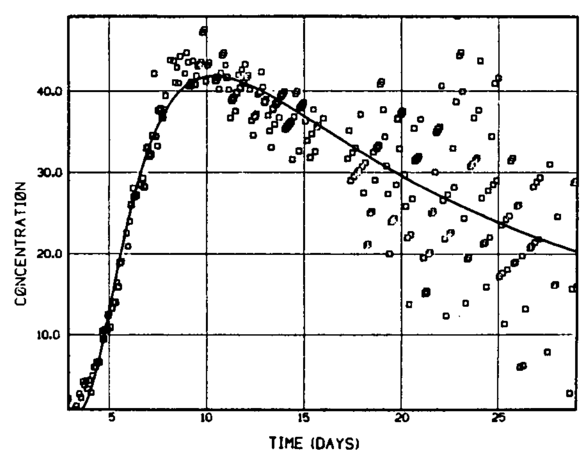


Figure 4: Wairakei (3/79) - CWK70 from WK107 Matrix diffusion model: single fracture fit

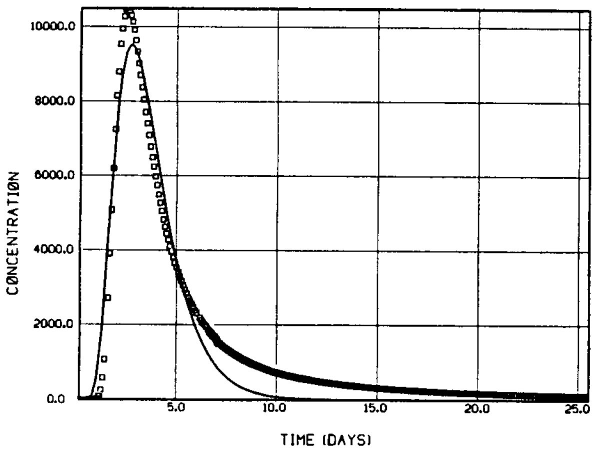


Figure 5: Wairakei (7/79) - CWK121 from WK101
Fossum model: single fracture fit

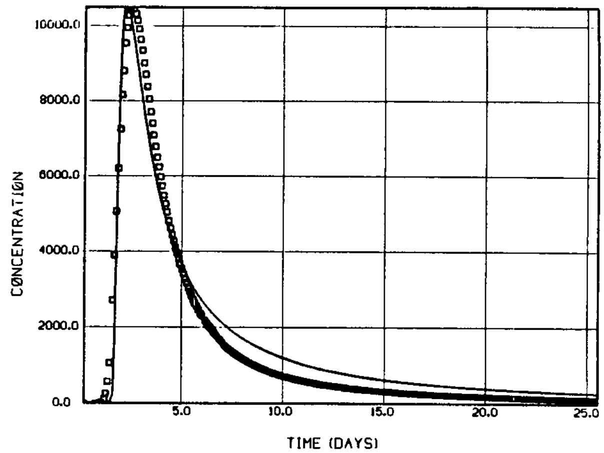


Figure 6: Wairakei (7/79) - CWK121 from WK101
Matrix diffusion model: single fracture fit

TABLE 1

| Production well | Injector-Producer Distance (meters) | Flow Fraction $\epsilon_j/E$ | Nonlinear Parameters $\alpha_1$ | $1/\alpha_2$ (days) | Minimum Flow Velocity (m/hr) | Fracture Width (mm) $\phi$=1% | $\phi$=5% |
|---|---|---|---|---|---|---|---|
| WK24 | 210 | 1.000 | 1.250 | 0.231 | 37.9 | 0.08 | 0.18 |
| WK30 | 240 | 0.811 | 1.370 | 4.367 | 2.3 | 0.32 | 0.71 |
| | | 0.189 | 1.270 | 3.212 | 3.1 | 0.29 | 0.66 |
| WK48 | 120 | 0.450 | 1.393 | 0.293 | 17.1 | 0.08 | 0.18 |
| | | 0.550 | 1.669 | 1.040 | 4.8 | 0.13 | 0.28 |
| WK55 | 220 | 1.000 | 2.578 | 2.671 | 3.4 | 0.13 | 0.29 |
| WK67 | 120 | 1.000 | 2.736 | 1.651 | 3.0 | 0.10 | 0.22 |
| WK68 | 120 | 1.000 | 2.049 | 2.919 | 1.7 | 0.17 | 0.39 |
| WK70 | 170 | 1.000 | 2.483 | 2.033 | 3.5 | 0.12 | 0.27 |
| WK81 | 175 | 1.000 | 1.535 | 3.659 | [illegible].0 | 0.26 | 0.58 |
| WK83 | 330 | 1.000 | 2.167 | 2.550 | 5.4 | 0.15 | 0.34 |
| WK108 | 80 | 1.000 | 1.685 | 6.782 | 0.5 | 0.32 | 0.72 |
| WK103 | 165 | 1.000 | 3.437 | 0.619 | 11.1 | 0.05 | 0.11 |
| WK116 | 350 | 0.259 | 0.920 | 4.696 | 3.1 | 0.49 | 1.09 |
| | | 0.741 | 3.844 | 0.626 | 23.4 | 0.04 | 0.10 |
| WK121 | 490 | 1.000 | 0.916 | 1.451 | 14.1 | 0.27 | 0.61 |
| | | 0.530 | 2.555 | 0.719 | 28.4 | 0.07 | 0.15 |
| | | -0.470 | 2.100 | 1.265 | 16.1 | 0.11 | 0.25 |

All production wells produce tracer injected at well WK107, except wells WK103, 116, and 121 which produce tracer injected at WK101.