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# WELL TEST ANALYSIS FOR NATURALLY FRACTURED RESERVOIRS

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## ABSTRACT

Pressure transient solutions for constant rate production and transient rate analysis for constant pressure production are presented for a naturally fractured reservoir. The results obtained for a finite no-flow outer boundary -- are surprising. Initially, the flow rate shows a rapid decline, then it becomes nearly constant for a certain period, and finally it falls to zero. Ignoring the presence of a constant flowrate period in a type-curve match can lead to erroneous estimates of the dimensionless matrix pressure and fracture pressure distributions are presented for both the constant rate and constant pressure production cases. Interference test for-constant rate production can be interpreted for long times by means of the line-source solution. For the constant pressure production case, the pressure away from the wellbore does not correlate well with the line source solution.

## INTRODUCTION

Naturally fractured reservoirs or reservoirs with double porosity behavior as they are commonly referred consist of heterogeneous porous media where the opening (fissures and fractures) vary considerably in size. Fractures and openings of large size form vugs and interconnected channels, whereas the fine cracks form block systems which are the main body of the reservoir. The porous blocks store most of the fluid in the reservoir and are often of low permeability, whereas the fractures have a low storage capacity and high permeability. Most of the fluid flow will occur through the fissures with the blocks acting as fluid sources.

These systems have been studied extensively in the petroleum literature. One of the first such studies was published by Pirson(1.953). In 1959 Pollard presented one of the first pressure transient models available for interpretation of well test data from two-porosity systems. The most complete analysis of transient flow in two-porosity systems was presented by Barenblatt and Zheltov (1960) - The Warren avad Root (1963) study is widely considered to be the forerunner of modern interpretation of two-porosity systems. The behavior of fractured systems has long been a topic of controversy. Warren and Root and Kazemi (1969) have indicated that the graphical technique proposed by Pollard is susceptible to error caused by approximations in the mathematical model-nevertheless, the Pollard method is still used. The most complete study of two-porosity systems appears to be the Mavor and Cinco- Ley Study in 1979. This study considers wellbore storage and skin effect, and also considers production, both at constant rate and at constant pressure. However, little information is presented concerning the effect of the size of the system on pressure build-up behavior.
In this paper a literature review is presented on the basic solutions which can be applied in dealing with pressure transient analysis naturally fractured reservoirs.

## PARTIAL DIFFERENTIAL EQUATIONS

The basic partial differential equations for fluid flow in a two-porosity system were presented by Warren and Root in 1963. The model has been extended by Mavor and Cinco Ley (1979) to include wellbore storage and skin effect. Da Prat (1981) extended the model and developed a method to determine the permeability thicknes product.Kh. Deruyck et all(1982) applied with success the Warren and Root model to study interference data from a geothermal field.
The basic partial differential equations are: Da Prat (1981).

$$\frac{\partial^2 p_{Df}}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_{Df}}{\partial r_D} = (1-\omega)\frac{\partial p_{DM}}{\partial t_D} + \omega\frac{\partial p_{Df}}{\partial t_D} \qquad (1)$$

and

$$(1-\omega)\frac{\partial P_{Dm}}{\partial t_D} = \lambda(p_{Df} - p_{Dm}) \qquad (2)$$

$P_{fD}$ and $t_D$ are defined in the nomenclature.

where $\omega$ is the dimensionaless fracture storage parameter:

$$\omega = \frac{(\phi c)_f}{\{(\phi c)_f + (\phi c)_m\}} \qquad (3)$$

and $\lambda$ controls the interporosity flow and is given by

$$\lambda = \alpha\frac{km}{k_f}\ r_w^2 \qquad (4)$$

A complete mathematical definition requires additional equations which represent the appropiate initial and boundary conditions.

The initial boundary conditions is:

$$p_{mD} = p_{fD}(r_D, 0) = 0 \qquad (5)$$

For a well producing at a constant pressure the inner boundary condition is:

$$p_{fD} - S\left(\frac{\partial p_{fD}}{\partial t_D}\right)_{r_D=1} = 1 \qquad (6)$$

Where S is the skin factor

For a well producing at a constant flow rate the condition is:

$$C_D \frac{\partial p_{fwD}}{\partial t_D} - \left(\frac{\partial p_{fD}}{\partial r_D}\right)_{r_D} = 1 \qquad (7)$$

where:

$$C_D = \frac{c}{2\pi h\{(\phi c)_f + (\phi c)_m\}\cdot r_w^2} \qquad (8)$$

The skin effect condition is:

$$p_{fwD} = p_{fD} - S\left(\frac{\partial p_{fD}}{\partial r_D}\right)_{r_D=1} \qquad (9)$$

Two outer boundary conditions are considerated in this study: an infinitely large reservoir and a closed outer boundary. For an infinitely large reservoir, we have:

$$\lim_{r_D \to \infty} p_{fD}(r_D, t_D) = 0 \qquad (10)$$

For the closed outer boundary, the condition is:

$$\frac{\partial p_D}{\partial r_D}\Big|_{r_D = r_{eD}} = 0 \qquad (11)$$

The dimensionless flow-rate into the wellbore is given by:

$$q_D(t_D) = -\left(\frac{\partial p_D}{\partial r_D}\right)_{r_D=1} \qquad (12)$$

where:

$$q_D = \frac{141.2\, q\mu B}{k_f h\,(pi - p_{wf})} \qquad (13)$$

METHOD OF SOLUTION

A common method for solving Eqs. 1 and 2 is to use the Laplace transformation. The equations are transformed into a system of ordinary differential equations which can be solved analytically. The resulting solution in the transformed space is a function of the Laplace variable s, and the space variable, $r_D$. To obtain the solution in real time and space the inverse Laplace tranform is used. In the present work, the inverse was found by using an algorithm for approximate numerical inversion of the Laplace space solution. This algorithm was presented by Stehfest (1970).

TRANSIENT PRESSURE SOLUTIONS-CONSTANT FLOWRATE PRODUCTION

The solutions for the dimensionless wellbore pressure from either an infinite or a closed outer boundary system have appeared in several places in the literature (see Mavor and Cinco-Ley (1979) and Da Prat (1981). Fig. 1 shows the solution for $p_{fwD}$ for an infinite system for two values of $\omega$ and several values of $\lambda$ ($C_D$= o and skin effect = o). At early times, $p_{fwD}$ depends on $t_D$ and $\omega$. For a given value of $\lambda$, as time increases a period is reached where in the pressure tries to stabilize due to flow from the matrix. After this transition period, the solution becomes the same as that for a homogeneous system. Fig. 2 shows the solution for the dimensionless wellbore pressure for a well located in a closed outer boundary system. At early times the solution depends on $t_D$ and $\omega$ (assuming skin effect = o and $C_D$= o) for any value of $\lambda$. Then the solution goes to a transition period to finally meet the solution for a homogeneous system. Fig. 3 shows the wellbore storage effect on the Horner plot for a two porosity system where $\omega$= 0.01 and $\lambda = 5.10^{-7}$. It is seen that low values of $C_D$ have a large impact on the initial straight line making impossible the evaluation of $\omega$. For large values of $C_D$, not only the initial straight line, but also the transition zone is obscured by wellbore storage.

The analysis of interference tests in two porosity - systems has been the subject of study for many years. These tests can be used to provide information such as mobility thickness product, $\frac{kh}{\mu}$, and the porosity-compressibility - thickness product, $\phi c_t h$. Kazemi (1969), based on the two-porosity model of Warren and Root, presented results for the fracture-pressure distribution in the reservoir. In Kazemi's work, the wellbore response to an interference test is dependent on the pressure variations in the fractures, rather than in the matrix. Hence, - this study is limited to the case where the wells used in the tests are completed in the fractures. Streltsova-Adams (1976) considered both fracture and matrix pressure distribution and pointed out the importance of differencing matrix flow and fracture flow in the analysis of test made on fractured formations. Recently Deruyck et al (1982) presented a systematic approach for analyzing interference tests in reservoir with double porosity behavior and field tests conducted in a geothermal reservoir are discussed to illustrate the method. Fig. 4 shows the solution for the dimensionless fracture pressure us $t_D/r_D^2$ for several values of $\omega$ and $\theta$. The parameter $\theta$ is equal to $\lambda\, r_D^2$. It was found to be a correlating group. Fig. 5 shows the solution for the dimensionless matrix pressure for several values of the parameters $\theta(= \lambda r_D^2)$ and $\omega$.

TRANSIENT RATE SOLUTIONS-CONSTANT PRESSURE PRODUCTION

Although decline curve analysis is widely used, methods specific to naturally fractured reservoirs do not appear to be available. Fig. 6 shows the dimensionless flowrate functions for a well produced at a constant pressure from a two-porosity system where $\omega$= 0.01 and $\lambda$= $10^{-5}$.

Fetkovich (1980) observed that for homogeneous systems at the onset of depletion (a type of pseudosteady state) all solutions for various values of $r_{eD}$ develop exponential rate decline, and converge to a single curve. This statement is not true for two-porosity systems as a shown in Fig. 6. It can be seen that the solutions do not converge to a single line. Log-log type - curve matching to analize rate-time data can be applied to naturally fractured systems. Howe - ver, the relationship between $q_D$ and $t_D$ is controlled by $\omega$ and $\lambda$, as well as by other parameters. Thus more than one type-curve may be necessary. Figs. 7 to 20 show the solutions for different values of $\omega$ and $\lambda$.

In the case of homogeneous systems, interference tests have been used with success in many cases, Earlougher (1977). For wells producing at constant inner pressure, Ehlig-Economides (1979) observed that, unlike the constant rate solution, the pressure distribution for cons - tant inner pressure does not correlate with the line-source solution. A different solution results for each value of $r_D$. Considering the homogeneous reservoir solution, a particular case of the naturally fractured reservoir solu - tion, it can be expected that the same depen - dence on $r_D$ applies for two-porosity systems. Fig. 21 shows $p_{fD}$ vs $t_D/r_D{}^2$ for the case of $r_D = 1000$, and several values of $\omega$ and $\lambda$. Fig. 22 shows the solution for the dimensionless matrix pressure $p_{mD}$ vs $t_D/r_D{}^2$, for a well produced at a constant inner pressure.

## INTERFERENCE EXAMPLE

The use of type-curve matching will be illus - trated with a simulated injection test. During an interference test, water was injected into a well for 400 hours. The pressure response in a well 250 ft away was observed during the injection process. Reservoir properties and the observed pressure data are given in table 1. - The pressure change was graphed as a function of time on tracing paper and then placed over Fig. 4 (see Fig. 23). From the match, $\omega$ and $\theta$ can be obtained as parameters. From Fig. 23, - $\theta= 1$ and $\omega= 0.01$. The fracture permeability is given by:

$$k_f = \frac{141.2\ qB\ \mu}{h} \left(\frac{p_{fD}}{\Delta p}\right)_M = \frac{141.2(-100)(1)(1)}{480} \frac{0.4}{(-10)}$$

$$= 1.2 \text{ md}$$

From the time - $t_D/r_D^2$ match, the total storativity is obtained:

$$\{(\phi c)_f + (\phi c)_m\} = \frac{0.000264\ k_f}{r^2} \left(\frac{t}{t_D/r^2{}_D}\right)$$

$$= \frac{0.000264\ (1.2)}{(250)^2} \left(\frac{100}{1.5}\right)$$

$$= 3.38\ 10^{-7}\ \text{psi}^{-1}$$

From $\omega= 1$ and the total storativity, the fracture storativity is:

$$(\phi c)_f = \{(\phi c)_f + (\phi c)_m\}$$

$$= (0.01)\{3.38\text{x}10^{-7}\} = 3.38\ 10^{-9}\ \text{psi}^{-1}$$

as

$$\theta = \lambda r^2{}_D = 1;$$

$$\lambda = \frac{\theta}{r^2} \quad \frac{1}{(1000)^2} = 10^{-6}$$

also from the relation $\theta = \alpha \frac{k_m}{k_f} r^2 =$

$$\alpha\, k_m = \frac{\theta\, k_f}{r^2} =$$

$$= \frac{(1)\ (1.2)}{(250)^2} = 1.9\text{x}10^{-5}\ \text{md/ft}^2$$

Thus, if km can be obtained from core analysis, then the shape factor $\alpha$ is equal to:

$$\alpha = \frac{1.9\text{x}10^{-5}}{k_m}\ \text{md - ft}^{-2}$$

The shape factor $\alpha$ can provide information - about the effective block size in the system.

## DISCUSSION AND CONCLUSIONS

This work presents basic solutions that can be used to analyse pressure or flowrate transient data from a naturally fractured reservoir. The model used assumes pseudo steady state flow - from matrix to fractures. According to the literature the solutions assuming transient flow have been presented by many authors: Raghavan and Ohaeri (1981) presented declining rate type curves for a constant producing pressure De - ruyck et al (1982) presented solutions for the pressure distribution throughout a reservoir with double porosity behavior considering boths pseudo-steady state flow and transient interporosity flow. The authors concluded based on field examples that the pseudo-steady state and the transient interporosity flow models are shown to yield consistent interpretations.

From the solution presented in this work the following conclusions can be derived:

1) The initial decline in flowrate is often not representative of the final state of depletion.

2) The fracture permeability $k_f$; total storativity,

   $\{(\phi c)_m + (\phi c)_f\}$; and the shape factor "$\alpha$" can be obtained from type curve mat - ching.

3) Both dimensionless matrix pressure and fracture pressure are necessary for proper analysis of interference tests.

4) For constant-rate production interference

tests can be analized at long times using the line-source solution.

5) In analyzing interference tests for constant-inner-pressure production, a different solution for the pressure distribution results for each value of radial distance, $r_D$. The pressure function does not correlate with the line-source solution.

NOMENCLATURE

$A$= Drainage area, $ft^2$
$B$= formation volume factor, RB/STB
$c$= compressibility, $psi^{-1}$
$h$= formation thickness, ft
$k$= permeability, md
$p_{ws}$= wellbore pressure during the build-up period, psi
$\bar{p}$= average reservoir pressure, psi
$p_D$= dimensionless wellbore pressure

$$\frac{k_f\, h}{141.2\, qB\mu}\,(p_i - p_w)$$

$p_{fD}$= dimensionless fracture pressure
$p_{mD}$= dimensionless matrix pressure
$q$= volumetric rate, B/D
$q_D$= dimensionless flowrate

$$\frac{141.2\, qB\mu}{k_{fh}(p_i - p_w)}$$

$r$= radius, ft
$r_w$= wellbore radius, ft
$r_D$= dimensionless radius, $r/r_w$
$r_e$= reservoir outer boundary radius, ft
$r_{eD}$= dimensionless outer boundary radius, $r_e/r_w$
$S$= skin effect
$t$= time, hours
$t_D$= dimensionless time

$$\frac{2.637\ 10^{-4}\, k_f\, t}{\{(\phi c)_m + (\phi c)_f\}\,\mu\, r_w^2}$$

$t_{AD} = \frac{t_D\, r_w^2}{A}$, dimensionless

$\Delta t$= shut-in time, hr

Subscripts

f= fracture
m= matrix
D= dimensionless

Greek

$\mu$= viscosity, cp
$\phi$= porosity, fraction
$\alpha$= interporosity, flow shape factor, $ft^{-2}$

$\lambda = \frac{\alpha\, k_m\, r_w^2}{k_f}$, dimensionless

$\omega = \frac{(\phi c)_f}{\{(\phi c)_f + (\phi c)_m\}}$, dimensionless

$\theta = \lambda r_D^2$, correlating group, dimensionless.

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**TABLE I**

q = —100 B/D  r = 250 ft
h = 480 ft  $B_w$ = 1,0 RB/STB
$p_i$ = 0  $\mu_w$ = 1 cp
$t_{iny}$ = 400 hr

| t hours | $P_w$ (psig) | $\Delta p = p_i - p_w$ (psi) |
|---|---|---|
| 0 | 0 | — |
| 4,7 | 11 | —11 |
| 6,7 | 11 | —11 |
| 11,5 | 12 | —12 |
| 16,5 | 12,5 | —12,5 |
| 26,5 | 13,5 | —13,5 |
| 46 | 15,5 | —15,5. |
| 65 | 17 | —17 |
| 98 | 19,5 | —19,5 |
| 135 | 22 | —22 |
| 200 | 26 | —26 |
| 265 | 29 | —29 |
| 400 | 33,5 | —33,5 |

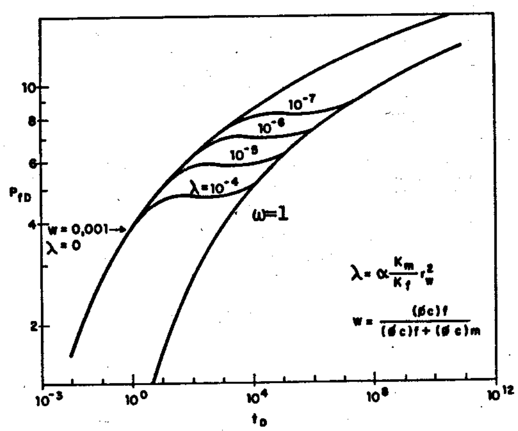


Fig. 1. $p_{fD}$ vs $t_D$ for a well produced at constant flowrate from an infinitely large system ($c_D$ =0, S = 0)

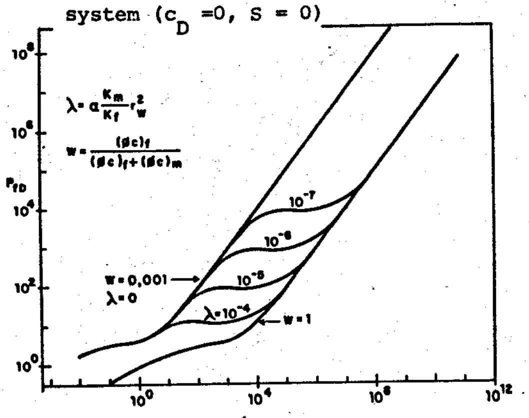


Fig. 2. $p_{fD}$ vs $t_D$ for a well produced at constant flowrate from a bounded, two-porosity system ($r_{eD}$ = 50)

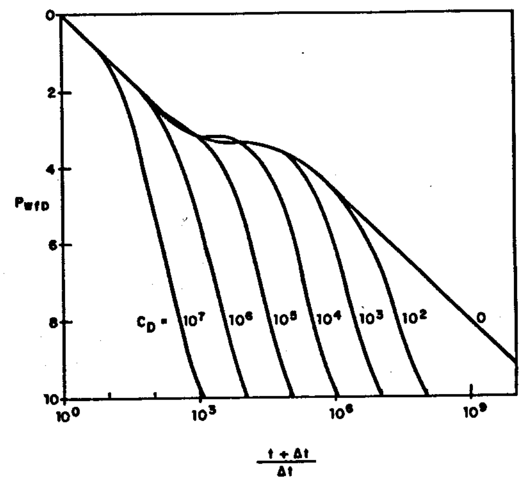


Fig. 3. Wellbore storage effect in horner build-up, infinite two porosity system ($\omega$= 0.01, $\lambda$ = 5.10 $^{-7}$, S = 0)

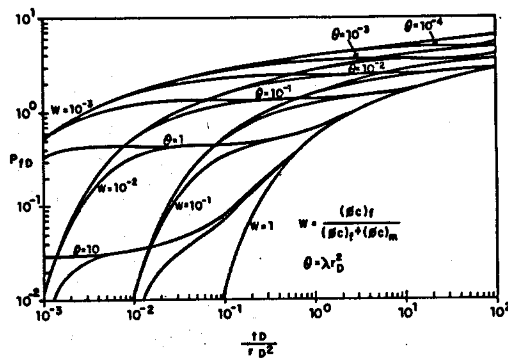


Fig. 4. $p_{fD}$ vs $t_D/r_D^2$ for a well producing at constant flowrate from an infinitely large two-porosity system.

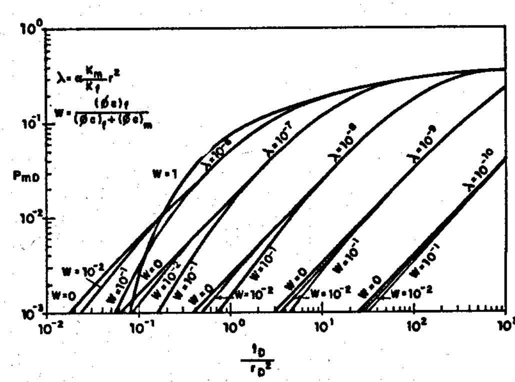


Fig. 5. $p_{mD}$ vs $t_D/r_D$2 for a well producing at a constant flowrate from a naturally fractured reservoir.

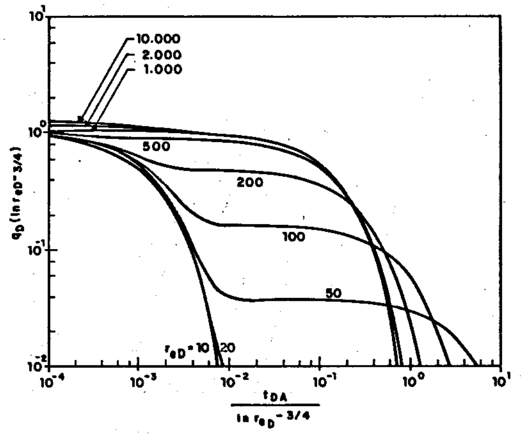


Fig. 6.
Dimensionless flowrate functions for a well produced at a constant pressure from a two-porosity system ($\omega$= 0.01, $\lambda$= $10^{-5}$).

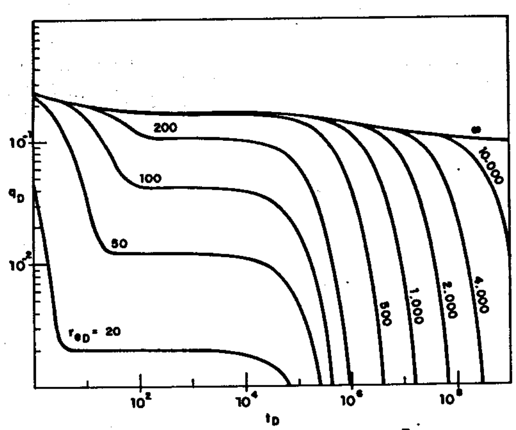


Fig. 7. $q_D$ vs $t_D$ $\omega = 0.001$, $\lambda = 10^{-5}$

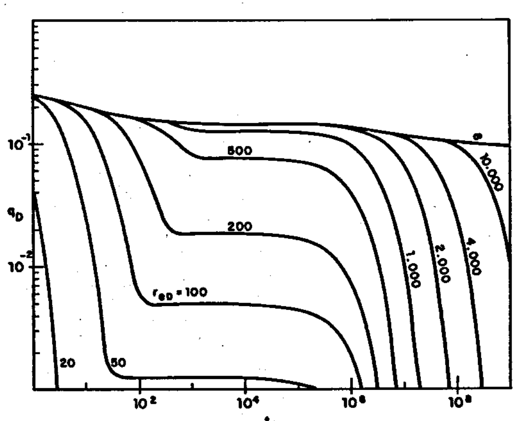


Fig. 8. $q_D$ vs $t_D$ $\omega = 0.001$, $\lambda = 10^{-6}$

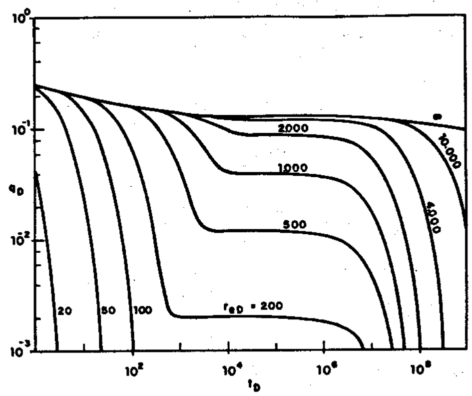


Fig. 9. $q_D$ vs $t_D$ $\omega = 0.001$, $\lambda = 10^{-7}$

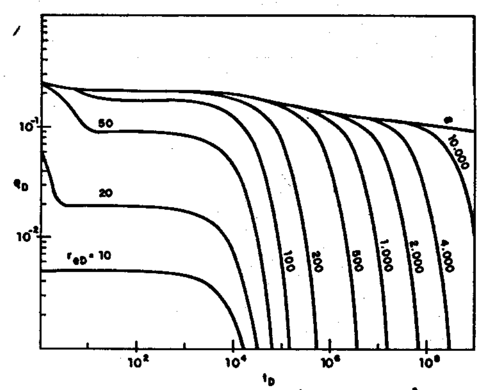


Fig. 10. $q_D$ vs $t_D$ $\omega = 0.001$, $\lambda = 10^{-4}$

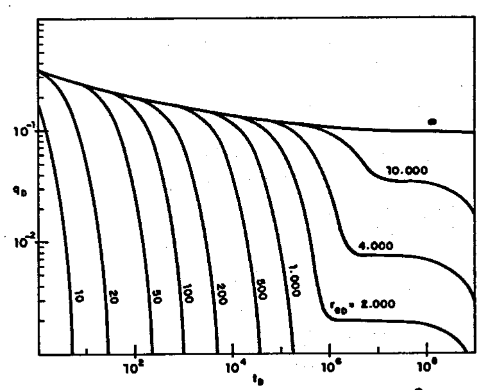


Fig. 11. $q_D$ vs $t_D$ $\omega = 0.01$, $\lambda = 10^{-9}$

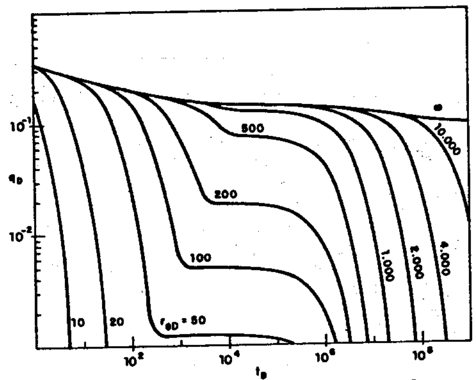


Fig. 12. $q_D$ vs $t_D$ $\omega = 0.01$, $\lambda = 10^{-6}$

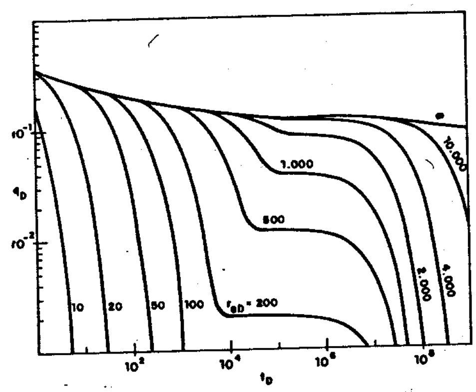


Fig. 13. $q_D$ vs $t_D$ $\omega = 0.01$, $\lambda = 10^{-7}$

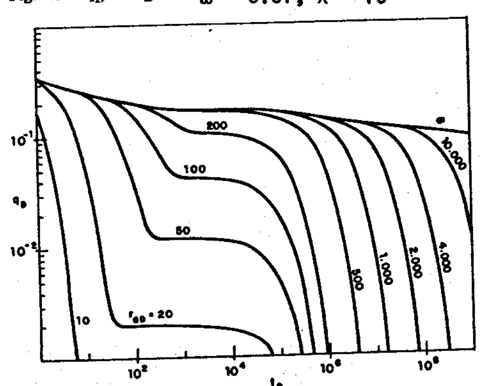


Fig. 14. $q_D$ vs $t_D$ $\omega = 0.01$, $\lambda = 10^{-5}$

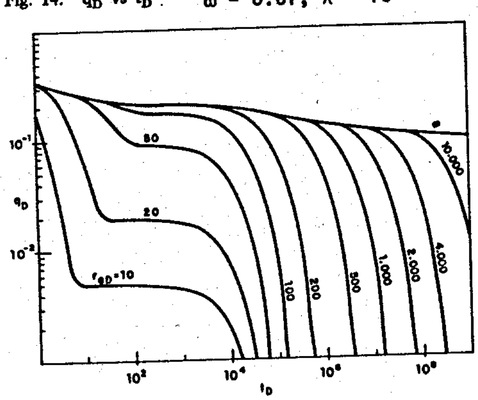


Fig. 15. $q_D$ vs $t_D$ $\omega = 0.01$, $\lambda = 10^{-4}$

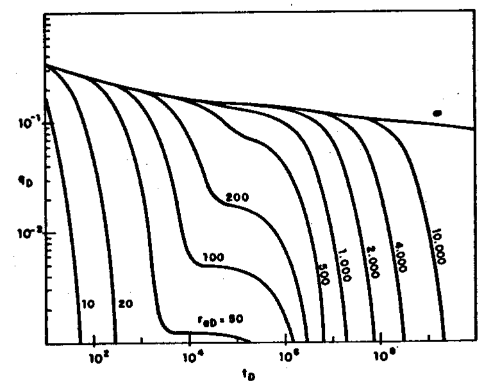


Fig. 16. $q_D$ vs $t_D$ $\omega = 0.1$, $\lambda = 10^{-6}$

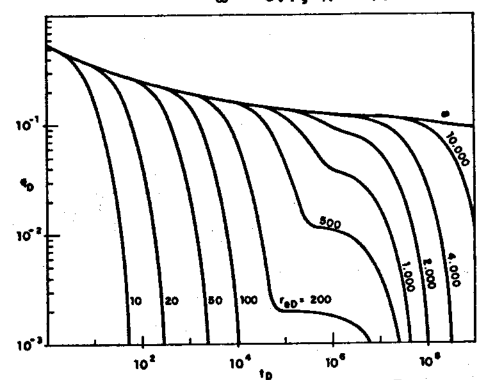


Fig. 17. $q_D$ vs $t_D$ $\omega = 0.1$, $\lambda = 10^{-7}$

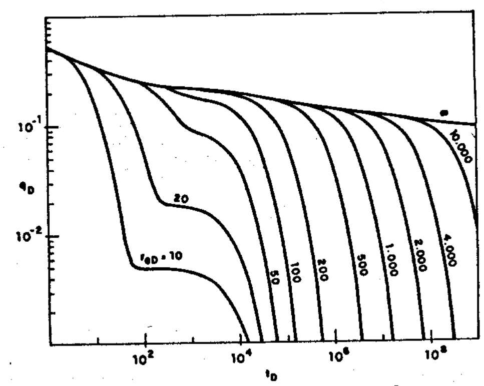


Fig. 18. $q_D$ vs $t_D$ $\omega = 0.1$, $\lambda = 10^{-4}$

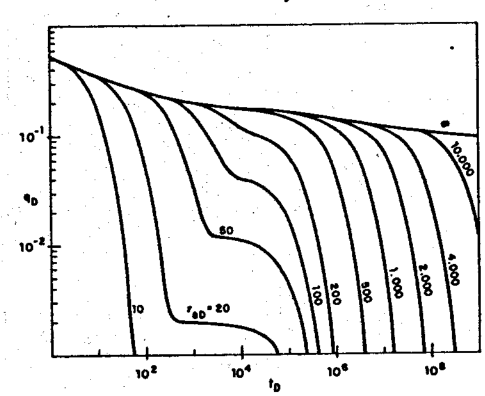


Fig. 19. $q_D$ vs $t_D$ $\omega = 0.1$, $\lambda = 10^{-5}$

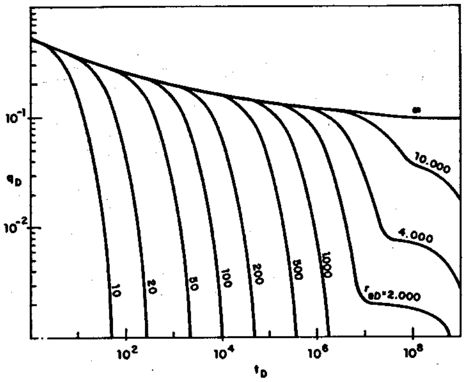


Fig. 20. $q_D$ vs $t_D$ $\omega = 0.1,\ \lambda = 10^{-9}$

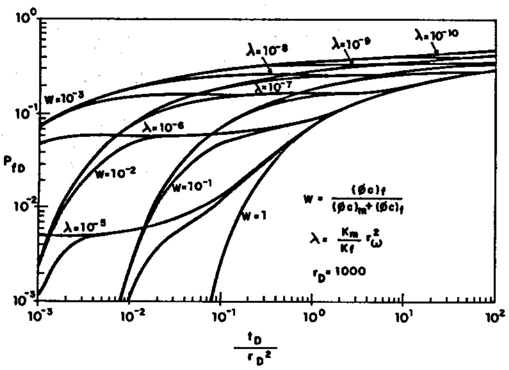


Fig. 21. $P_{fD}$ vs $t_D/r_D2$ for a well produced at constant inner pressure from an infinitely large, two porosity system.

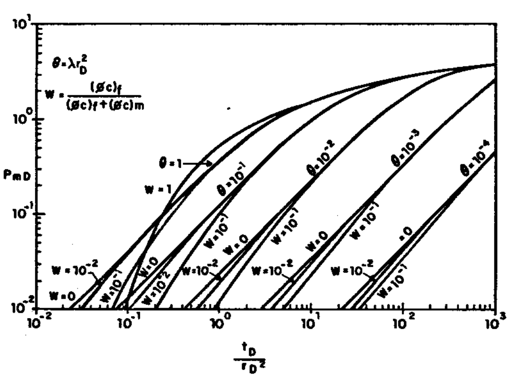


Fig. 22. $p_{mD}$ vs $t_D/r_D^2$ for a well producing at at constant flowrate from a naturally fractured reservoir.

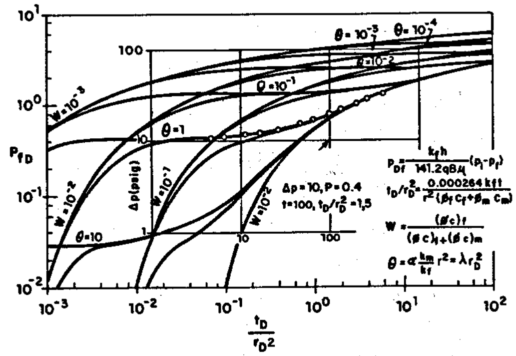


Fig. 23. Type-curve matching for interference example.