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A NUMERICAL SIMULATION MODEL FOR VERTICAL FLOW IN GEOTHERMAL WELLS

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Abstract

This paper presents a numerical simulation model for vertical flow in geothermal wells. The model consists of equations for the conservation of mass, momentum, and energy, for thermodynamic state of water, for friction losses, for slip velocity relations, and of the criteria for various flow regimes. A new set of correlations and criteria is presented for two-phase flow to improve the accuracy of predictions; bubbly flow - Griffith and Wallis correlation, slug flow - Nicklin et al. one, annular-mist flow - Inoue and Aoki and modified by the author. The simulation method was verified by data from actual wells.

Introduction

Numerical simulation of flow in geothermal wells are useful for evaluating downhole pressures, temperatures, enthalpies, and steam qualities from those at well heads, for predicting producible steam and water flow rates by production wells on the basis of data about the reservoir characteristics obtained from wildcat wells, and for designing proper casing programs for wells.

Simulation methods for flow in oil wells have been applied to flow in geothermal wells with a heat transfer equation by several authors [1]-[3]. However, we found that they are not accurate enough for two-phase flow especially in a well with a small diameter of 3 or 4 inches.

A number of correlations for two-phase flow discussed in existing literature have been examined and a set of correlations has been selected to improve the accuracy of calculated values. A new relationship has been developed to deal with the annular-mist flow regime.

Flow equations

Flow in geothermal wells has a variety of configurations, such as subcooled water flow, water-steam mixture flow, and superheated steam flow. In the description of flow equations, subcooled water flow and superheated steam flow are regarded as particular cases of two-phase flow with steam fraction 0 and 1 respectively. Two-phase vertical flow is expressed by the separate and slipping flow model (Figure 1). Water and steam phases are assumed to be in local thermodynamic equilibrium. The steady flow equations are given as follows.

Mass conservation equation

$$\rho_g v_g f_g + \rho_\ell v_\ell (1 - f_g) = W_s \tag{1}$$

Momentum conservation equation

$$\begin{aligned}&[\rho_g v_g{}^2 f_g + \rho_\ell v_\ell{}^2 (1 - f_g) - P \\ &\qquad + \left\{\tau + (\rho_g f_g + \rho_\ell (1 - f_g))g\right\} dz] A \\ &= [\rho_g^\circ v_g^{\circ\,2} f_g^\circ + \rho_\ell^\circ v_\ell^{\circ\,2}(1 - f_g^\circ) - P^\circ] A^\circ\end{aligned} \tag{2}$$

Energy conservation equation

$$\begin{aligned}&[\rho_g v_g f_g (\tfrac{1}{2} v_g{}^2 + h_g) + \rho_\ell v_\ell (1 - f_g)(\tfrac{1}{2} v_\ell{}^2 + h_\ell) \\ &\qquad + (W_s g + Q) dz] A \\ &= [\rho_g^\circ v_g^\circ f_g^\circ (\tfrac{1}{2} v_g^{\circ 2} + h_g^\circ) \\ &\qquad + \rho_\ell^\circ v_\ell^{\circ 2} (1 - f_g^\circ)(\tfrac{1}{2} v_\ell^{\circ 2} + h_\ell^\circ)] A^\circ\end{aligned} \tag{3}$$

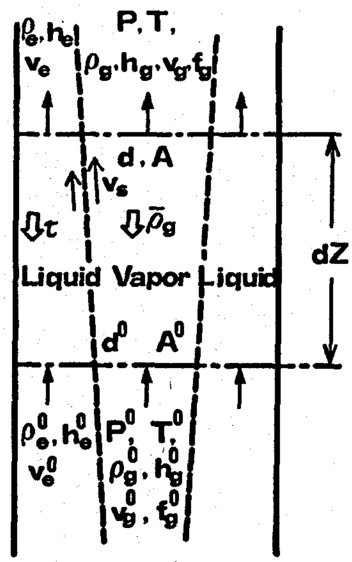

Figure 1. Separate and slipping two-phase flow model.

The acceleration term in Eq.2 has often been neglected by other authors [1]-[3], except in the annular-mist flow regime, however, in case of geothermal wells acceleration is important for slag flow, as well.

Dependent unknown variables of the equations are temperature T, depth increment dz, and velocity v_g or v_ℓ for single-phase flow, steam fraction f_g, depth increment dz, velocities v_g and v_ℓ for two-phase flow.

In addition to the above three conservation equations, equations of the friction loss gradient, of the slip velocity relationships, and of the fluid state, are required.

Flow regimes

The configuration of steam-water two-phase flow is complicated and usually divided into three or four flow regimes [1]-[3]. In the present work, three flow regimes are defined: bubbly flow, slug flow, and annular-mist flow. The correlations employed for each regime are as follows.

Bubbly flow	Griffith and Wallis
Slug flow	Nicklin et al.
Annular-mist flow	Inoue and Aoki as modified by the author

The boundary of bubbly and slug flow regimes is determined by the criteria proposed by Duns and Ros [6], and that of slug and annular-mist by the criteria presented by the author. The experimental results on flashing flow [9] have shown that the slug-annular transition occurrs in the steam fraction range of 0.7-0.8. When steam quality is less than 0.3, a steam fraction of 0.6 is chosen as the criterion for the slug-annular boundary to obtain a smooth transition between the regimes when using the present correlations.

Bubbly flow

$$(q_g/q_t)\le L_B \tag{4}$$

Slug flow

$$(q_g/q_t)>L_B,\quad \begin{array}{ll} f_g<0.6 & (\text{when } x<0.3)\\ f_g<0.75 & (\text{when } x\ge 0.3)\end{array} \tag{5}$$

Annular-mist

$$(q_g/q_t)>L_B,\quad \begin{array}{ll} f_g\ge 0.6 & (\text{when } x<0.3)\\ f_g\ge 0.75 & (\text{when } x\ge 0.3)\end{array} \tag{6}$$

where

$$L_B=1.071-7.14v_t^2/gD,\ L_B\ge 0.13$$

Friction loss gradient

The friction loss gradient is basically calculated by a Fanning type of friction law.

$$\tau=\lambda\rho v^2/2D \tag{8}$$

where the friction factor λ is obtained by entering the Moody diagram with the appropriate Reynolds number N_{Re} and relative roughness of tube ε/D.

Single phase flow

The density, velocity and viscosity values of the phase must be entered in Eq.8 and in the Reynolds number $N_{Re}=vD/\nu$.

Bubbly flow

As the gas bubbles are dispersed inside the liquid and rise with small slip velocities, they have little effect on the friction losses.

$$\tau=\lambda\rho_\ell v_\ell^2/2D \tag{9}$$

where λ is determined as a function of $N_{Re}=v_\ell D/\nu_\ell$.

Slug flow

The friction losses are mainly caused by liquid slugs, at the center of which liquid velocities are nearly equal to the mean fluid velocity.

$$\tau=\lambda\rho_\ell(1-f_g)v_t^2/2D \tag{10}$$

where λ is determined as a function of $N_{Re}=v_\ell D/\nu_\ell$.

Annular-mist flow

According to Inoue and Aoki [7], the friction loss gradient is calculated by multiplying the friction loss gradient associated with the specific liquid velocity by the factor R, as shown in Eq.11 and 12.

$$\tau=R\lambda\, W_s^2/\rho_\ell/2D \tag{11}$$

$$R=1+250[x/(1-x)] \tag{12}$$

Their equations give rather large values for friction losses compared to observed pressure drops in geothermal wells. Instaed of them the following equations are used in the present work.

$$\tau_\ell=\lambda_\ell\rho_\ell v_\ell^2(1-f_g)^2/2D \tag{13}$$

$$\tau_g=\lambda_g\rho_g v_g^2 f_g^2/2D \tag{14}$$

$$\tau=\gamma\tau_g x+\tau_\ell(1-x) \tag{15}$$

where λ_ℓ and λ_g are determined as a function of $N_{Re\ell}=v_\ell(1-f_g)D/\nu_\ell$ and $N_{Reg}=v_g f_g D/\nu_g$ respectively. γ is about 0.7.

Slip velosity

Bubbly flow

Experimental results [9] indicate that the slip velocities, v_g-v_ℓ, are independent of the tube size and are approximately constantsat 20-30 cm/s when tube diameters are greater than five times the effective diameter of the bubbles. In the present work a constant slip velocity of 24cm/s suggested by Griffith and Wallis [4] is used.

Slug flow

Gas slug velocity has been given by Nicklin et al. [5]. Its relationship is rewritten as follows.

$$v_s=[v_g+1.75\ (gD)^{\frac{1}{2}}]\ /6(1-f_g) \tag{16}$$

Annular-mist flow

Inoue and Aoki [7] have proposed a slip velocity relationship that is valid from the slug flow regime to the annular. It is given as follows.

$$v_s = \alpha[v_\ell(1-f_g)]^\beta x \qquad (17)$$

where

$$\alpha \approx \rho_\ell/\rho_g$$
$$\beta \approx 1 \qquad (18)$$

Eq. 17 can be rewritten as follows.

$$v_s = (W_s/\rho_g)x(1-x) \qquad (19)$$

Thermodynamic properties of the fluids

The densities ρ_g, ρ_ℓ, the enthalpies h_g, h_ℓ, the kinematic viscosities ν_g, ν_ℓ are obtained from the equations prepared by the International Formulation Committee of the Sixth International Conference on the Properties of steam.

Computational procedure

The flow chart of the numerical simulator is presented in Figure 2. The outline of the computational procedure follows.

1. Input the casing geometries and the flow conditions either at the well head or at the well bottom: pressure, temperature, steam flow rate and water flow rate.

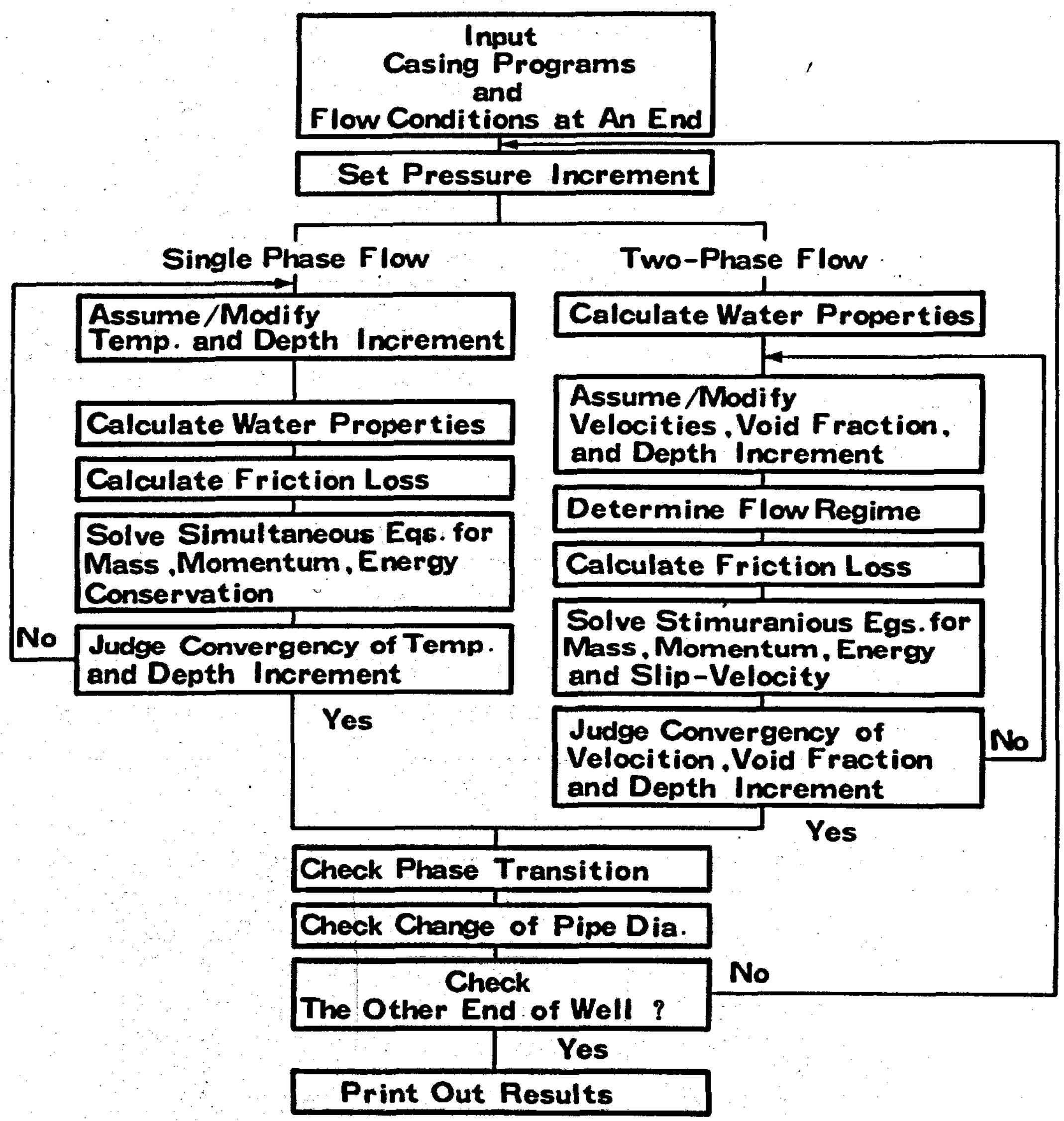

Figure 2. Flow chart of simulation for flow in geothermal wells.

2. Set the pressure increment and determine thermodynamic properties of the fluids.
3. Assume a flow regime, calculate the friction loss gradient and chose the appropriate slip velocity relationship.
4. Solve the coupled equations for mass, momentum, energy conservation and slip velocity by means of the Newton-Raphson method.
5. Check whether the flow regime determined from the calculated values of steam and water flow rates, and of steam fraction, is the same as the assumed one. When they differ, repeat the procedures 3 - 5. When they agree, return to 2 and go to the next pressure increment.
6. Repeat the procedures 2 - 5 until reaching the other end of the well.

Verification of model

To test the validity of the model, the numerical simulator developed was applied to well KE1-3 which was drilled in the Kirishima geothermal field by Nippon Steel Corp. and Nittetsu Mining Co., and to HGP-A in Hawaii.

Table 1 presents the completion characteristics of the KE1-3, and Table 2 the well head conditions of KE1-3 during the well tests. During discharge a Kuster pressure instrument was put down into the well and remained for a few minutes at every one hundred meters to measure pressure. Figure 3 shows the predicted and measured pressures at KE1-3. The calculated results indicate that the subcooled water flow is at deep levels and above it apper the slug and annular flow. Although the predicted pressures near the well head are slightly lower than measured ones, as a whole they agree well.

The other test was performed using the data published on HGP-A in Hawaii [8]. Since HGP-A has high steam quality, it was interesting to test the validity of the correlation for the annular-mist regime which had never been examined in the high quality ranges. The well head conditions of HGP-A during flashing are referenced and shown in Table 3. HGP-A is cased by a pipe (9×5/8 in.) to a depth of 2500 ft and below it is liner pipe (7×5/8 in.). Its bottom is at a depth of 6450 ft. Figure 4 shows the predicted and actual pressures on HGP-A. The agreement is fairly good.

Conclusion

1. A numerical simulation model has been presented for vertical flow in geothermal wells. The algorism is based on the coupled equations for mass, momentum, and energy conservation. Friction losses and slip velocities are calculated to the phase and the flow regime at each point in a well.
2. A new set of correlations has been proposed to improve the accuracy of the predicted values for two-phase flow. The Griffih and Wallis correlation is used for bubbly flow regime, the Nicklin et al. correlation for slug flow regime, and the Inoue and Aoki correlation, which has been modified by the author, for annular-mist flow regime. A new criteria on the boundary of the slug and annular-mist flow regimes has also been presented.
3. The method has been tested for validity on three cases of a water-dominated actual well, and on four cases of a vapor-dominated well. Although in all cases the heat losses to the formation were not considered, good agreement between the predicted and observed values was obtained.

Table 1. Completion characteristics of KE1-3.

Cased Depth m	Liner Length m	Casing Pipe I.D. mm	Slotted Liner I.D. mm	Feed zone Depth m
885	316	102.3	67	1170

Table 2. Well head conditions of KE1-3 during well test.

Test	Pressure (kg/cm^2G)	Steam Flow RATE (tonne/hr)	Water Flow RATE (tonne/hr)	Enthalpy (kcal/kg)
1	1.40	3.96	17.14	223
2	4.20	1.75	13.47	211
3	2.74	3.11	16.56	222

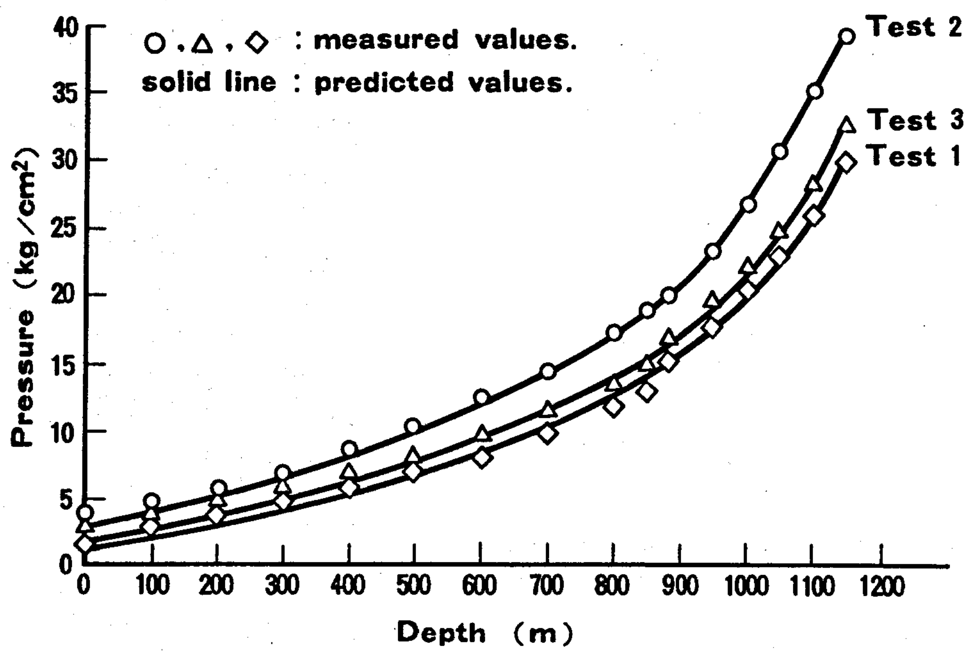

Figure 3. Comparison of measured pressures and predicted pressures on KE1-3 in the Kirishima field.

Table 3. Well head conditions of HGP-A during flashing.

Total Mass Flow Rate (tonne/hr)	Steam Flow Rate (tonner/hr)	Pressure (kg/cm^2G)	Temp. (°C)
44.9	31.8	3.8	149
42.2	29.9	7.0	170
38.1	26.3	16.7	205
34.5	22.7	26.4	226

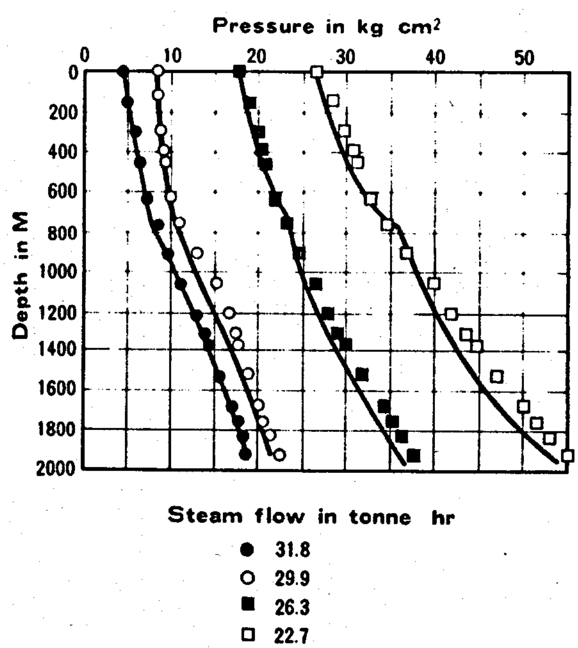

Figure 4. Comparison of measured pressures (symbols) and predicted pressures on HGP-A in Hawaii.

Nomenclature

A	= flow area of pipe, m^2
D	= pipe diameter, m
dz	= depth increment, m
f_g	= void fraction, volumetric gas fraction, -
g	= acceleration of gravity, m/s^2
h_g	= specific enthalpy of gas, J/kg
h_ℓ	= specific enthalpy of liquid, J/kg
N_{Re}	= Reynolds number, -
P	= pressure, Pa
Q	= heat loss to surroundings, J/sm
q_g	= volumetric flow rate of gas, m^3/s
q_ℓ	= volumetric flow rate of liquid, m^3/s
q_t	= volumetric flow rate of fluid, m^3/s
T	= temperature, °C
v_g	= velocity of gas, m/s
v_ℓ	= velocity of liquid, m/s
v_s	= slip velocity, m/s
v_t	= mean velocity of two-phase flow, m/s
W_s	= mass flux of fluid, kg/m^2s
x	= quality, -
ε	= pipe wall roughness, m
λ	= Moody friction factor,
μ_g	= dynamic viscosity of gas, Ns/m
μ_ℓ	= dynamic viscosity of liquid, Ns/m
ν_g	= kinematic viscosity of gas, m^2/s
ν_ℓ	= kinematic viscosity of liquid, m^2/s
ρ_g	= density of gas, kg/m^3
ρ_ℓ	= density of liquid, kg/m^3
τ	= friction loss gradient, N/m

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